

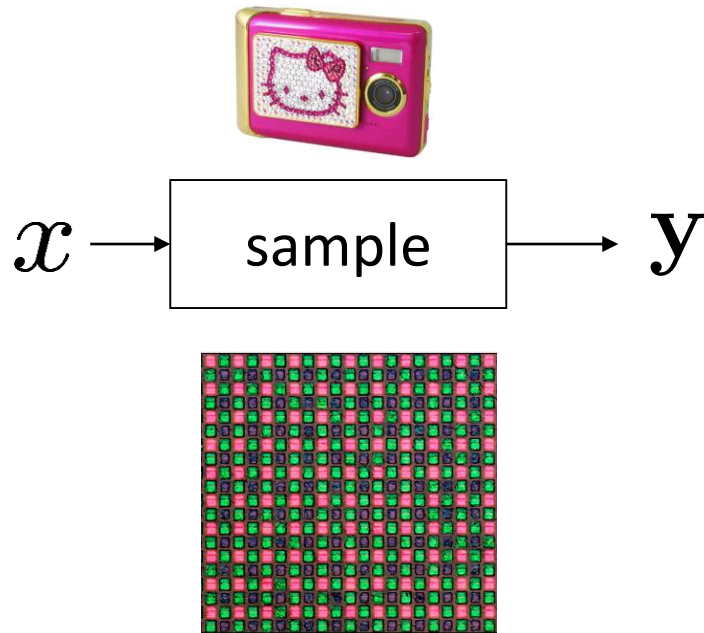
Compressive Imaging

Aswin Sankaranarayanan

(Computational Photography – Fall 2017)

Traditional Models for Sensing

- Linear (for the most part)
- Take as many measurements as unknowns

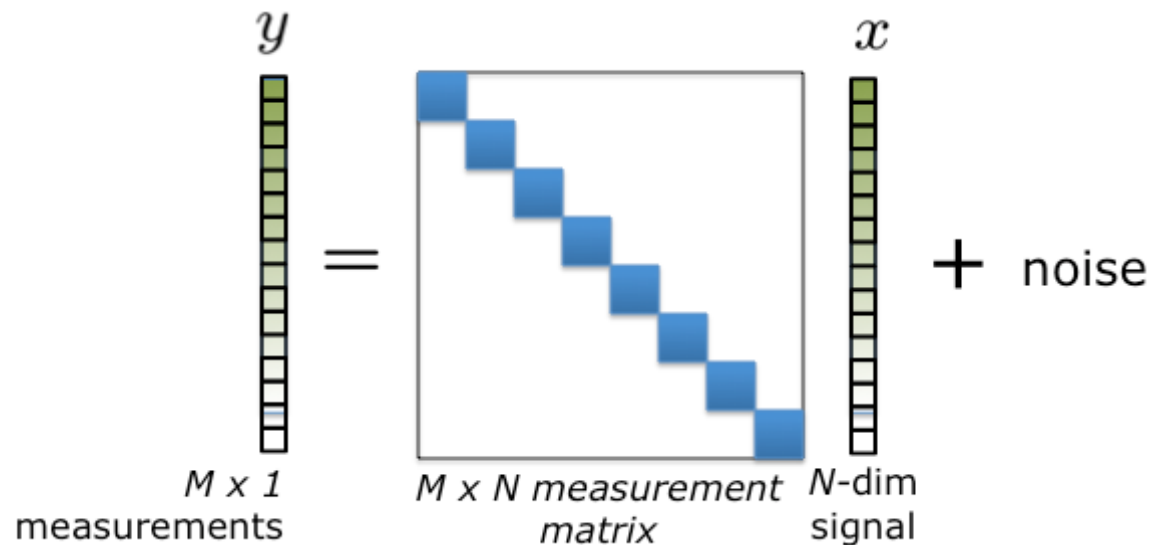
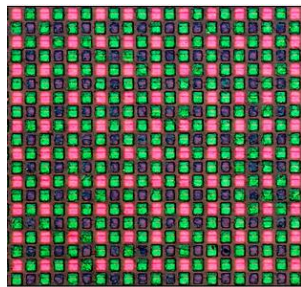


Traditional Models for Sensing

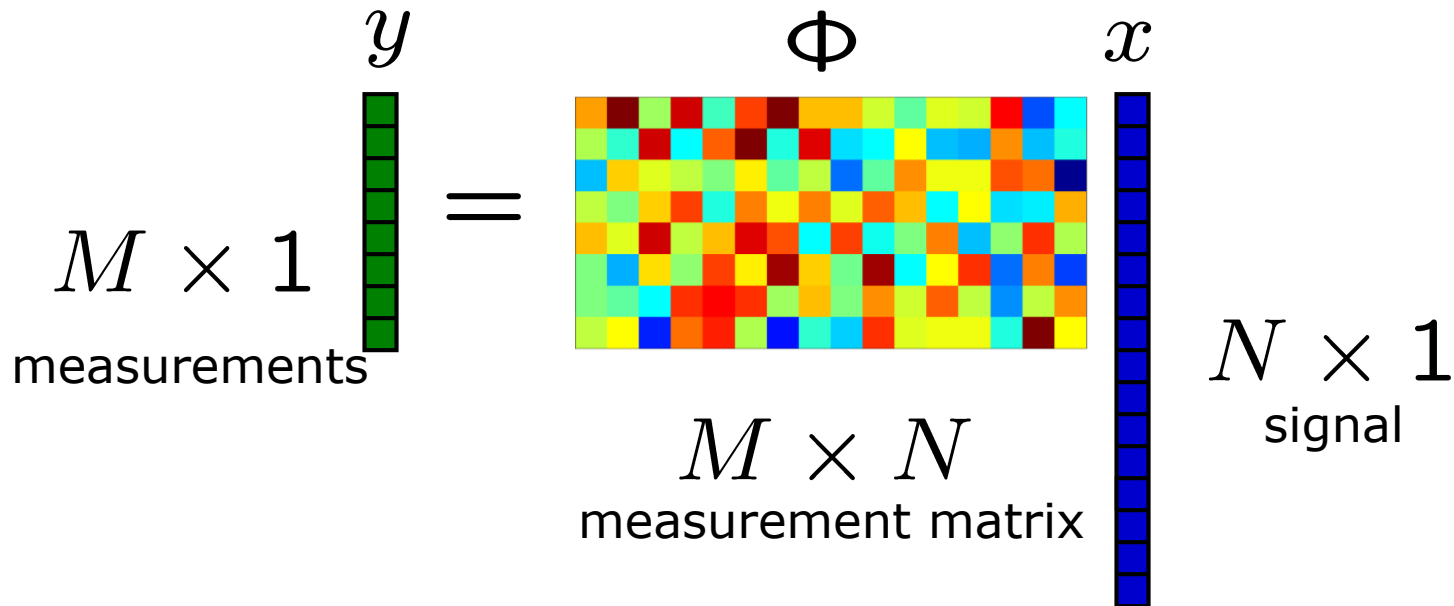
- Linear (for the most part)
- Take as many measurements as unknowns



Typically, $M \geq N$



Under-determined problems



The diagram illustrates an under-determined system of linear equations. On the left, a vertical column of 10 green squares represents the vector y , with the label $M \times 1$ and "measurements" below it. In the center is an equals sign. To the right of the equals sign is a 10x10 grid of colored squares (red, yellow, green, blue) representing the measurement matrix Φ , with the label $M \times N$ and "measurement matrix" below it. To the right of the matrix is a vertical column of 10 blue squares representing the vector x , with the label $N \times 1$ and "signal" below it. The labels y , Φ , and x are placed above their respective visual representations.

$$\begin{matrix} y \\ M \times 1 \\ \text{measurements} \end{matrix} = \begin{matrix} \Phi \\ M \times N \\ \text{measurement matrix} \end{matrix} \begin{matrix} x \\ N \times 1 \\ \text{signal} \end{matrix}$$

Fewer measurements than unknowns!

An infinite number of solutions to such problems

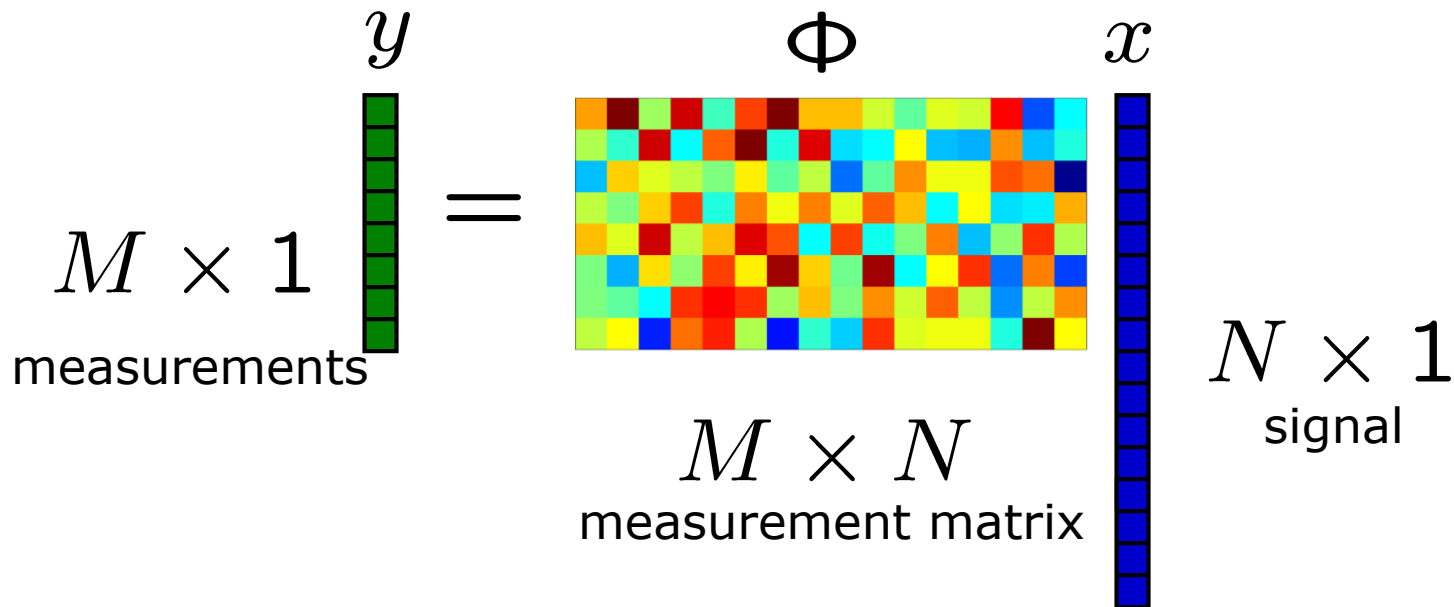


Credit: Rob Fergus and Antonio Torralba



Credit: Rob Fergus and Antonio Torralba

Under-determined problems



The diagram illustrates an under-determined system of linear equations. On the left, a vertical column of 10 green squares represents the vector y , with the label $M \times 1$ measurements below it. In the center is an equals sign followed by a 10x10 grid of colored squares (red, yellow, cyan, blue, and green) representing the measurement matrix Φ , with the label $M \times N$ measurement matrix below it. To the right of the grid is a vertical column of 10 blue squares representing the vector x , with the label $N \times 1$ signal below it. The labels y , Φ , and x are positioned above their respective visual representations.

$$\begin{matrix} y \\ M \times 1 \\ \text{measurements} \end{matrix} = \begin{matrix} \Phi \\ M \times N \\ \text{measurement matrix} \end{matrix} \begin{matrix} x \\ N \times 1 \\ \text{signal} \end{matrix}$$

Fewer measurements than unknowns!

An infinite number of solutions to such problems

Is there any “hope” of solving these problems ?

Complete the sentences

I cnt blv I m bl t rd ths sntnc.

Wntr s cmng. n wt, wntr hs cm.

Hy, I m slvng n ndr-dtrmnd lnr systm.

how: ?

Complete the matrix

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

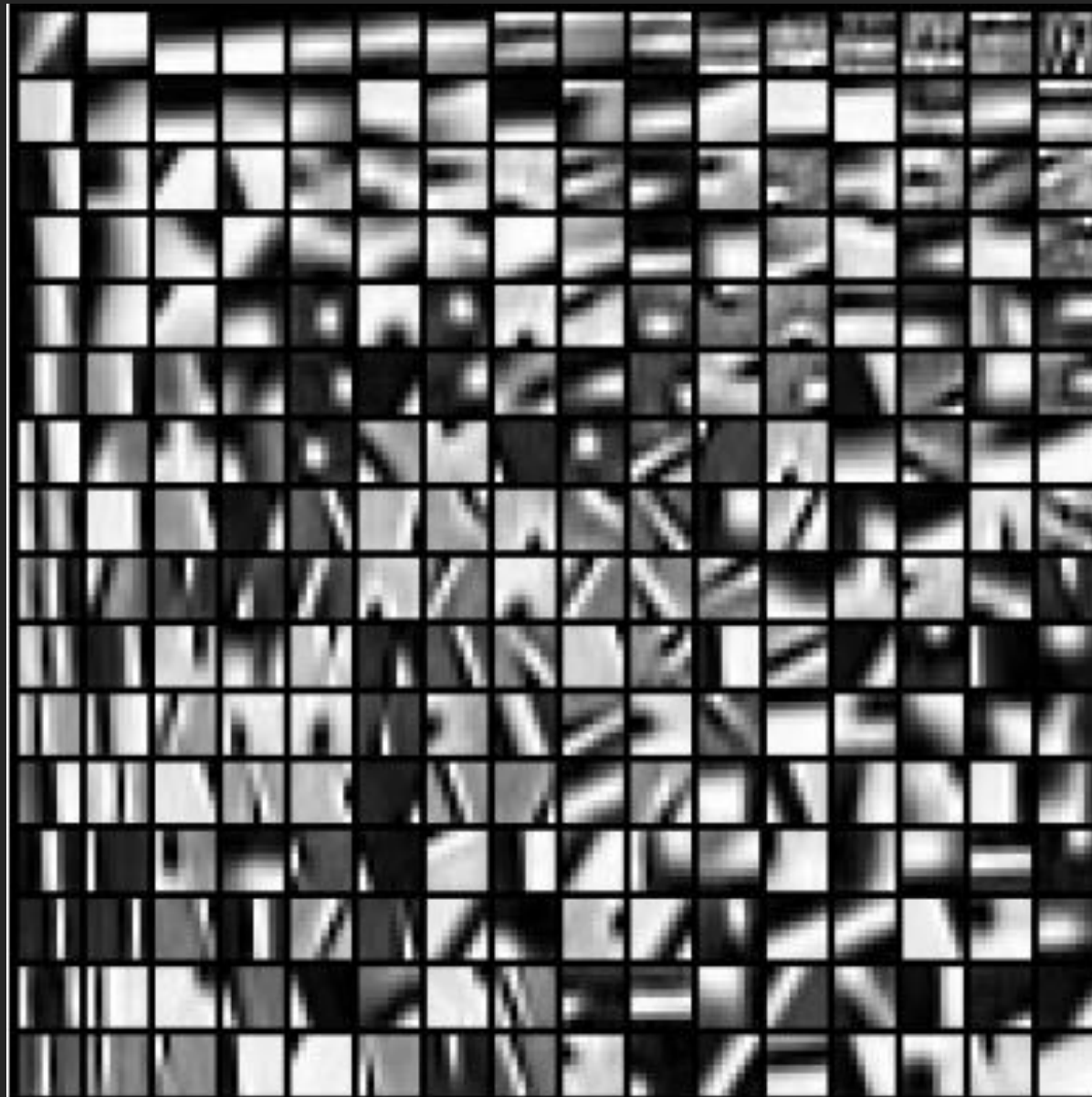
How: ?

Complete the image



Model ?

Image Dictionaries

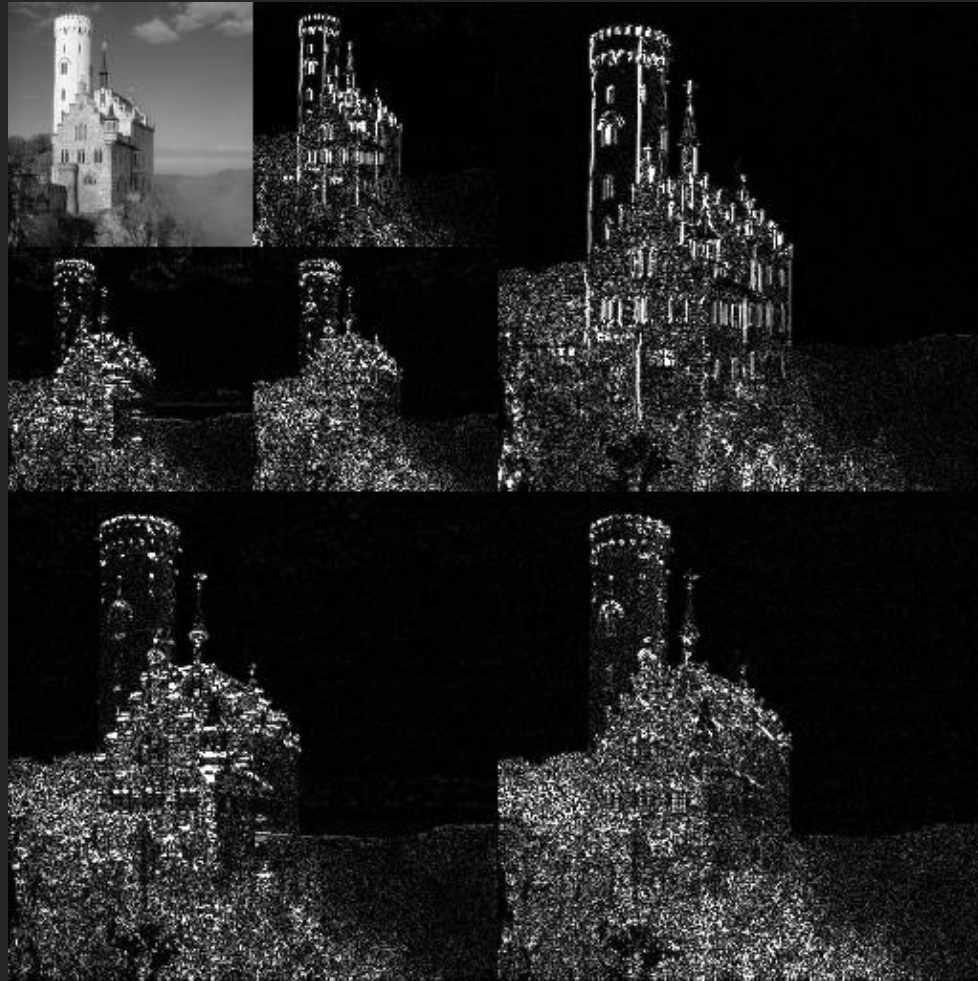


Real data has structure



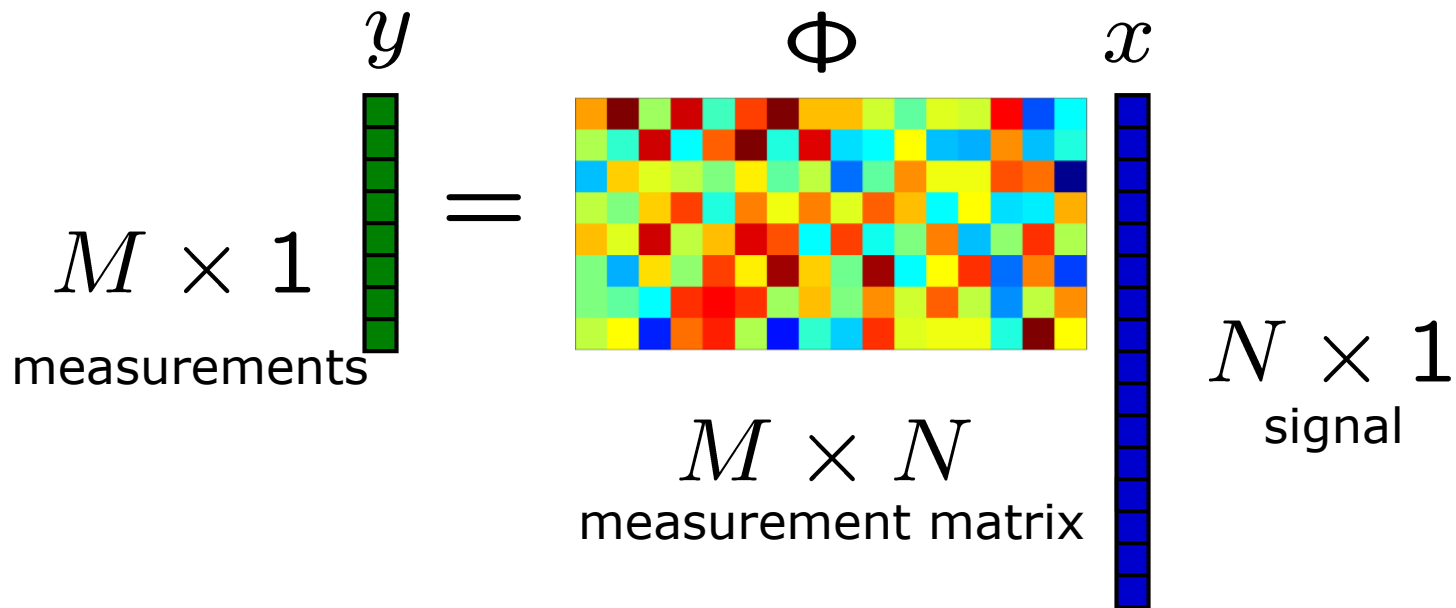
Image gradients are sparse!

