Pinhole cameras

15-463, 15-663, 15-862
Computational Photography
Fall 2017, Lecture 14

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework 4 is out.
  - Due October 26th.
  - Bilateral filter will take a very long time to run.
  - Make sure to sign up for a camera and a team.
  - Drop by Yannis' office to pick up cameras any time.

- Yannis has extra office hours on Wednesday, 2-4pm.
  - You can come to ask questions about HW4 (e.g., "how do I use a DSLR camera?").
  - You can come to ask questions about final project.

- Project ideas are due on Piazza on Friday 20th.

Overview of today’s lecture

- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- Camera matrix.
- Perspective.
- Orthographic camera.

Slide credits

Most of these slides were adapted from:

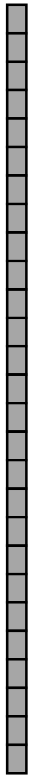
- Kris Kitani (15-463, Fall 2016).

Some slides inspired from:

- Fredo Durand (MIT).

Some motivational imaging experiments

Let’s say we have a sensor…



digital sensor
(CCD or CMOS)

… and an object we like to photograph

real-world
object

digital sensor
(CCD or CMOS)

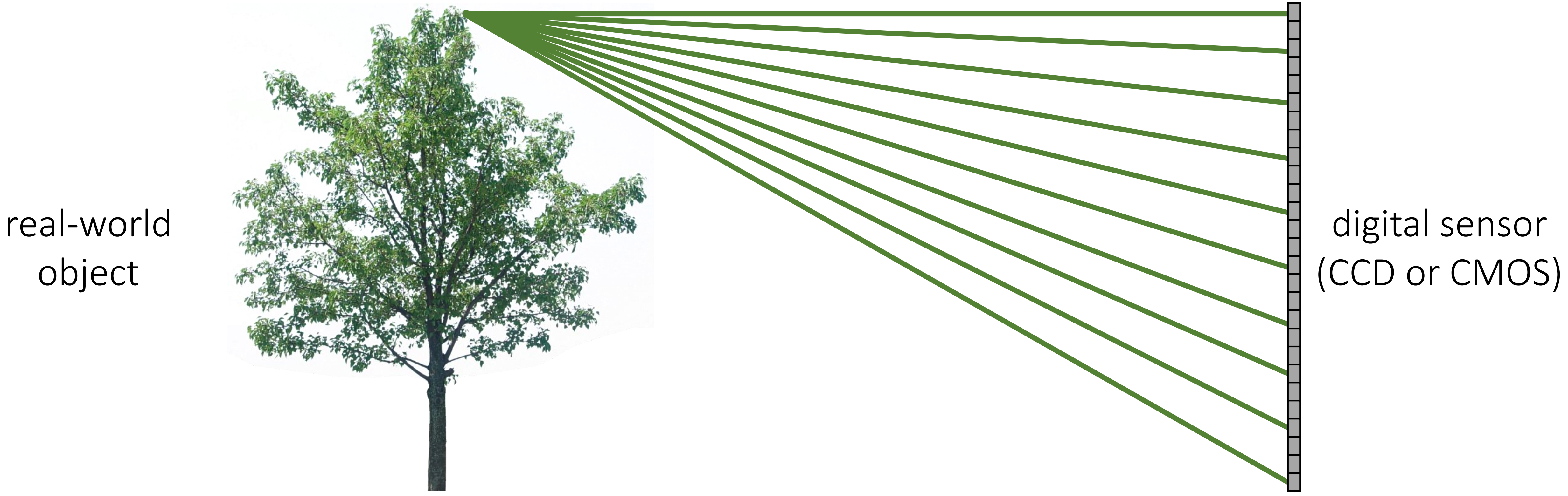
What would an image taken like this look like?

Bare-sensor imaging

real-world
object

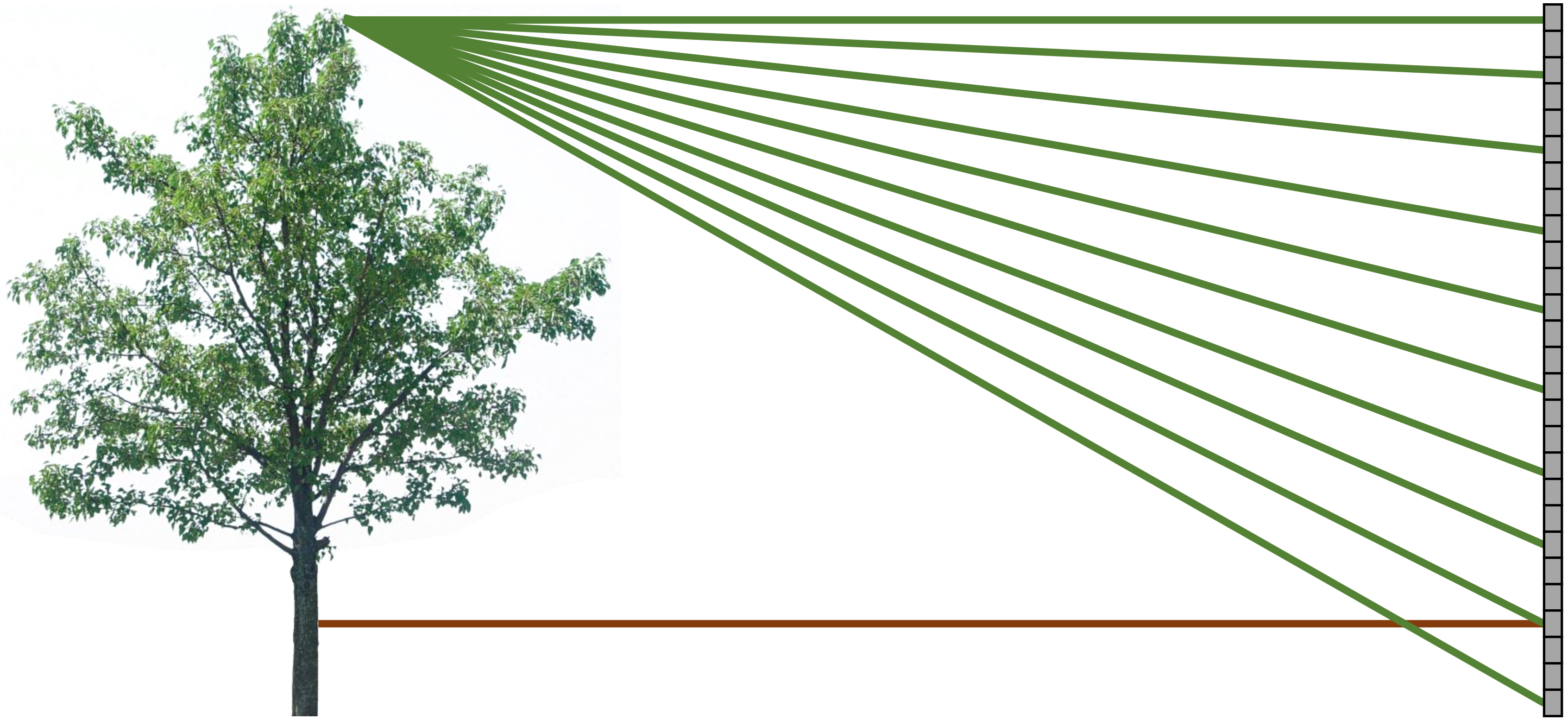
digital sensor
(CCD or CMOS)

Bare-sensor imaging



Bare-sensor imaging