Real data has structure



Real world images: Only a few **non-zero** coefficients in a transformation

Compressive Sensing



The diagram illustrates the compressive sensing equation $y = \Phi x$. On the left, a vertical green bar represents the measurements vector y , with dimensions $M \times 1$ and the label "measurements" below it. In the center is an equals sign. To the right of the equals sign is a colorful grid representing the measurement matrix Φ , with dimensions $M \times N$ and the label "measurement matrix" below it. To the right of the matrix is a vertical blue bar representing the signal vector x , with dimensions $N \times 1$ and the label "signal" below it. The symbol Φ is placed above the matrix grid.

$$\begin{matrix} M \times 1 \\ \text{measurements} \end{matrix} \begin{matrix} y \\ \text{green bar} \end{matrix} = \begin{matrix} \Phi \\ M \times N \\ \text{measurement matrix} \end{matrix} \begin{matrix} x \\ N \times 1 \\ \text{signal} \end{matrix}$$

A toolset to solve **under-determined systems** by exploiting **additional structure/models** on the signal we are trying to sense.

Compressive Sensing

$$\begin{array}{c}
 M \times 1 \\
 \text{measurements}
 \end{array}
 \begin{array}{c}
 y \\
 \begin{array}{|c|} \hline \text{colored squares} \\ \hline \end{array}
 \end{array}
 =
 \begin{array}{c}
 A \\
 \begin{array}{|c|} \hline \text{colored grid} \\ \hline \end{array} \\
 M \times N
 \end{array}
 \begin{array}{c}
 N \times 1 \\
 \text{Sparse signal}
 \end{array}
 \begin{array}{c}
 x \\
 \begin{array}{|c|} \hline \text{mostly white, some colored} \\ \hline \end{array}
 \end{array}
 + \text{noise}
 \begin{array}{c}
 K \\
 \text{nonzero entries}
 \end{array}$$

- Suppose measurement matrix A satisfied certain conditions
- $M \geq c_1 K \log(N/K)$
- All K -sparse signals x can be recovered
 - In the absence of noise, the recovery is exact!

Compressive Sensing: Big Picture

- If signal has structure, exploit it to solve under-determined problem
- **Structure**: Refers to a lower-dimensional parametrization of the signal class
 - Sparsity in a basis (like Fourier or wavelets)
 - Sparsity of gradients
 - Low-rank, low-dim smooth manifold
 - Any set with a projection operator
- Number of measurement is often **proportional** to the dim of the low-dim parameters
- Range of recovery techniques
- (Take **18-898G** next semester for a deep dive)

High-speed videography using CS

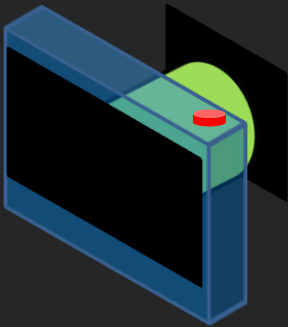
Key papers

Veeraraghavan et al., *Coded strobing, PAMI 2011*

Reddy et al., *P2C2, CVPR 2011*

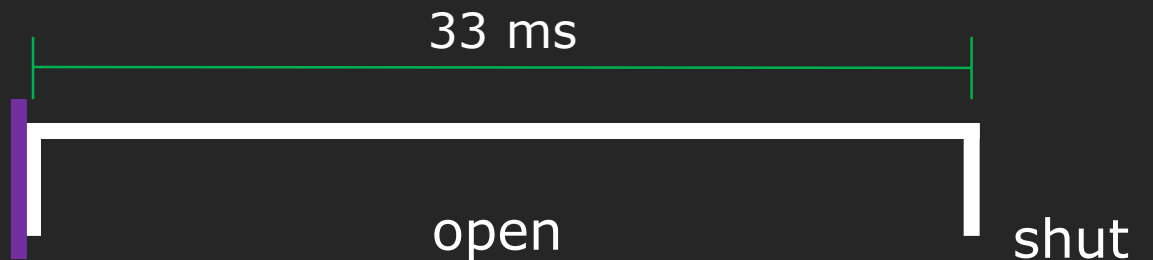
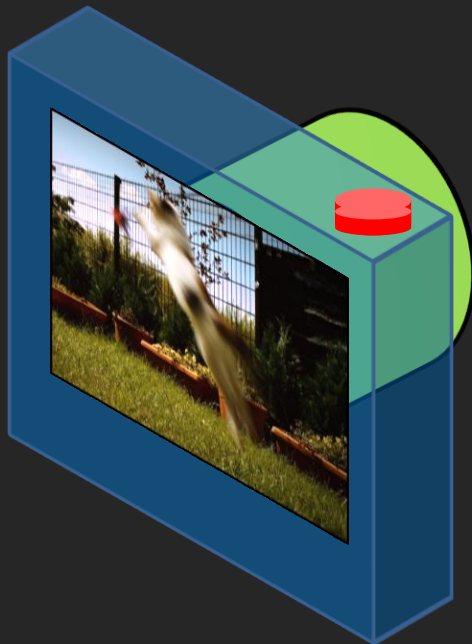
Hitomi et al., *Coded exposure, ICCV 2011*

Image Formation Model

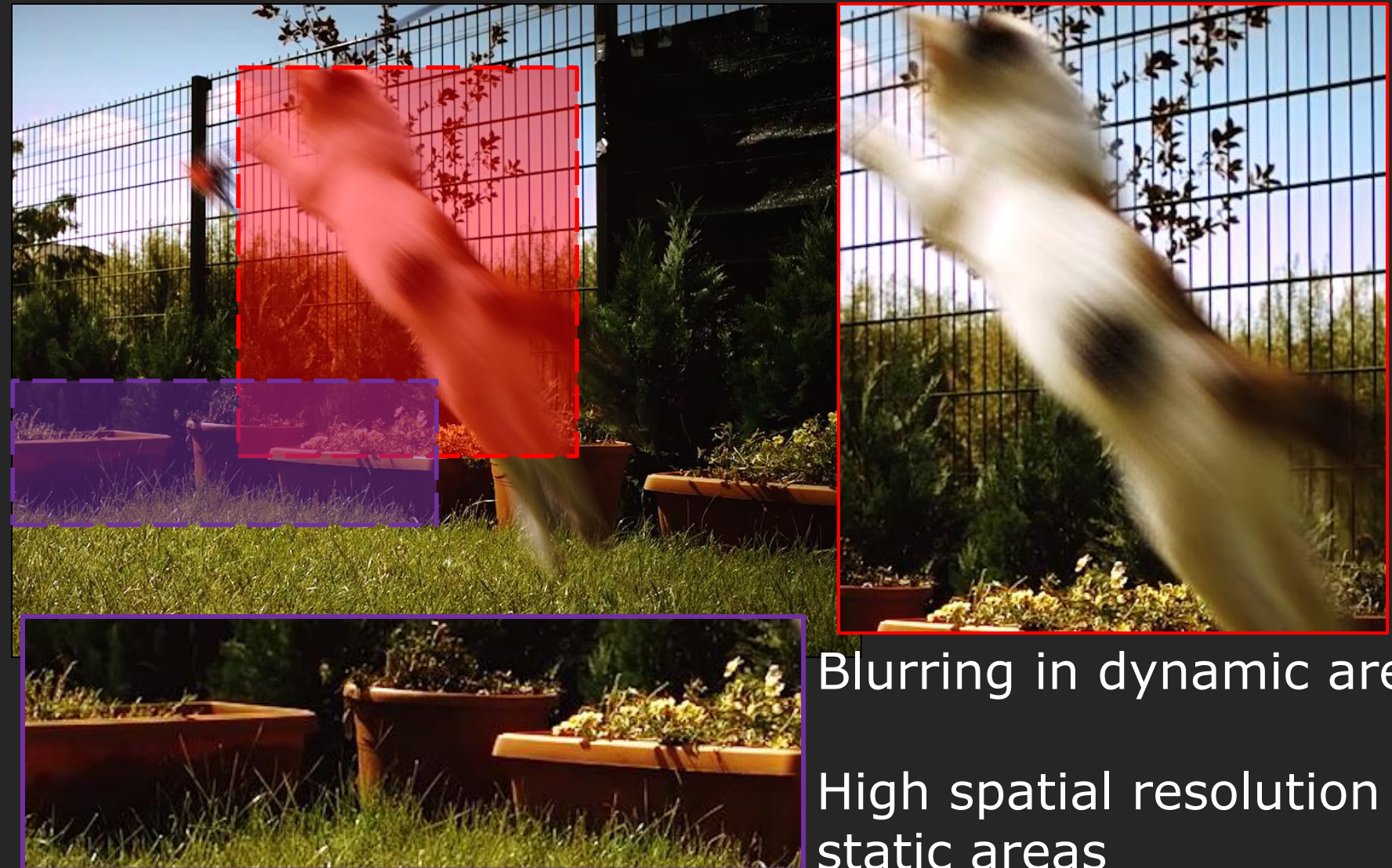


Low-speed capture
works well for static scenes

High-speed scenes



High-speed scenes



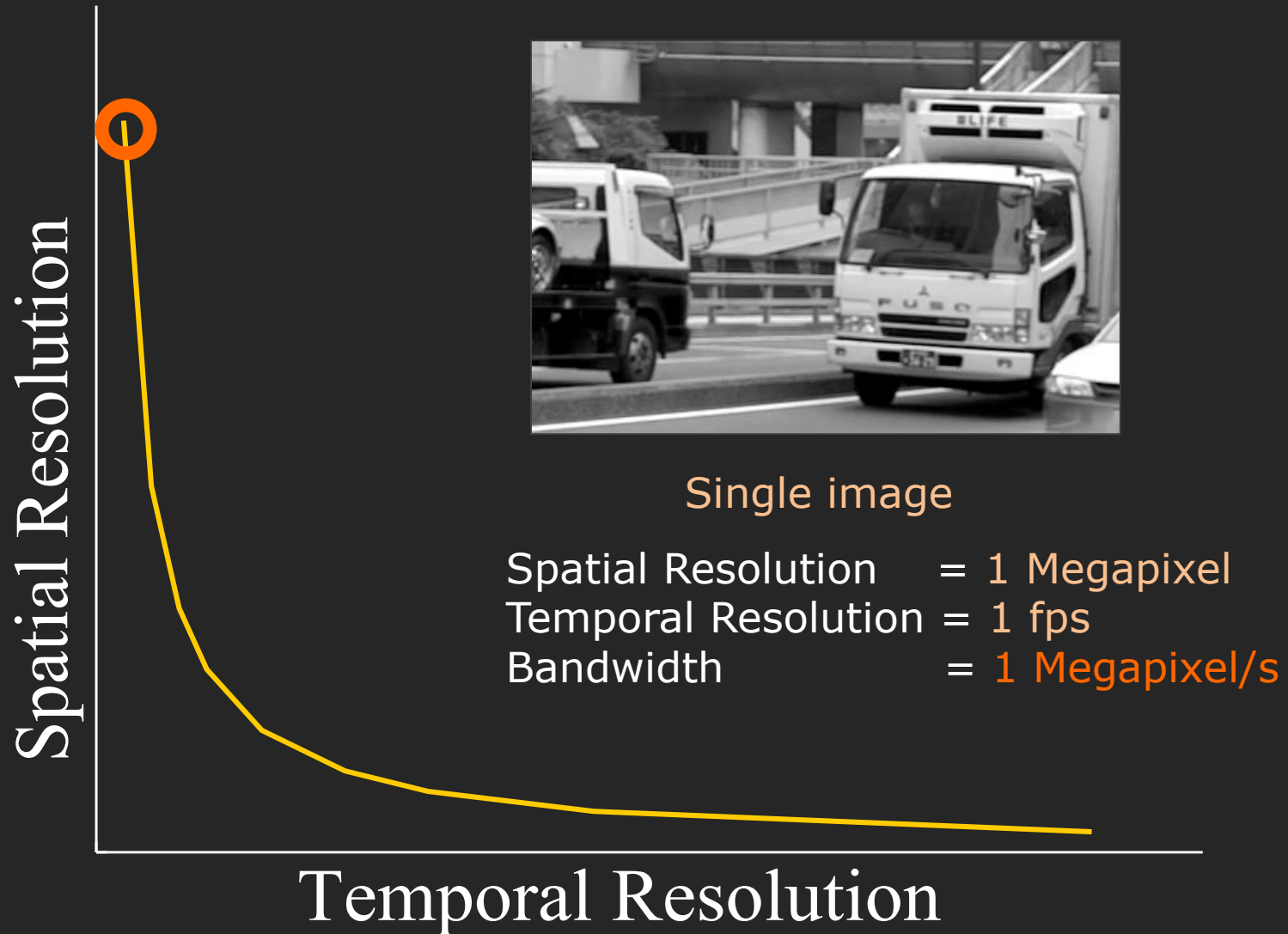
Blurring in dynamic areas

High spatial resolution in static areas

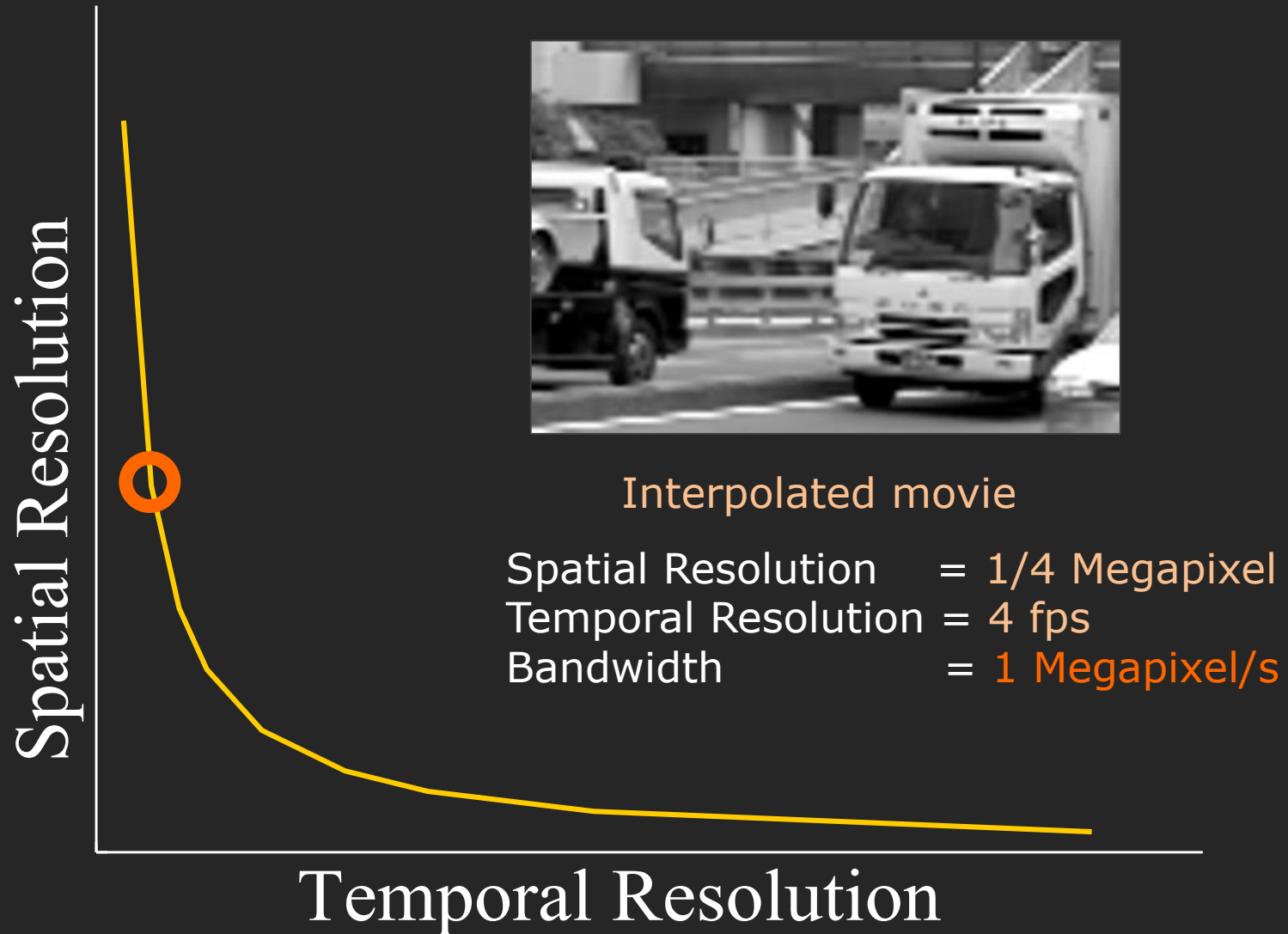
High speed scenes



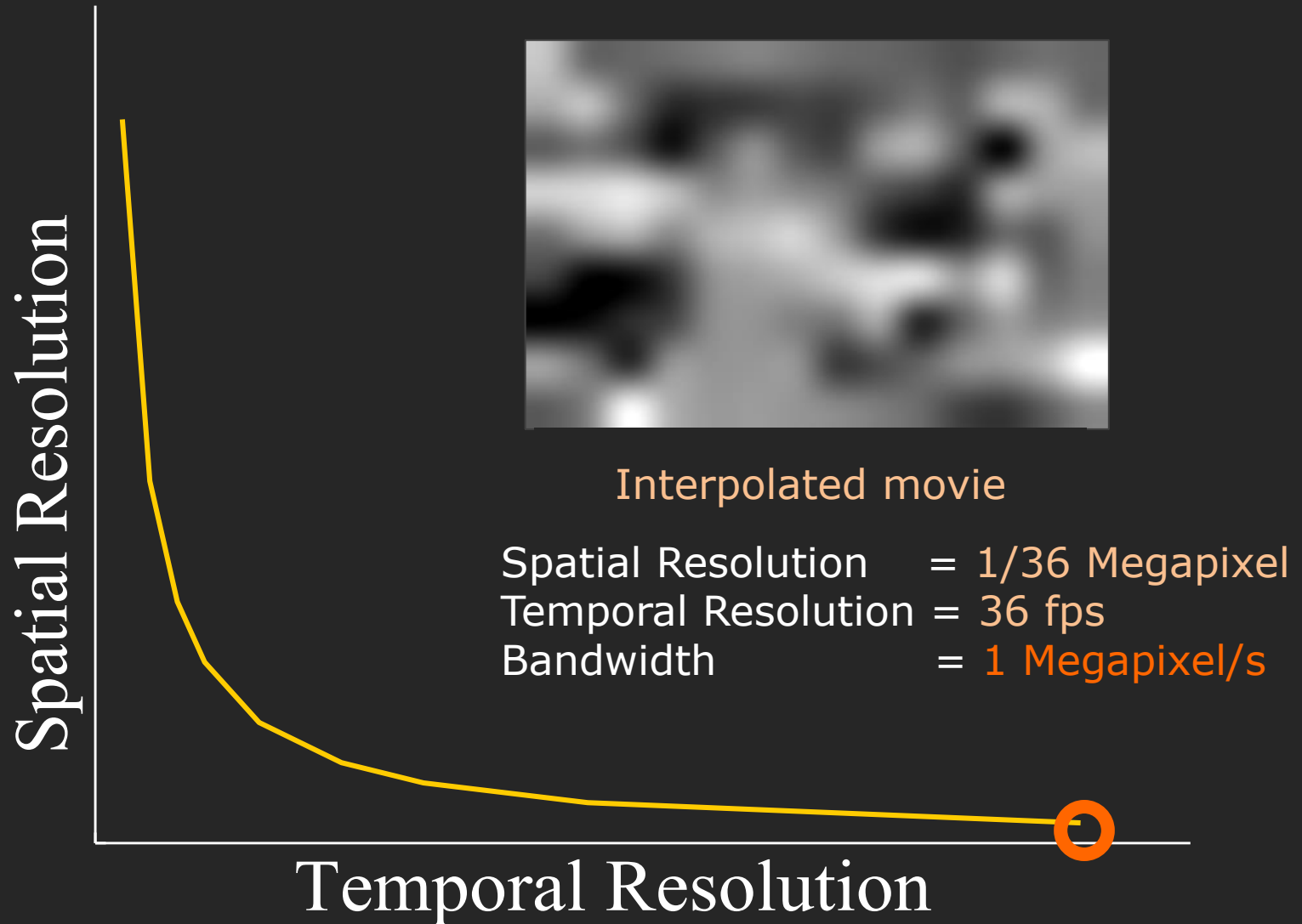
Spatio-Temporal Resolution Tradeoff



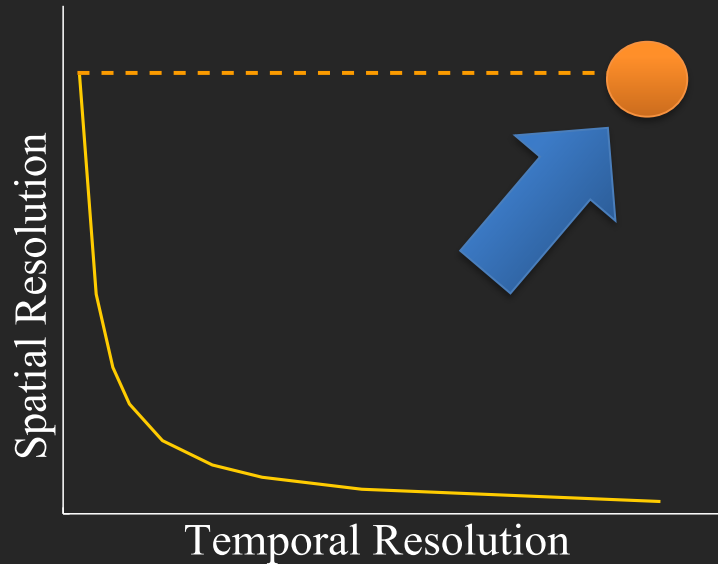
Spatio-Temporal Resolution Tradeoff



Spatio-Temporal Resolution Tradeoff



Spatio-Temporal Resolution Tradeoff



High-speed, High-res Video

Challenges

1. Bandwidth of data
2. Light throughput

From this photo ...



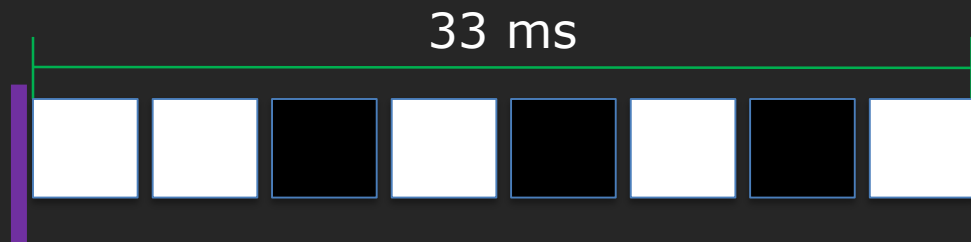
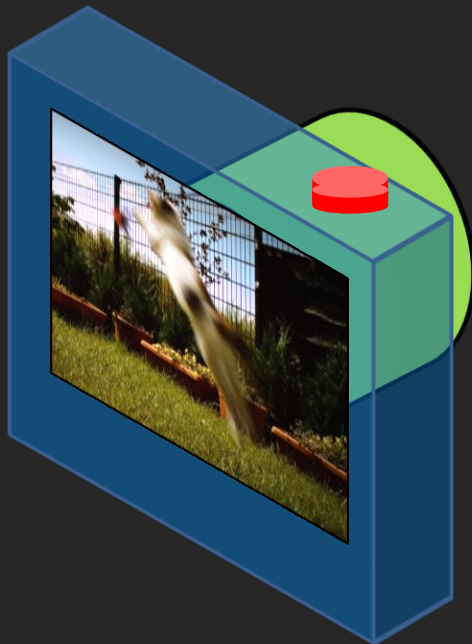
... to this one

Credit: Edgerton

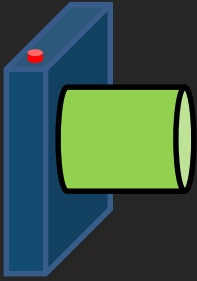


Harold Edgerton - "Moving Skip Rope", 1952. - Silver gelatin print. - Promised gift of the Harold and Esther Edgerton Family Foundation
© MIT 2010. Courtesy of MIT Museum.

Idea 1: Multiplexing in Time



Idea 1: Multiplexing in Time



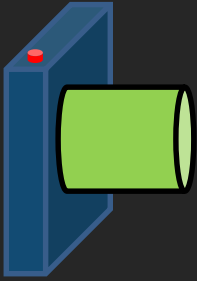
Optical
coding



Benefits

1. Bandwidth of data remains the same
2. Light throughput is not significantly reduced

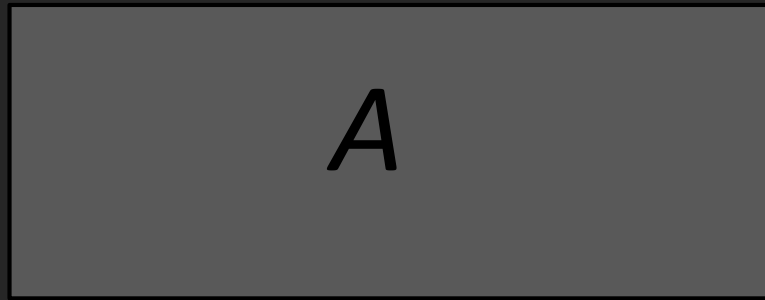
Idea 1: Multiplexing in Time



Optical
coding



=



Challenge:

More unknowns than measurements

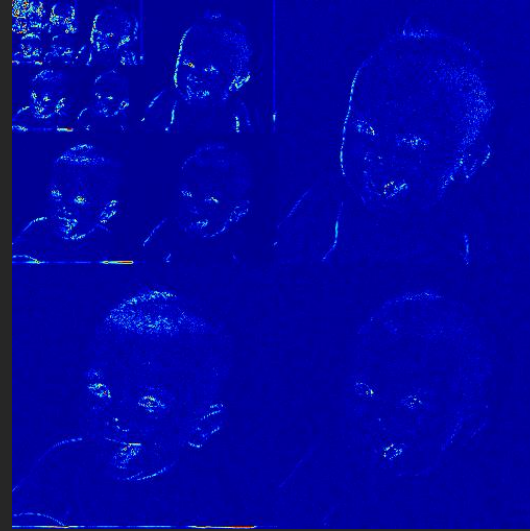
How do we recover ?

Idea 2: Signal Models

- Real-world signals are highly *redundant*

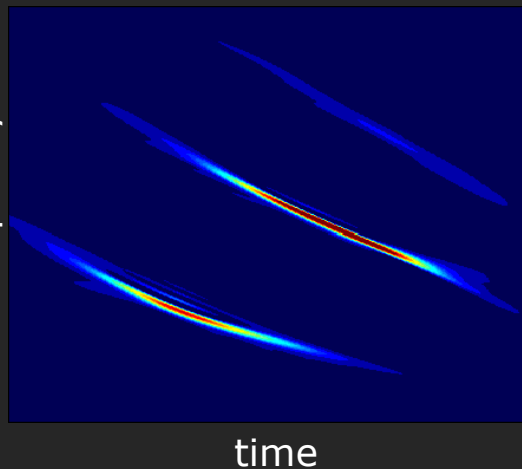
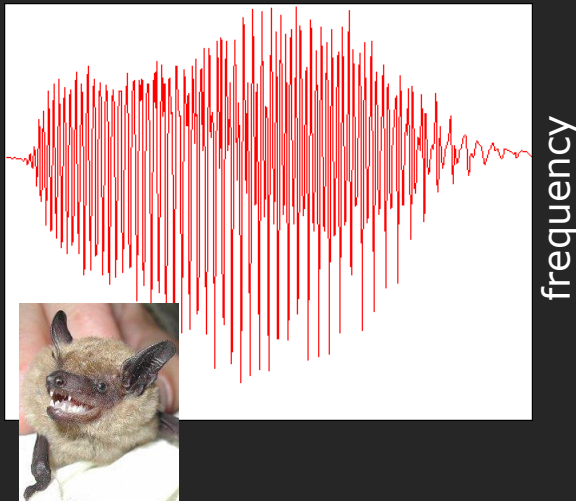
Sparsity

N
pixels



$K < N$
large
wavelet
coefficients
(blue = 0)

N
wideband
signal
samples



$K < N$
large
Gabor (TF)
coefficients

Idea 2: Signal Models

- Real-world signals are highly redundant
- Models
 - Sparse gradients
 - Sparse in transform: Wavelets, Fourier
 - Low rank: PCA, Union-of-subspaces
- Key idea: **Constrain** the solution space!
 - Number of *degrees of freedom* significantly lesser than ambient dimensionality

Periodic signals



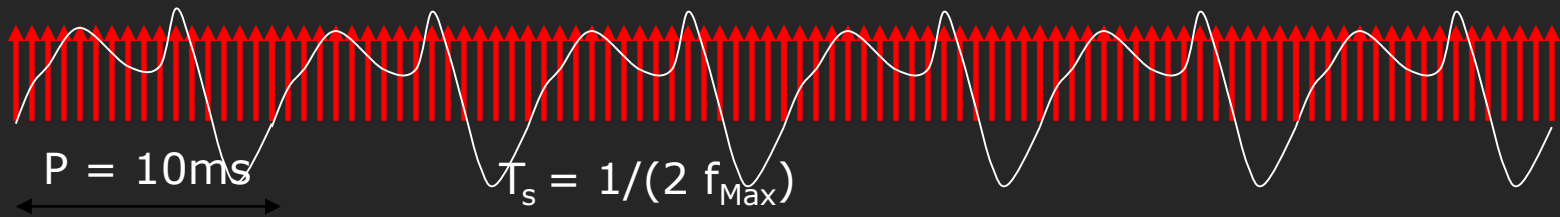
Bottling line



Toothbrush

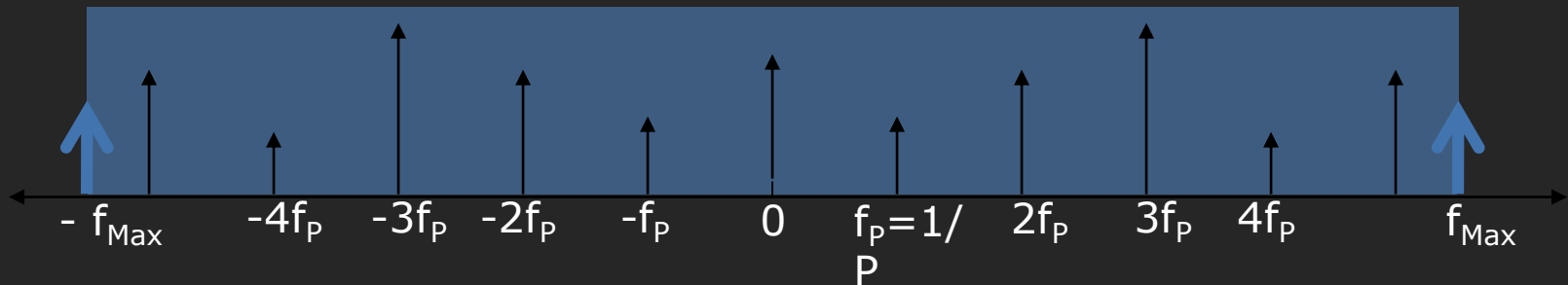
High-speed Camera

Nyquist Sampling of $x(t)$ – When each period of x has high frequency variations, Nyquist sampling rate is high.

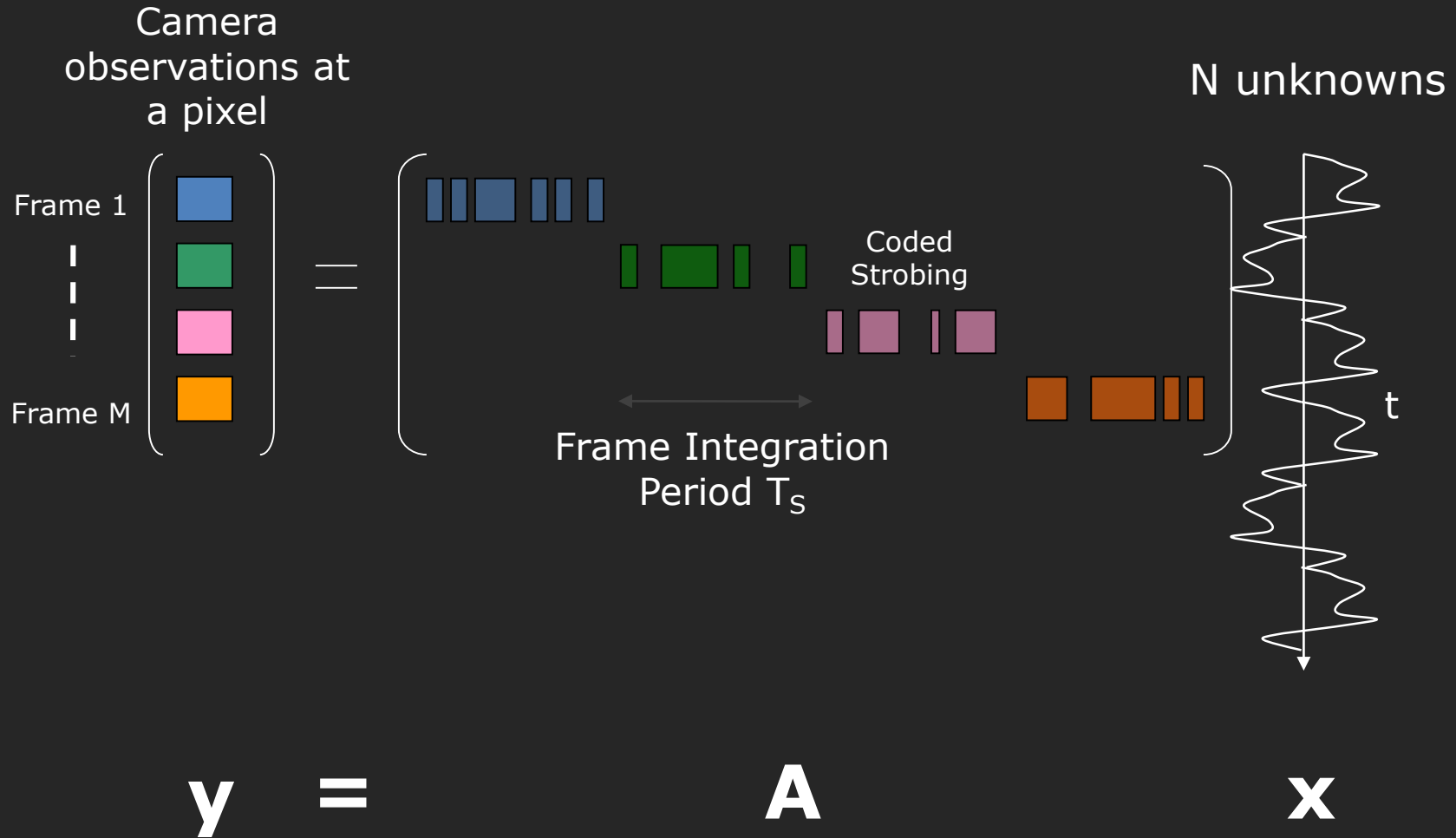


Periodic signal has regularly spaced, sparse Fourier coefficients.

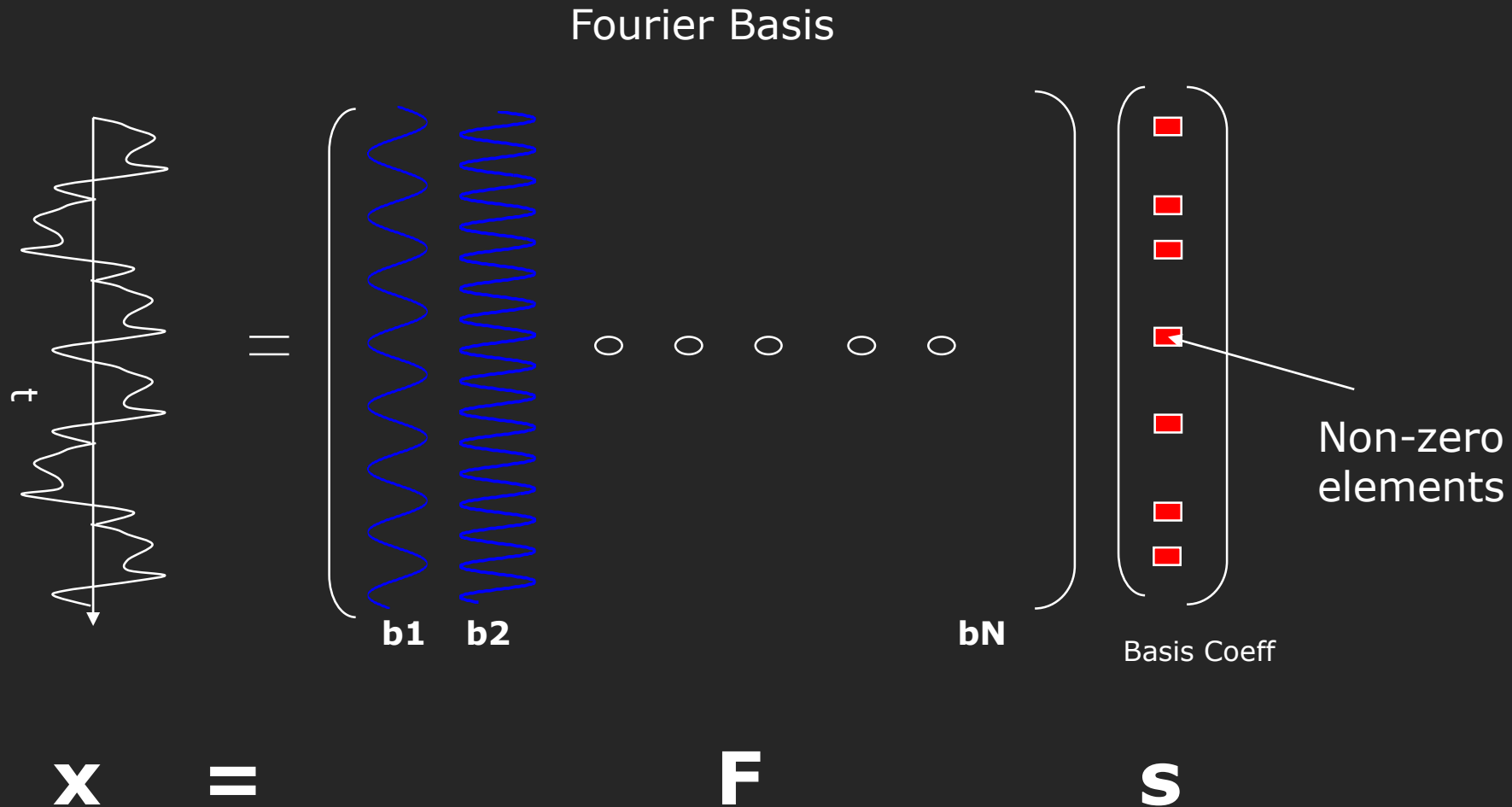
Is it necessary to use a high-speed video camera? Why waste bandwidth?



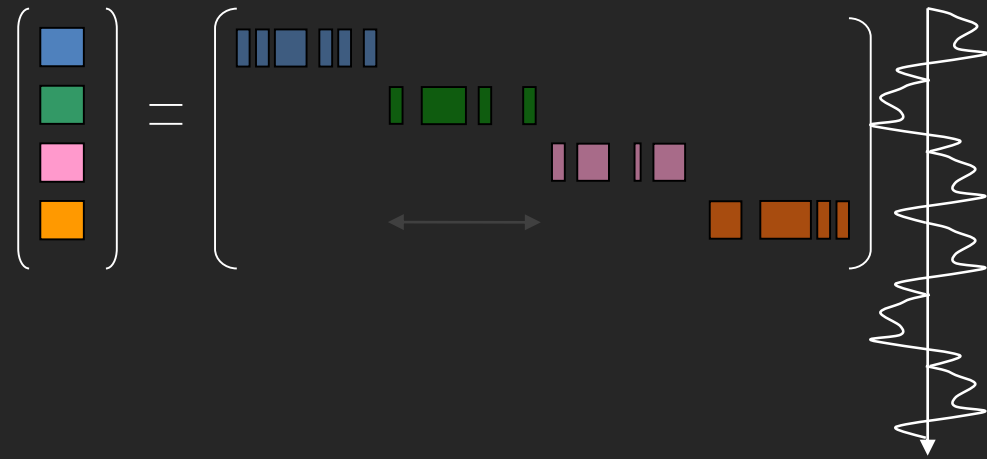
Solving for the video



Solving for the video



Solving for the video



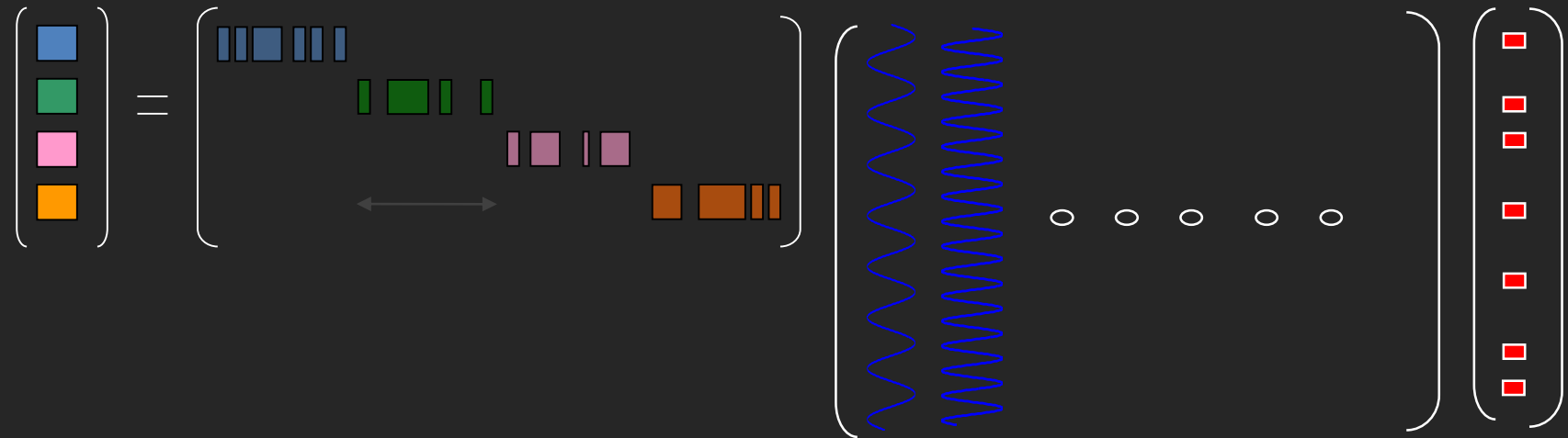
y =

A

F

s

Solving for the video



The diagram illustrates the equation $y = A F S$ for video reconstruction. On the left, a column vector y is represented by four colored squares (blue, green, pink, orange) stacked vertically. This is followed by an equals sign. To the right of the equals sign is a large matrix A , depicted as a block matrix with four colored blocks (blue, green, pink, orange) arranged diagonally. A double-headed arrow indicates a relationship between the blocks. To the right of A is a matrix F , shown as a column of five blue wavy lines, followed by five small white circles. Finally, on the right, is a column vector S , represented by eight small red squares stacked vertically.

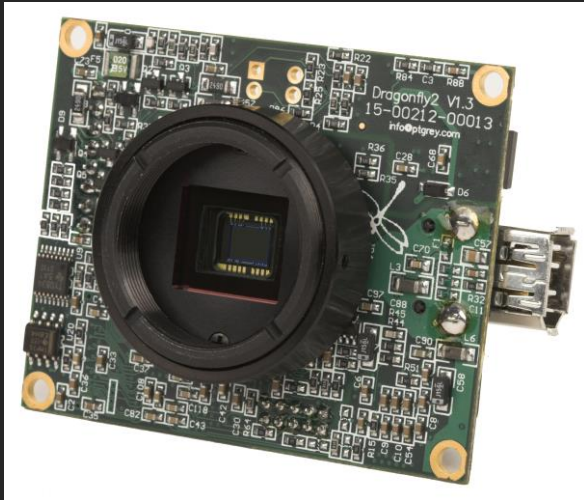
y =

A

F

S

Implementation



PGR Dragonfly2
(25 fps)



FLC Shutter
Can flutter at 250us

Toothbrush (simulation)



20fps normal camera



20fps coded strobing camera



Reconstructed frames



1000fps hi-speed camera

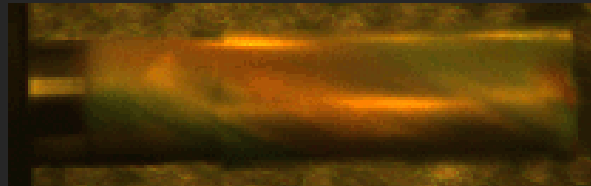
Mill Tool

Mill tool rotating at 50Hz



Normal Video: 25fps

Mill tool rotating at 50Hz



Coded Strobing Video: 25fps

Mill tool rotating at 50Hz



Reconstructed Video at 2000fps

Optical super-resolution

Key papers

Duarte et al., Single pixel camera, *SPM 2008*

Wang et al., *LiSens*, *ICCP 2015*

Chen et al., *FPA-CS*, *CVPR 2015*

Example

Video sensing in infrared

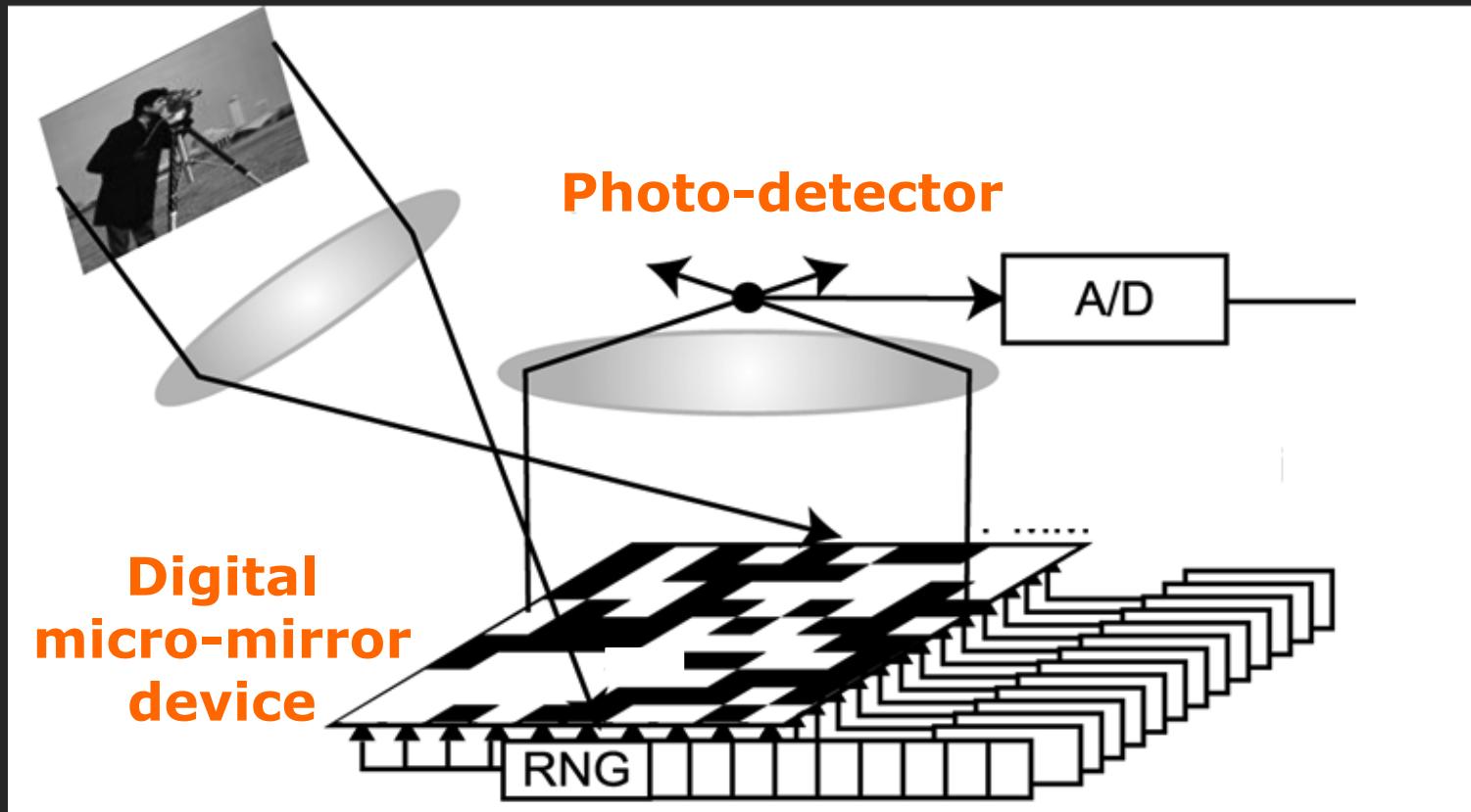
- Sensing in infra-red has applications in night-vision, astronomy, microscopy, etc.
- Materials that sense in certain infrared bands are extremely **costly**
 - A 64 x 64 sensor costs upwards of USD 2000
 - 1 Megapixel sensor costs > USD 100k

Table 1. Approximate per-pixel price of detector elements in various spectral bands.

Spectral band	Detector technology	Approx. per-pixel price (\$/pix)
mmW/THz	Multiple	10^2 – 10^4
LWIR	HgCdTe	$< 10^1$
	Bolometer	10^{-2}
MWIR	InSb/PbSe	10^{-1}
SWIR	InGaAs/PbSe	10^{-1}
NIR/VIS/NUV	Si	$< 10^{-6}$
MUV	Si (thinned)	$< 10^{-3}$
EUV	Si-PIN/CdTe	10^2 – 10^3
Soft-xray	Si (thinned)	10^{-2}
	Si-PIN/CdTe	10^2 – 10^3
Hard-xray/gamma	Multiple	10^2 – 10^4

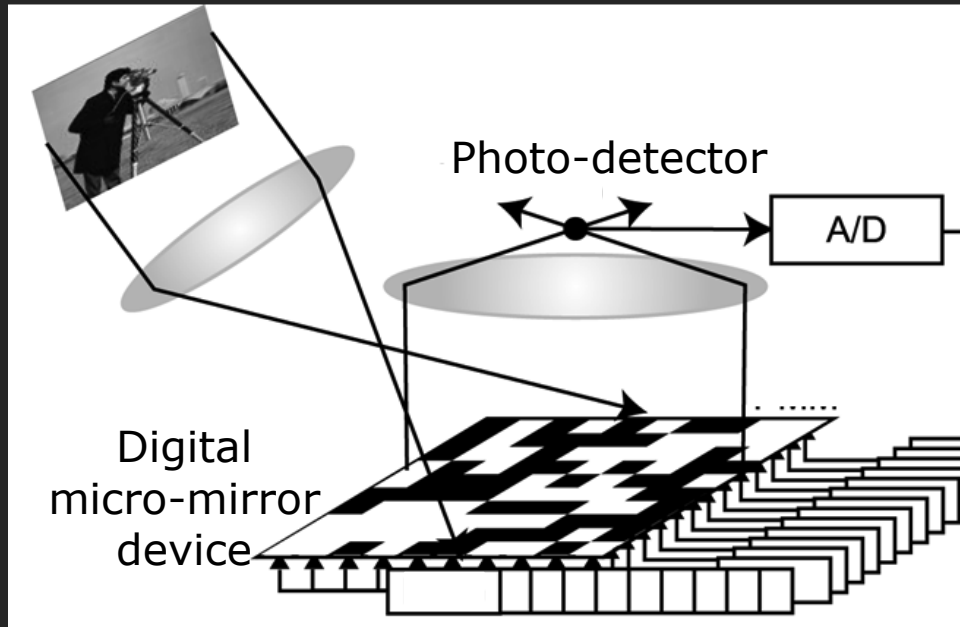
Can we super-resolve a low-resolution sensor ?

- Spatial light modulation
 - Introduce a **high-resolution mask** between scene and sensor

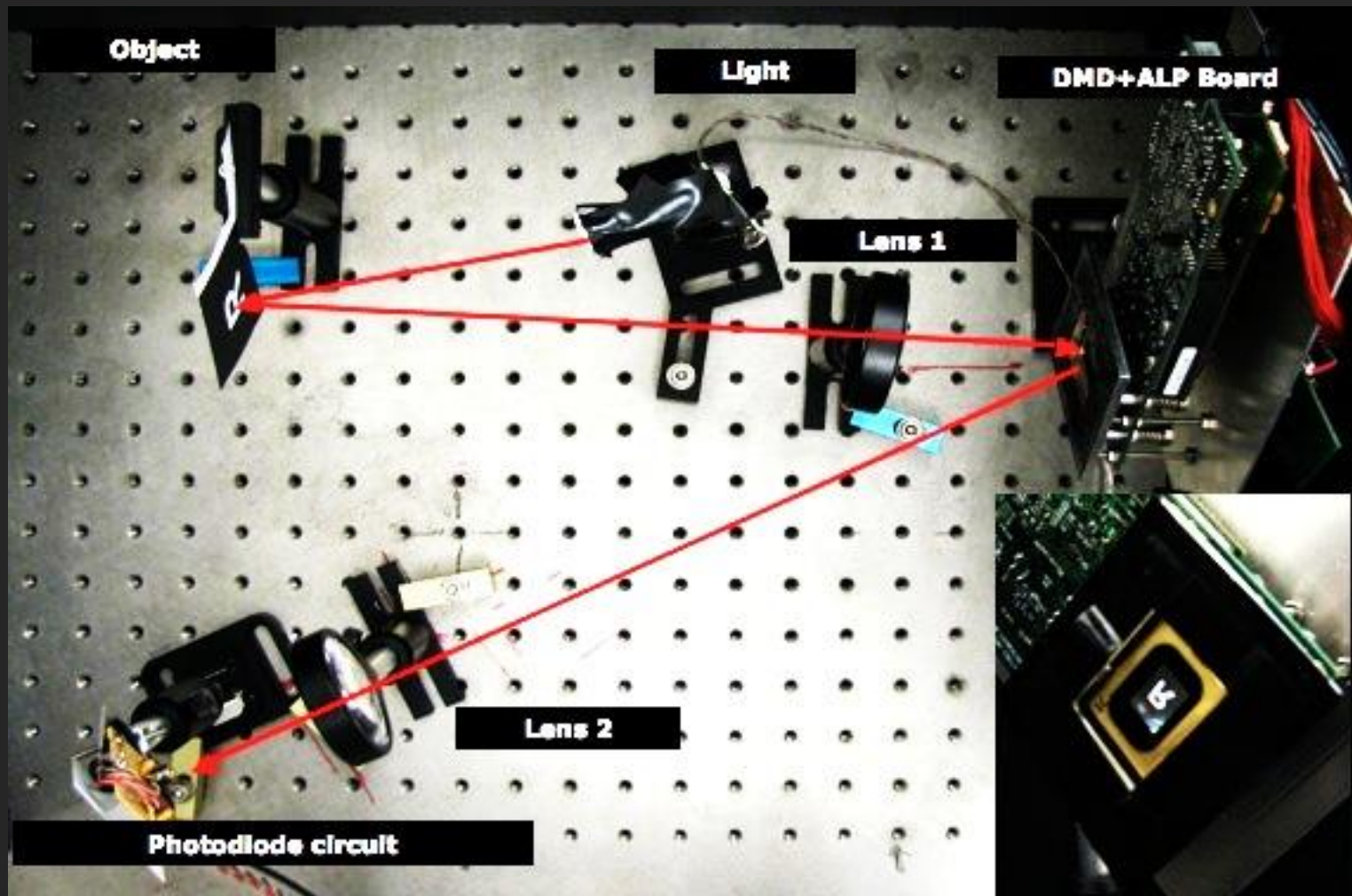


Single pixel camera

- Each pattern of micro-mirrors yield **ONE** compressive measurement
- A **single** photo-detector tuned to the wavelength of interest
- Resolution of the camera is that of the DMD, and not the sensor



CS-MUVI on SPC



Single pixel camera setup

CS-MUVI: IR spectrum

InGaAs Photo-detector (Short-wave IR)

SPC sampling rate: 10,000 sample/s

Number of compressive measurements: **M = 16,384**

Recovered video: **N = 128 x 128 x 61**. Compression = **61x**



Recovered Video



Results

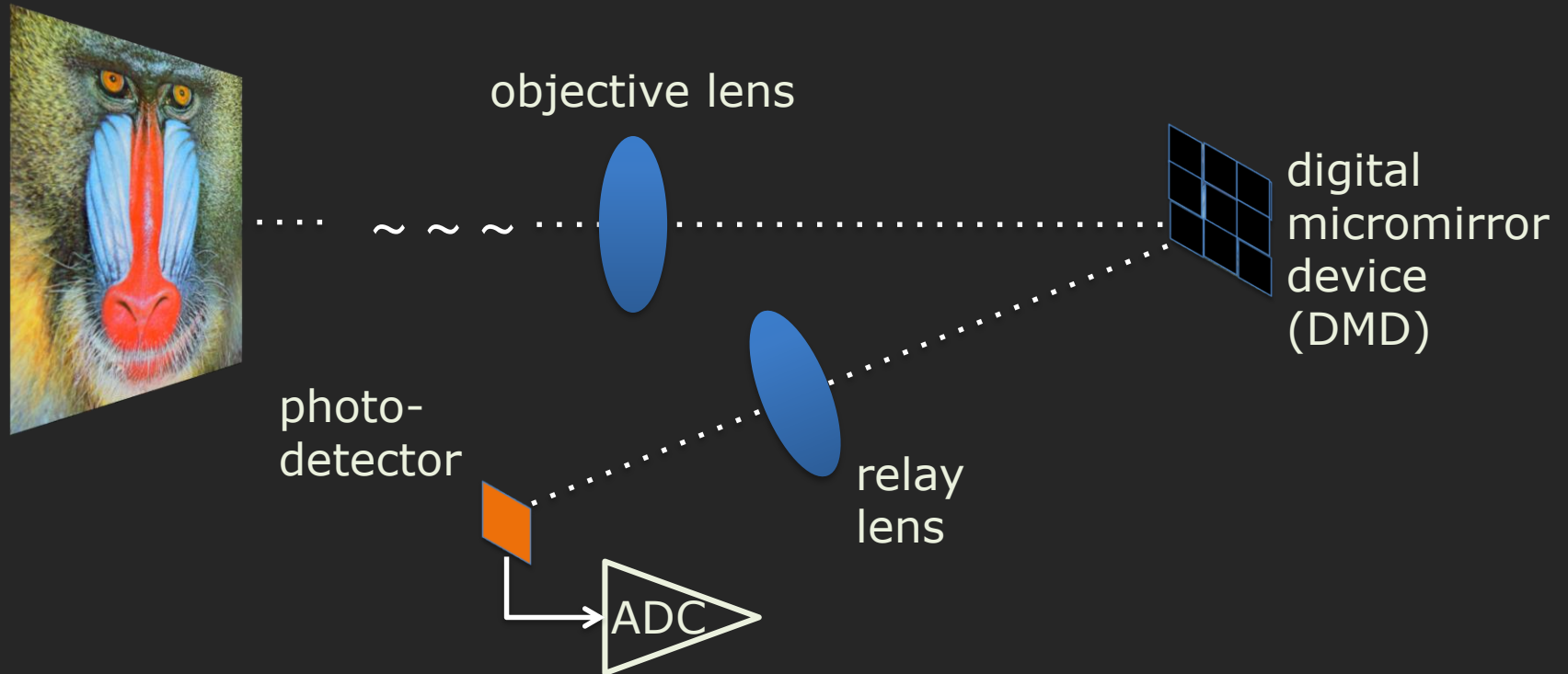
- Real data acquired using a single pixel camera
- Sampling rate: 10,000 Hz
- Number of compressive measurements: 65536
- Total duration of data acquisition: 6 seconds
- Reconstructed video resolution: 128x128x256



Final estimate (6 different videos)

Motivation

SPC has very low measurement rate



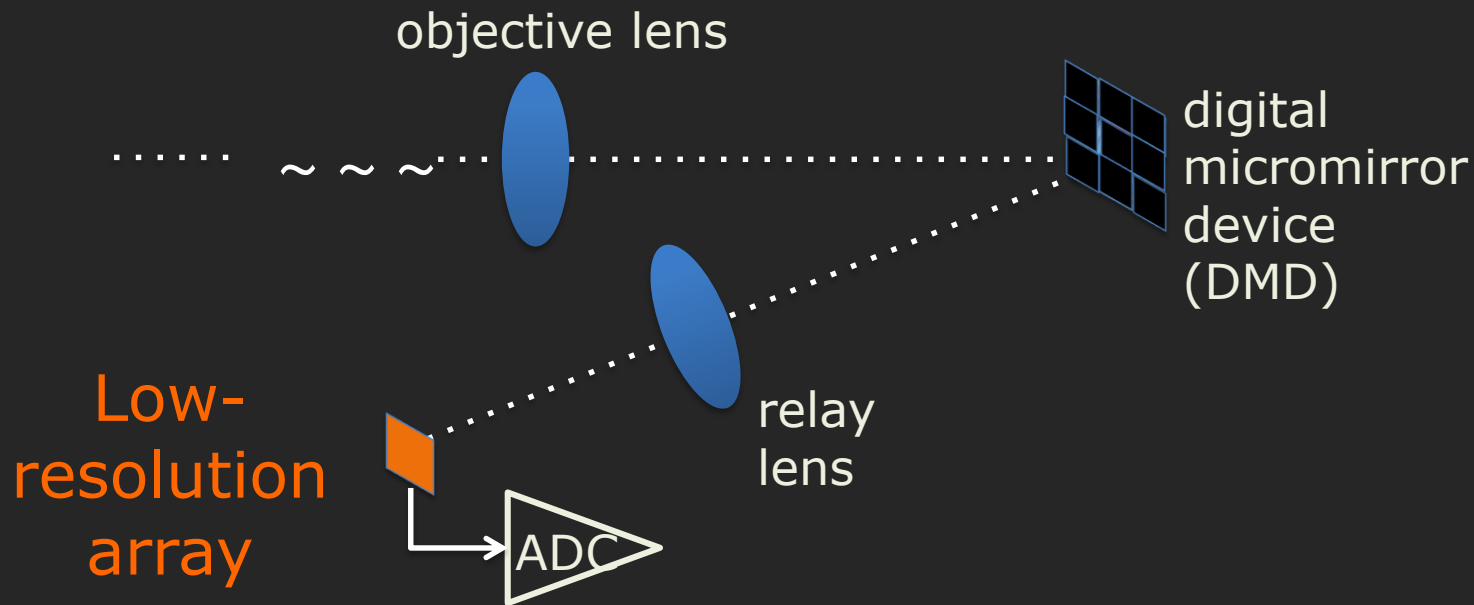
DMD --- R_{DMD} patterns/sec (typically, in 10s kHz)

ADC --- R_{ADC} samples/sec (typically, in 10s MHz)

Measurement rate of the SPC = $\min(R_{\text{ADC}}, R_{\text{DMD}})$

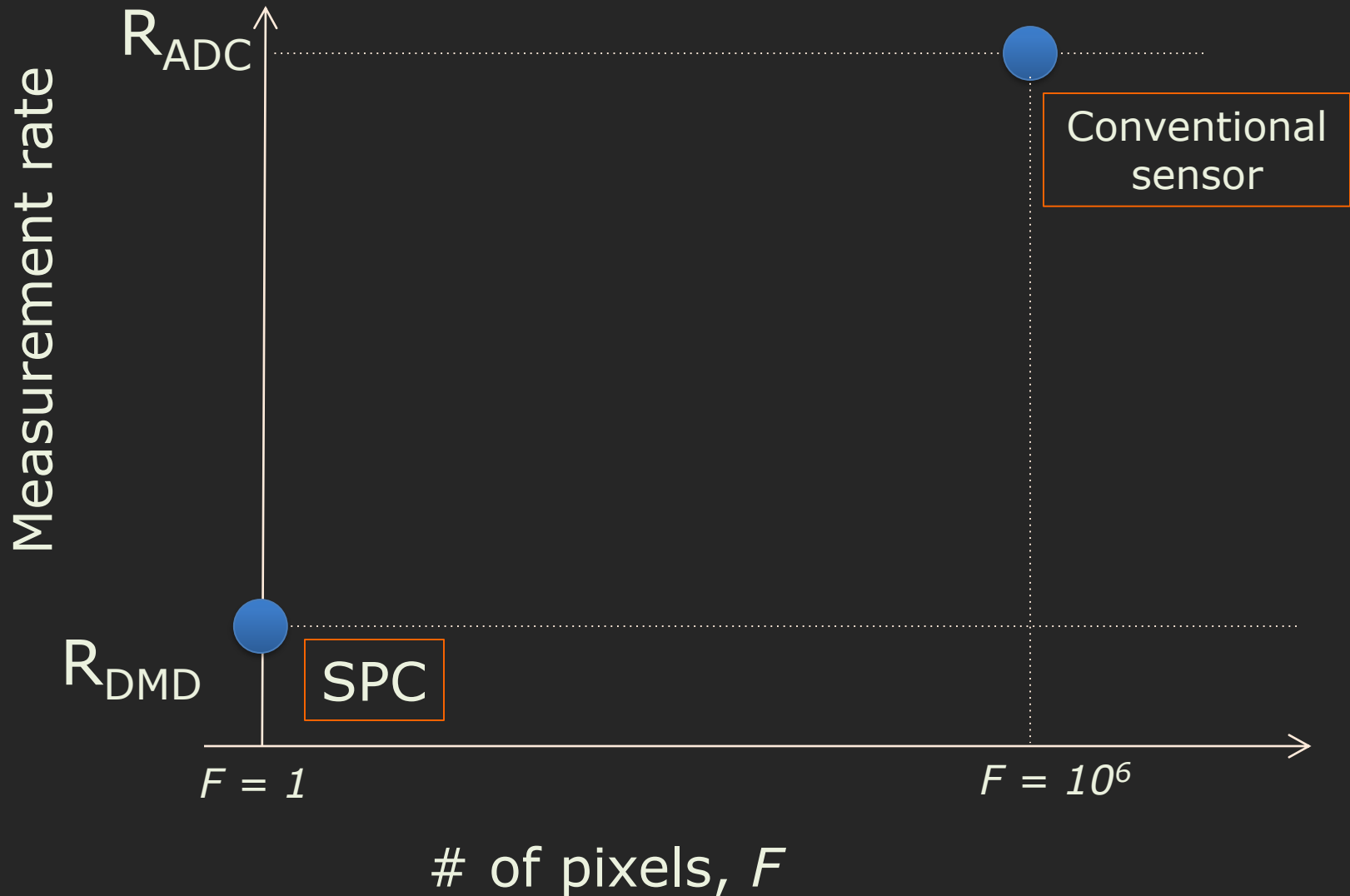
Parallel Compressive Imaging

- Use multiple pixels or a low-resolution sensor array

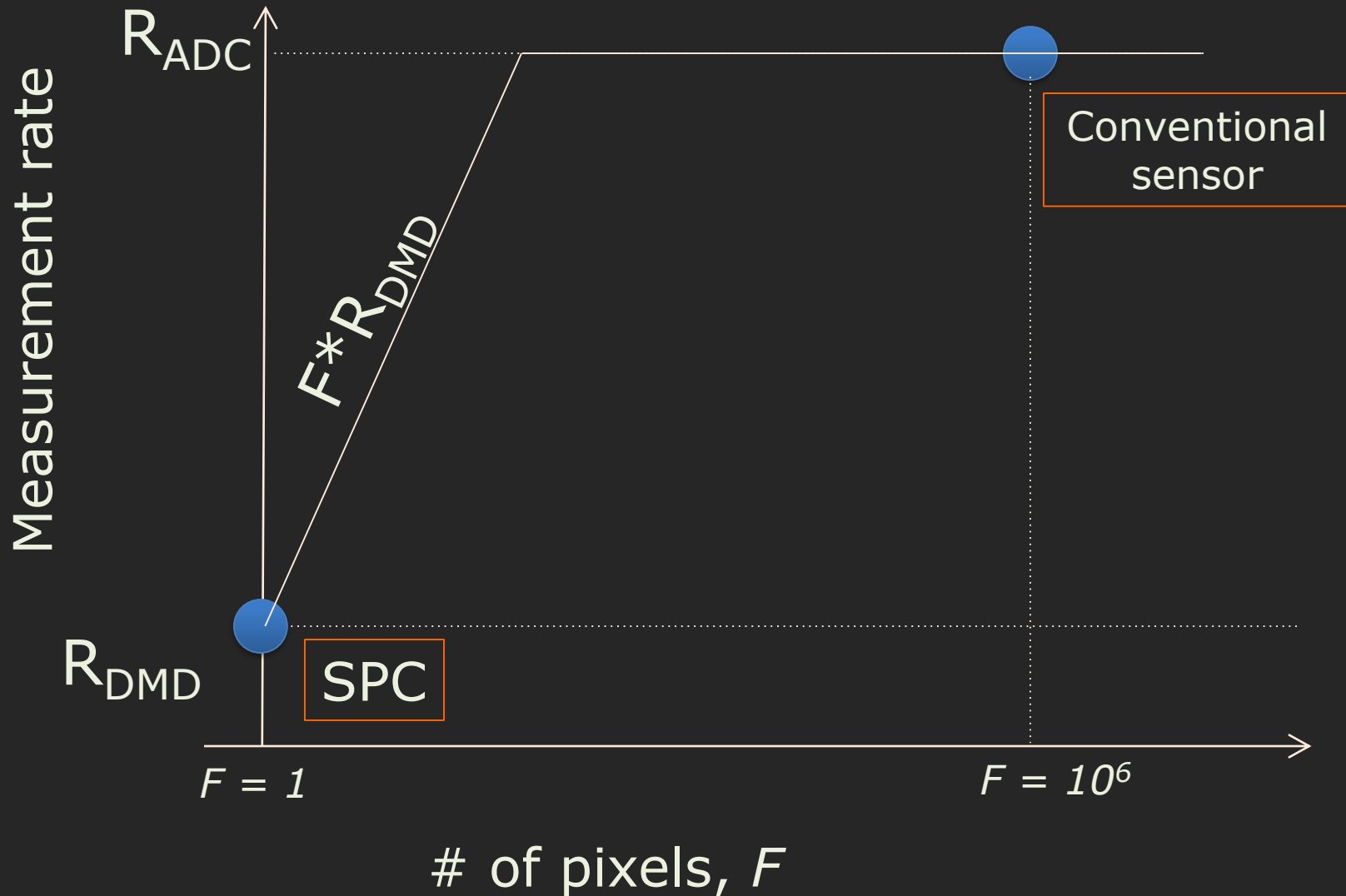


- How do we decide the specifications of the low-resolution sensor ?
 - Number of pixels, geometry, etc ...

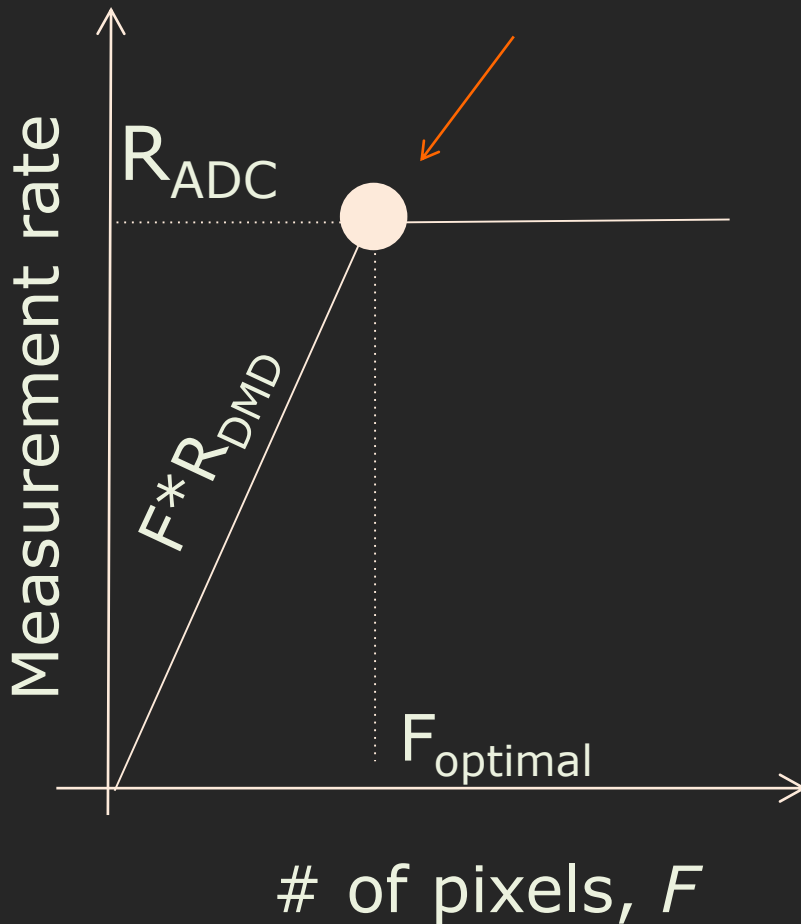
Measurement rate



Measurement rate



Optimal # of pixels



$$F_{optimal} = \frac{R_{ADC}}{R_{DMD}}$$

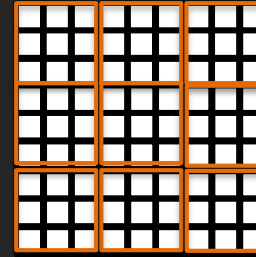
$F_{optimal}$ is typically in 1000s of pixel for today's DMDs and ADCs

Implications.

Measurement rate of a conventional sensor but with a fraction of the number of pixels! (less than 0.1% pixels)

Two Prototypes

- Focal plane array-based CS (FPA-CS)
 - SWIR
 - Map DMD onto a low-resolution 2D sensor
 - Each pixel on the sensor observes a 2D patch of micromirrors on the DMD

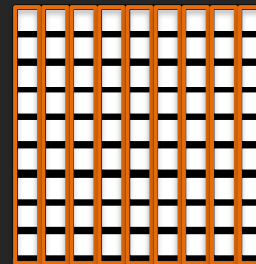


DMD



Low-resolution sensor

- Line-sensor based Compressive Imager (LiSens)
 - Map DMD onto a line-array sensor
 - Each pixel on the sensor observes a line of micromirrors on the DMD



DMD



Line-sensor

FPA-CS

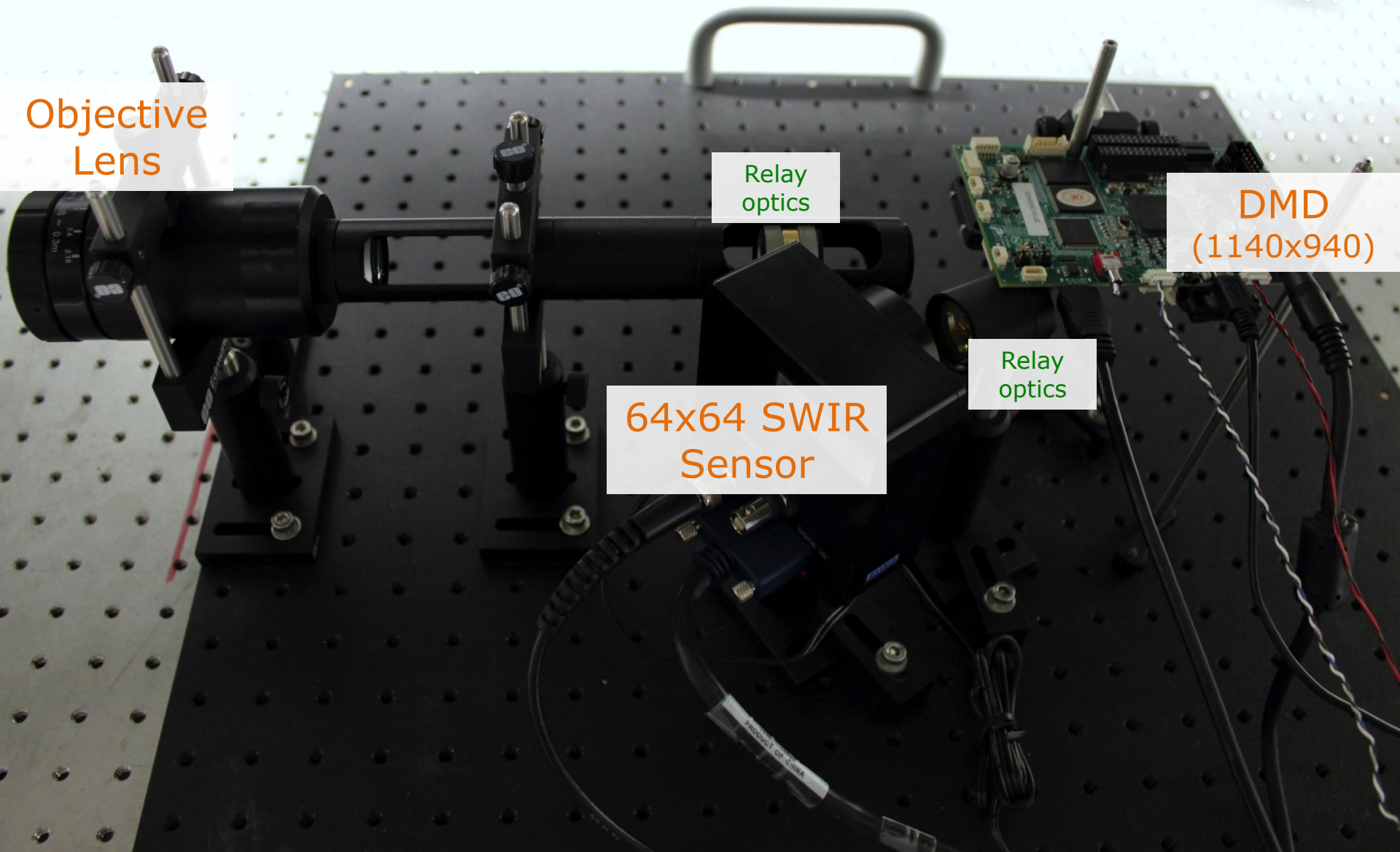
Objective
Lens

Relay
optics

DMD
(1140x940)

Relay
optics

64x64 SWIR
Sensor



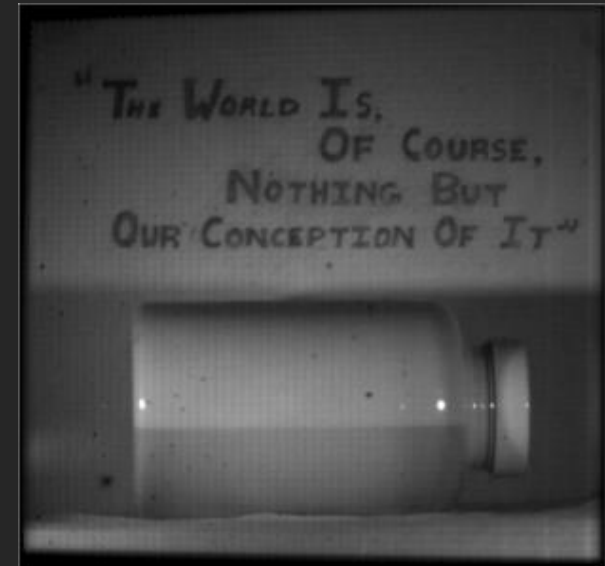
FPA-CS Results



Scene
(seen in a visible
camera)

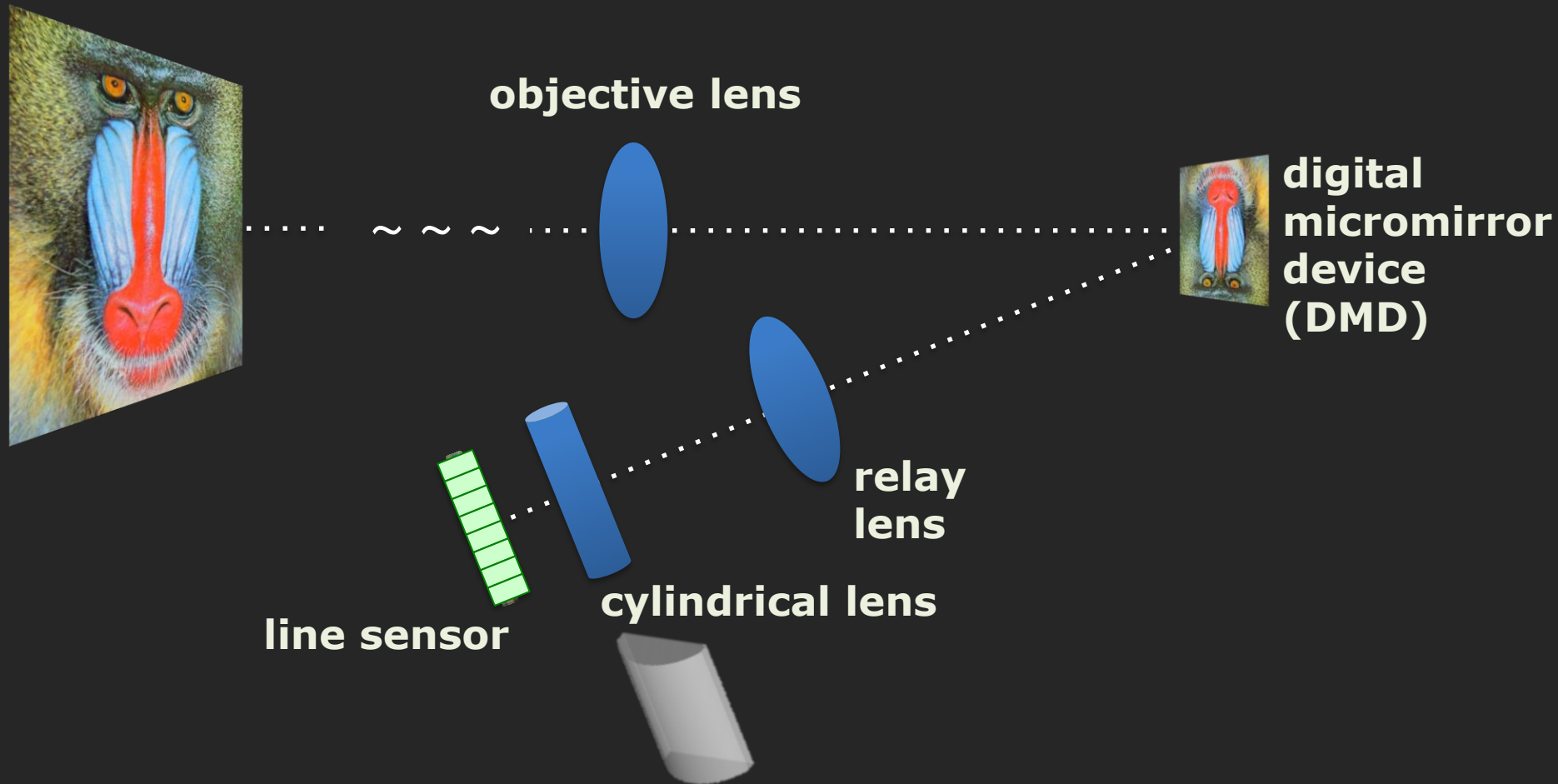


Image seen by 64x64
SWIR sensor



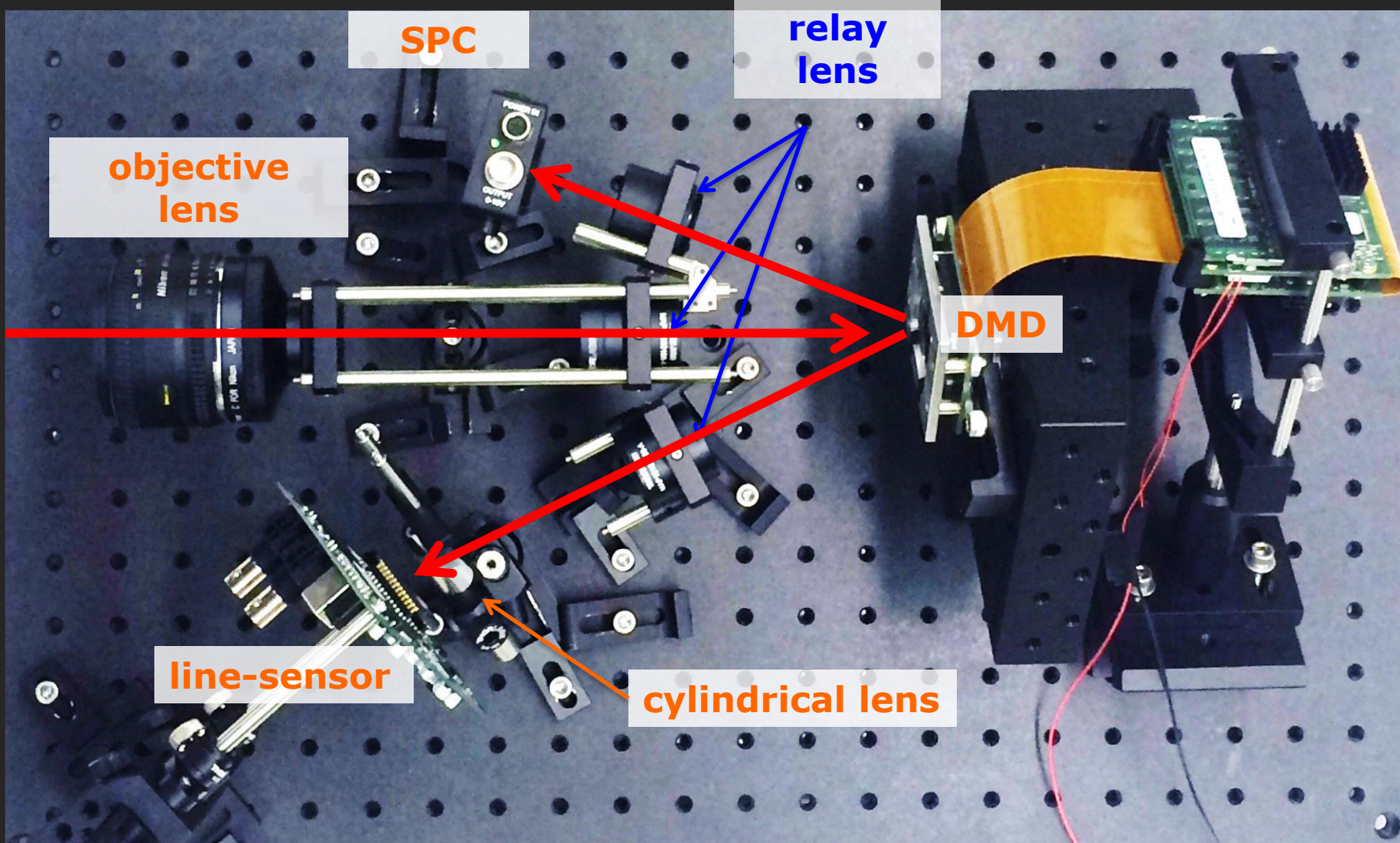
Super-resolved image
by FPA-CS
architecture

Line-Sensor-based compressive camera (LiSens)



1. Use a linear array of pixels (a line-sensor)
2. Add a cylindrical lens

Hardware prototype



Measurement rate: 1 MHz

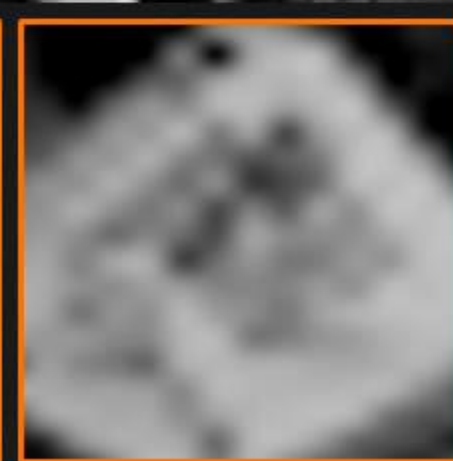
Comparison against SPC

Capture duration: **880ms** 440ms 220ms 110ms

LiSens
0.8M meas.



SPC
16k meas.





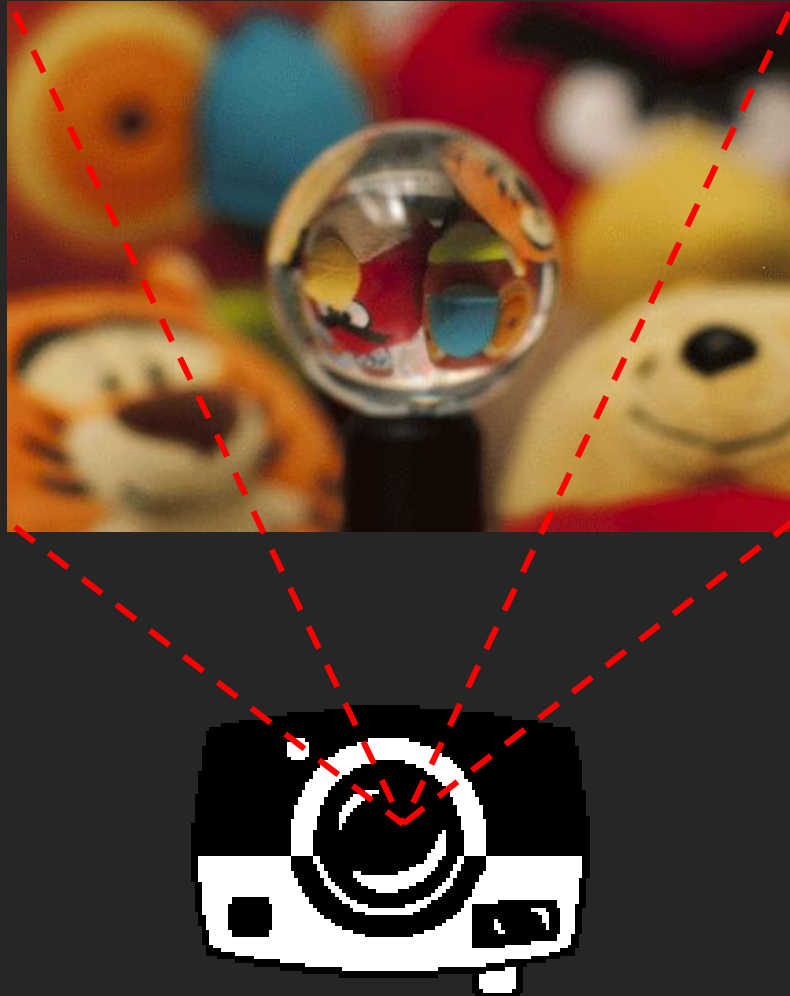
Compressive Light Fields

Key Papers

- Marwah et al., *Compressive coded apertures*, SIGGRAPH 2013
- Tambe et al., *Compressive LF videos*, ICCV 2013
- Ito et al., *Compressive epsilon photography*, SIGGRAPH 2014

Epsilon Photography

Capture stack of photographs by varying camera parameters **incrementally**



Ex 1 - Epsilon photography applied to exposure



Exposure Bracketting for HDR

Ex 2 – Epsilon photography applied to focus



Focus stack

Ex 3 – Epsilon photography applied to aperture and focus



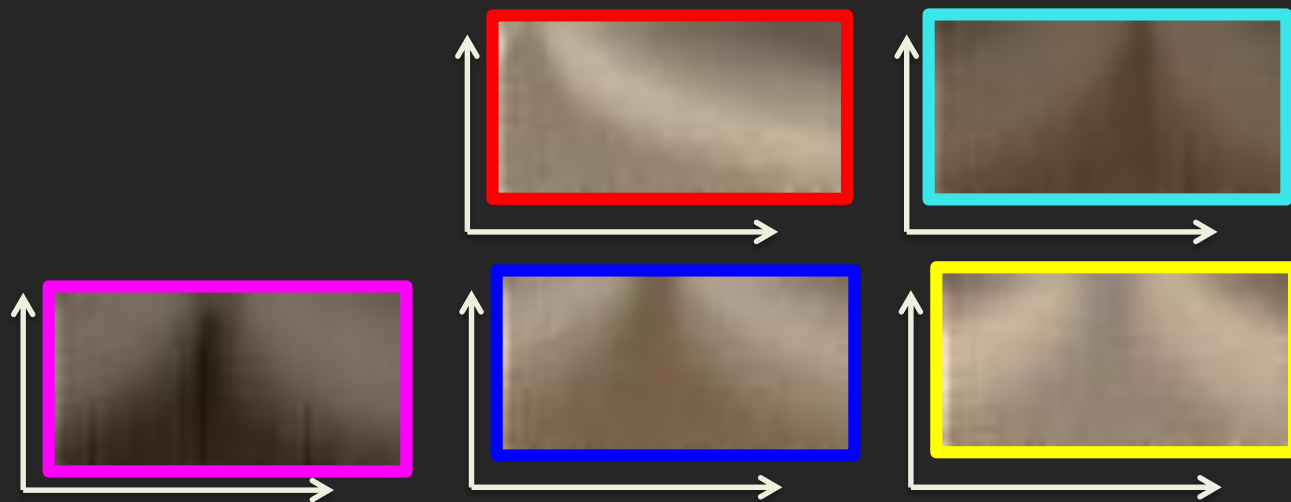
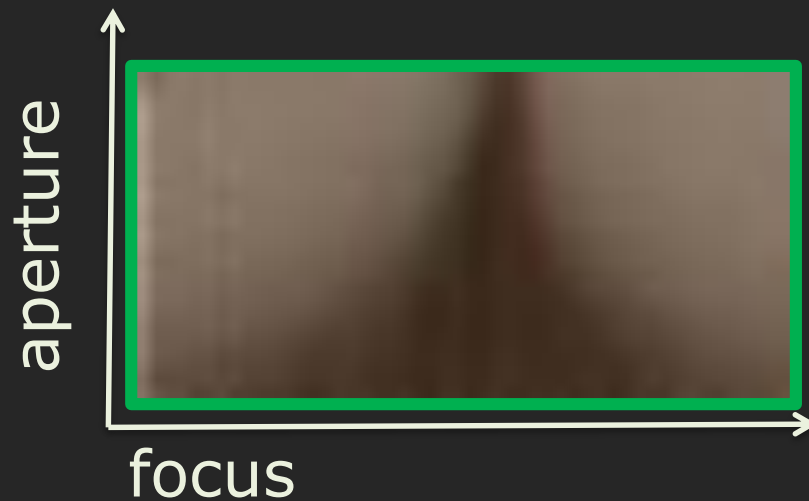
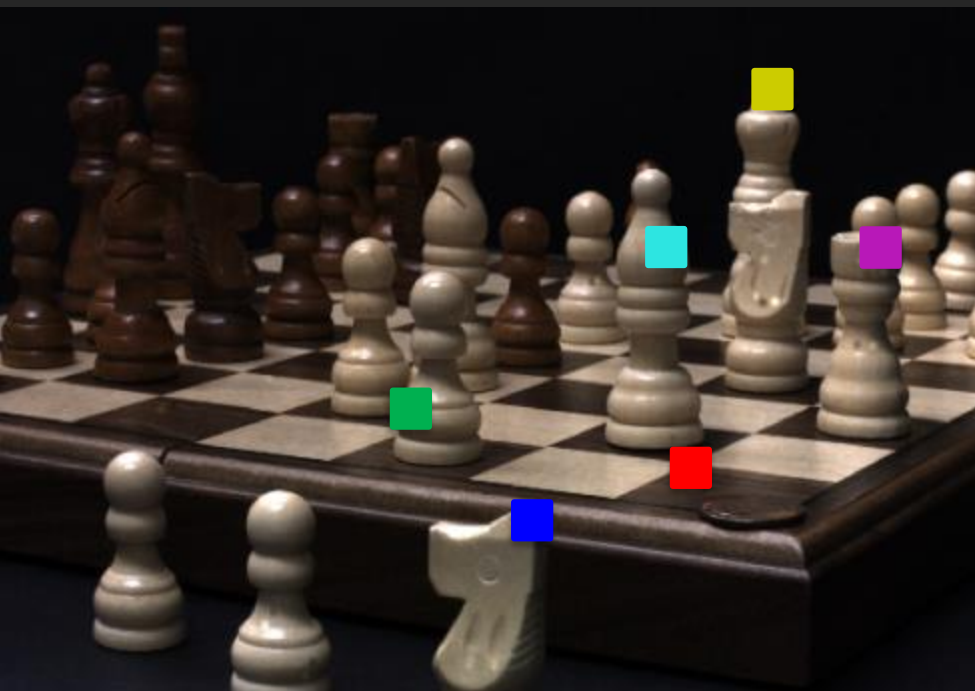
Confocal stereo
Per-pixel depth estimation

Confocal Stereo



Confocal stereo
Per-pixel depth estimation

Aperture Focus Images





Pros and Cons

- Pros
 - Per-pixel operations (for the most part)
- Cons
 - Too many images
 - Need texture (problem for everybody passive)
 - Align ?

Epsilon Photography

Capture stack of photographs by varying
camera parameters incrementally

Extremely slow!

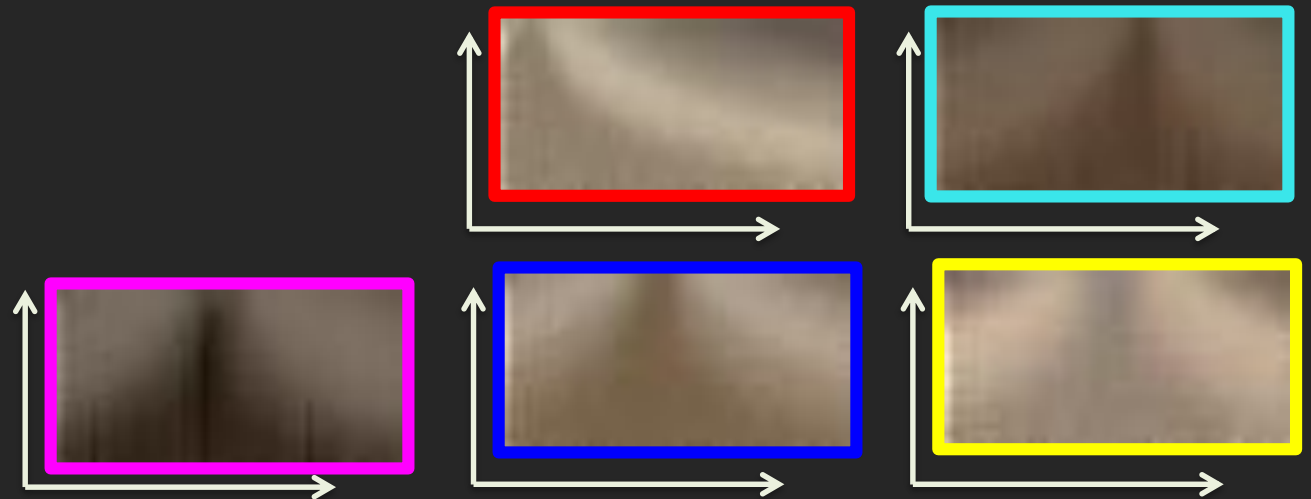
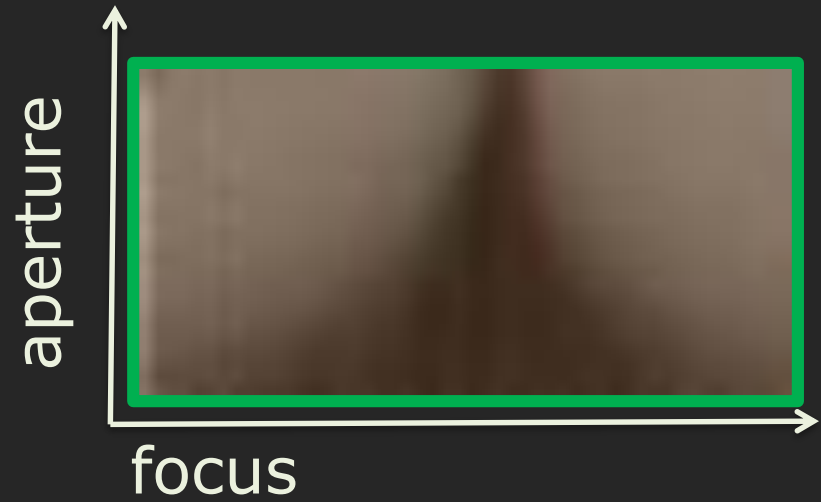
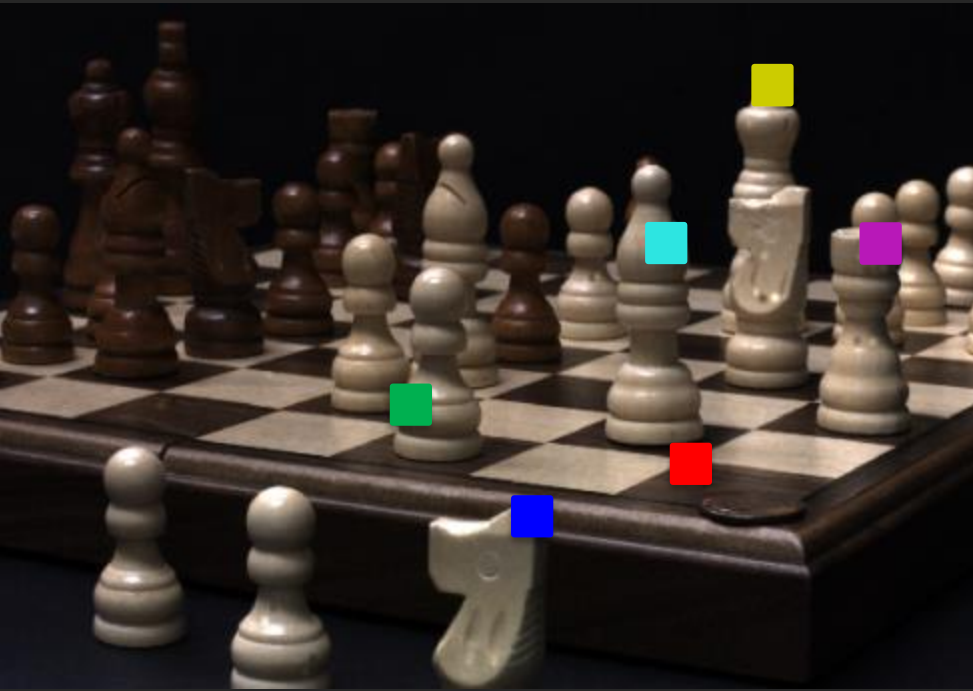
Compressive Epsilon Photography



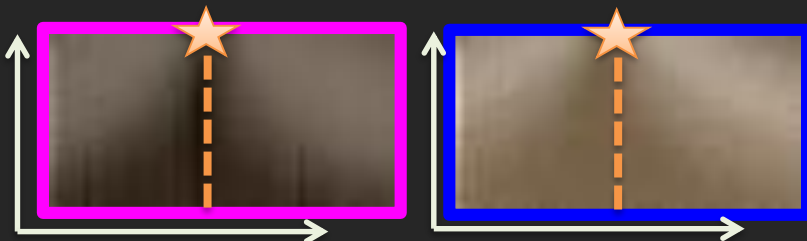
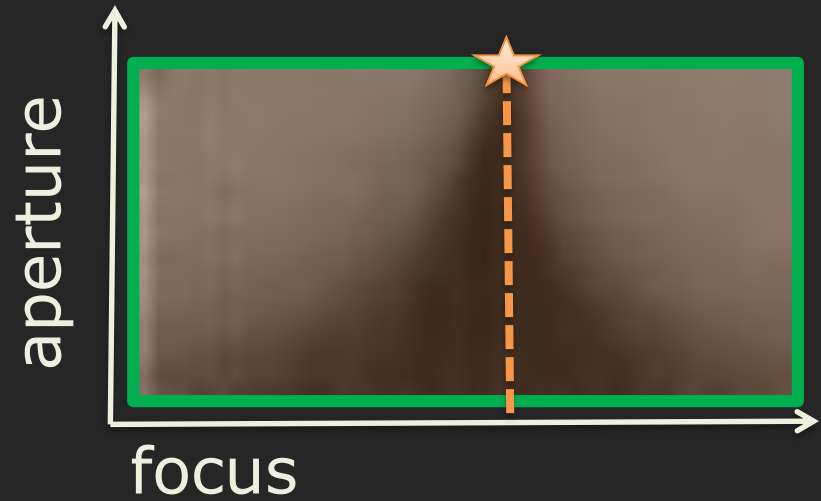
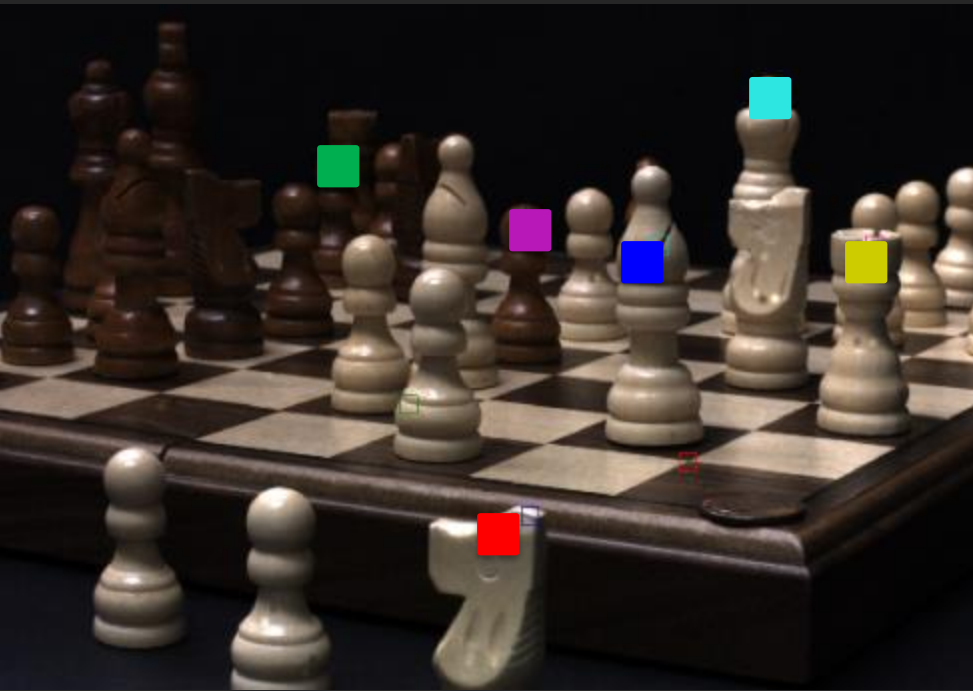
Compressive Epsilon Photography



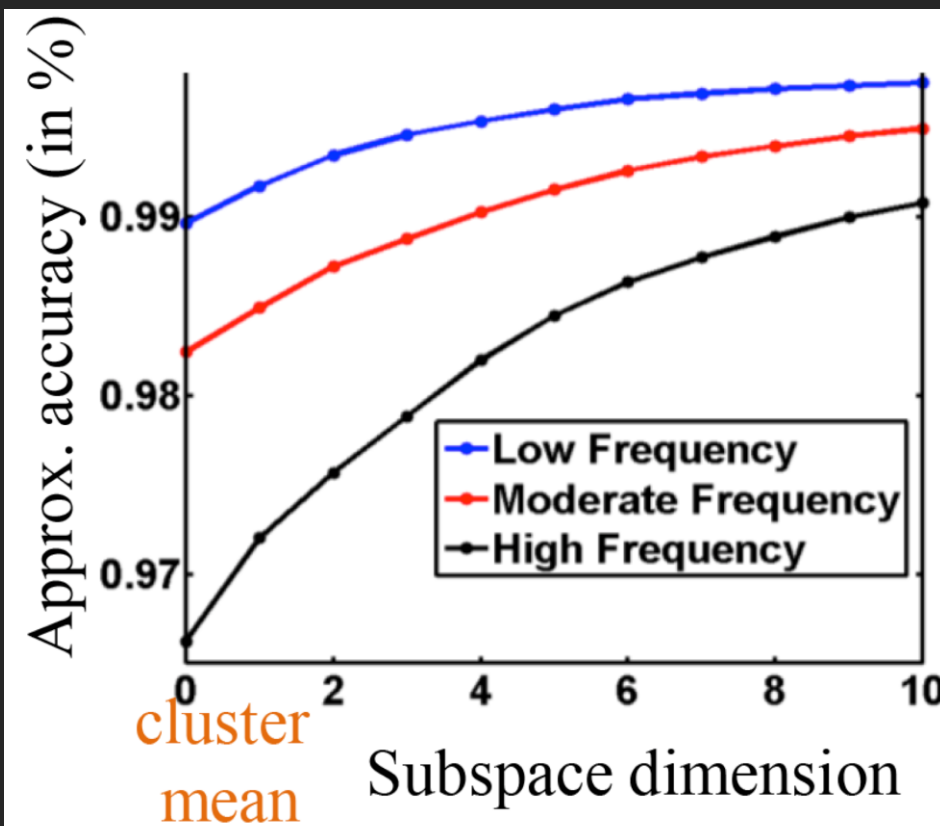
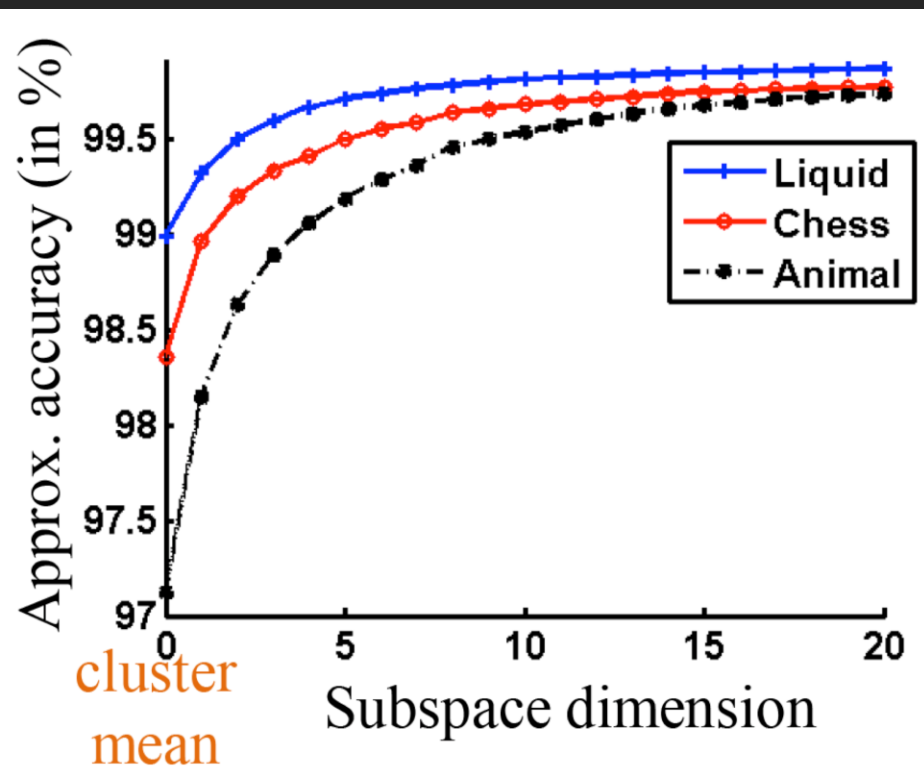
Redundancies in Focus-Aperture Stacks



Redundancies in Focus-Aperture Stacks



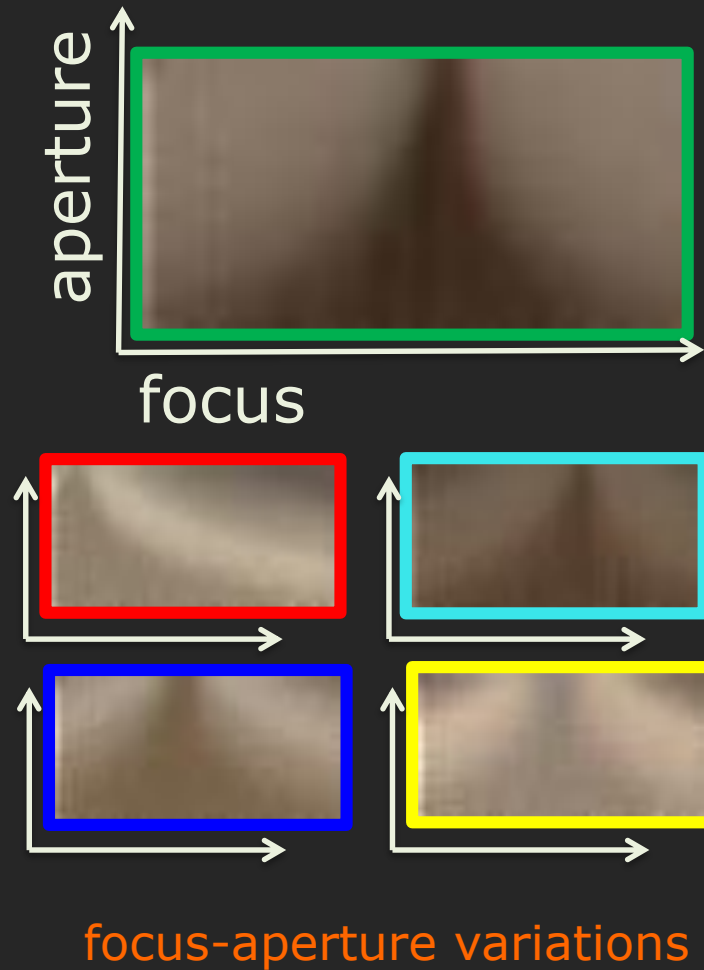
Redundancies in Focus-Aperture Stacks



Per-pixel models

- Key idea: Model intensity variations observed at an *individual pixel*
- Advantages
 - No smoothing. *Spatial resolution can be preserved*
 - Parallel recovery at each pixel
- Disadvantages
 - Lack of spatial constraints

Gaussian Mixture Models



Observation: Structure of EP intensity profiles tied to depth at a pixel

Problem formulation

Given a few images captured with pre-selected parameters

+

per-pixel GMM of intensity variations

recover the entire epsilon photography intensity profile at each pixel.

Linear inverse problem

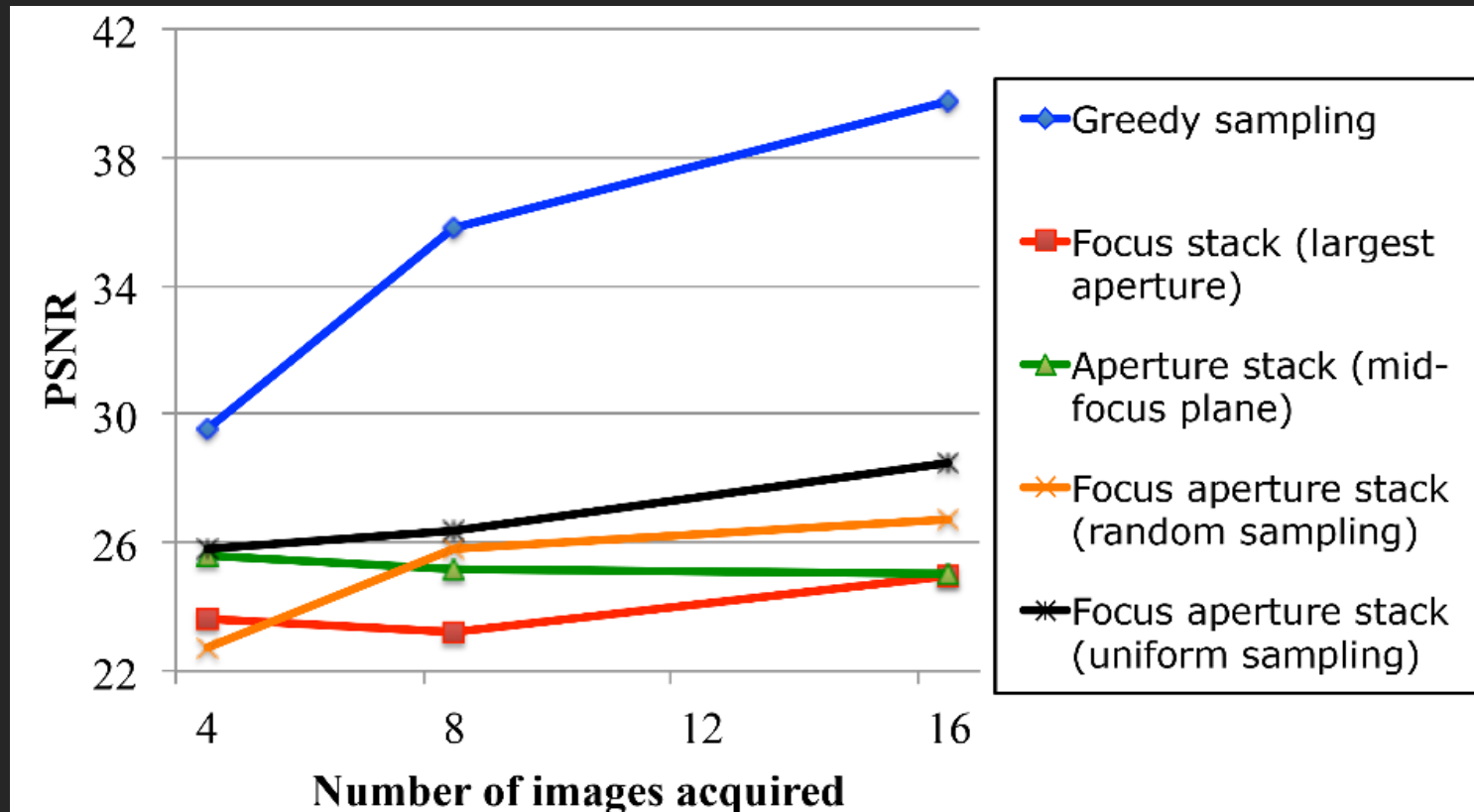
Lots of solvers

We use a maximum likelihood estimator

Advantages of the GMM model

Analytical bounds on performance.

Can **greedily** pre-select camera parameters that maximize average reconstruction performance



Advantages of the GMM model

Small aperture leads to large DOF and provides textural cues

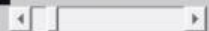


Large aperture leads to small DOF and provides depth cues



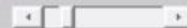
Chess

File Edit View Insert Tools Desktop Window Help



Fluffy

File Edit View Insert Tools Desktop Window Help



Animals

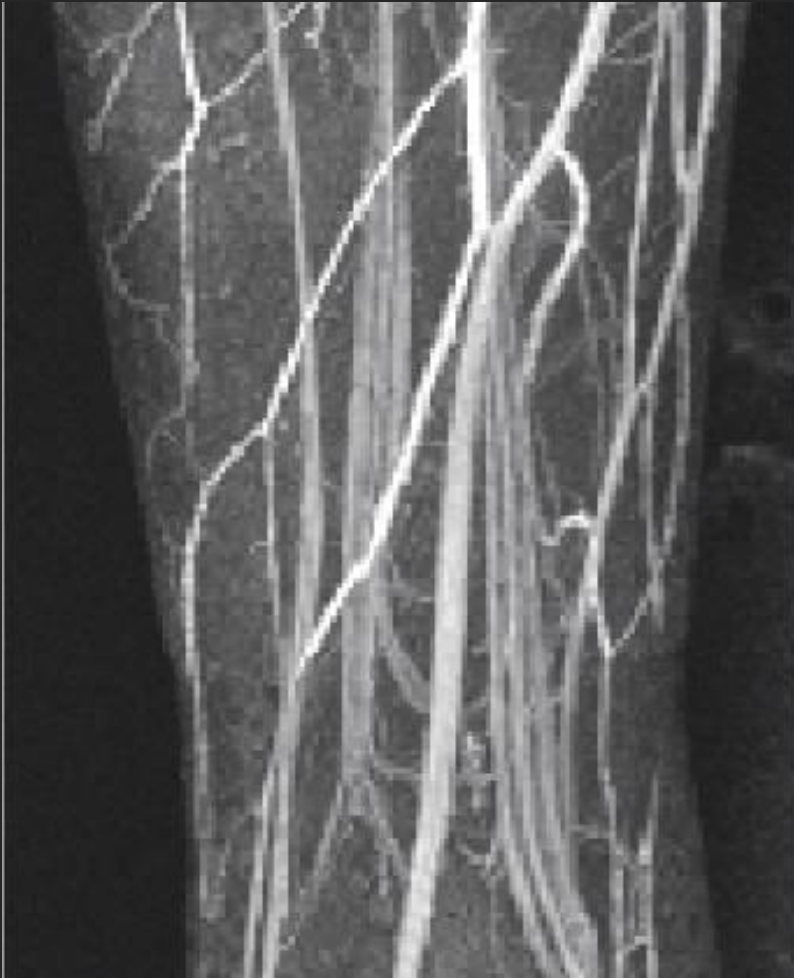
File Edit View Insert Tools Desktop Window Help



CS Summary

- Three questions
 - Is sensing **costly** ? (how?)
 - Is there a **sparsifying/parsimonious** representation ?
 - Acquire some sort of **randomized** measurements ?

A simple case study: MRI



MRI obtains samples
in Fourier space

Taking lesser
samples

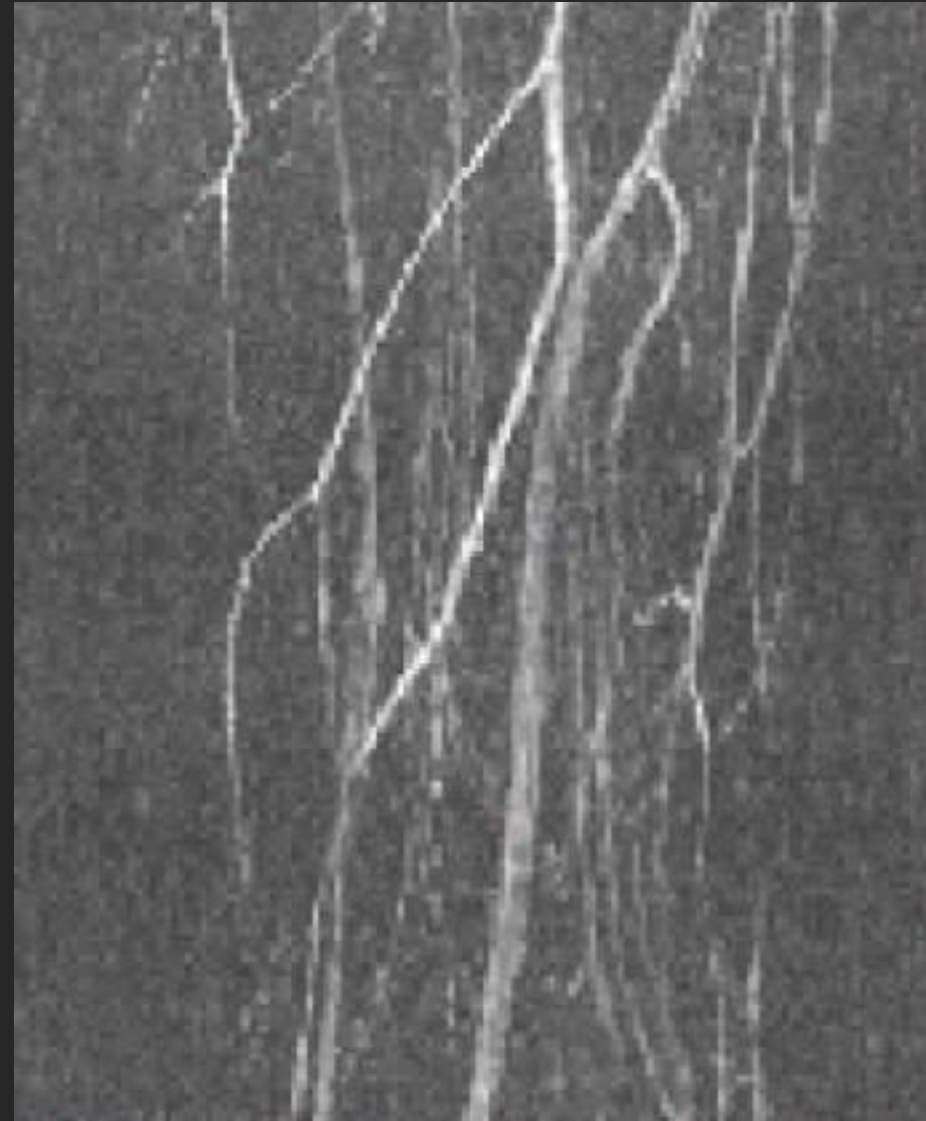
==

higher speed of
operation, less time
etc.

MRI

without a signal model

From 10 times lesser
number of
measurements



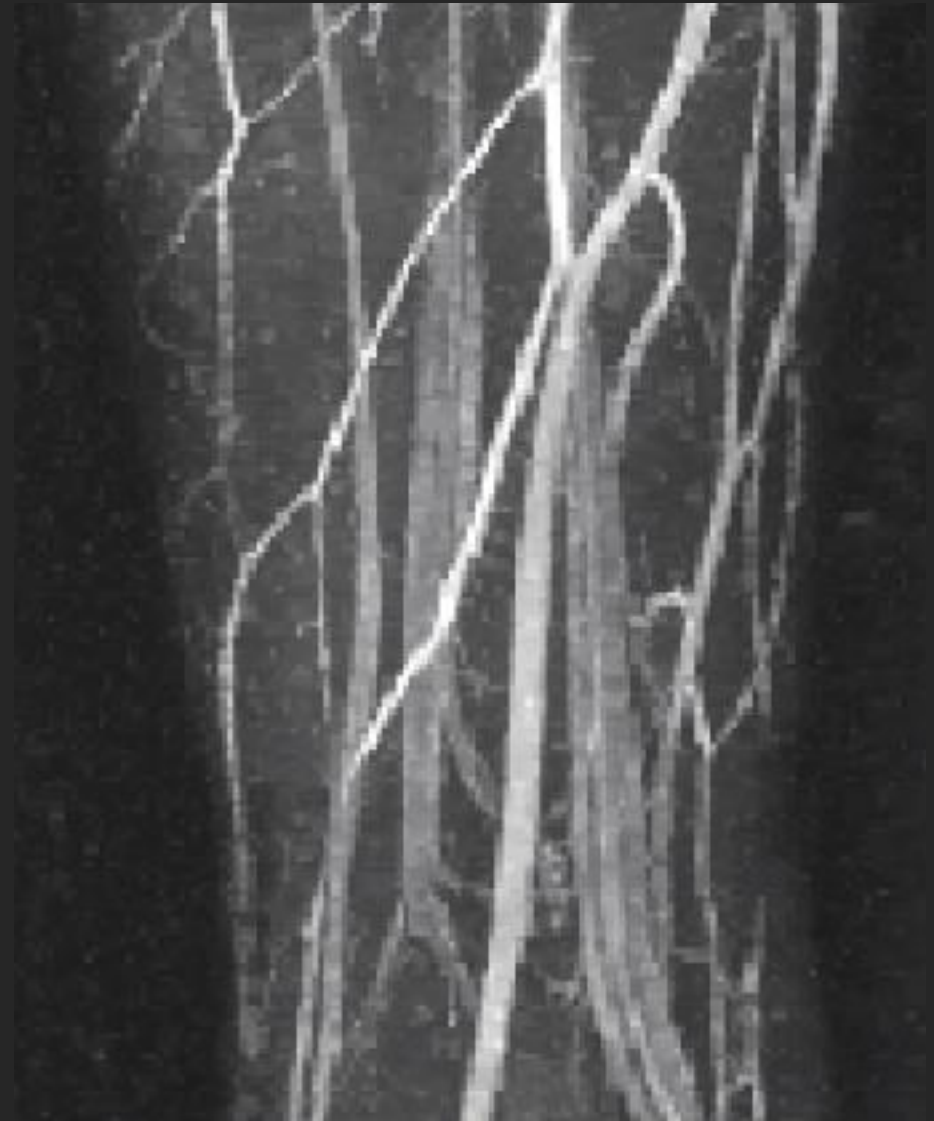
Lustig et al., 2008

MRI + CS

with signal model

From 10 times lesser
number of
measurements

The recovery is *exact*,
provided some
conditions are satisfied



Summary

- CS provides the ability to sense from far-fewer measurements than the signal's dimensionality
- Implications
 - Fewer pixels on the sensor
 - Shorter acquisition time
 - Slower rate of acquisition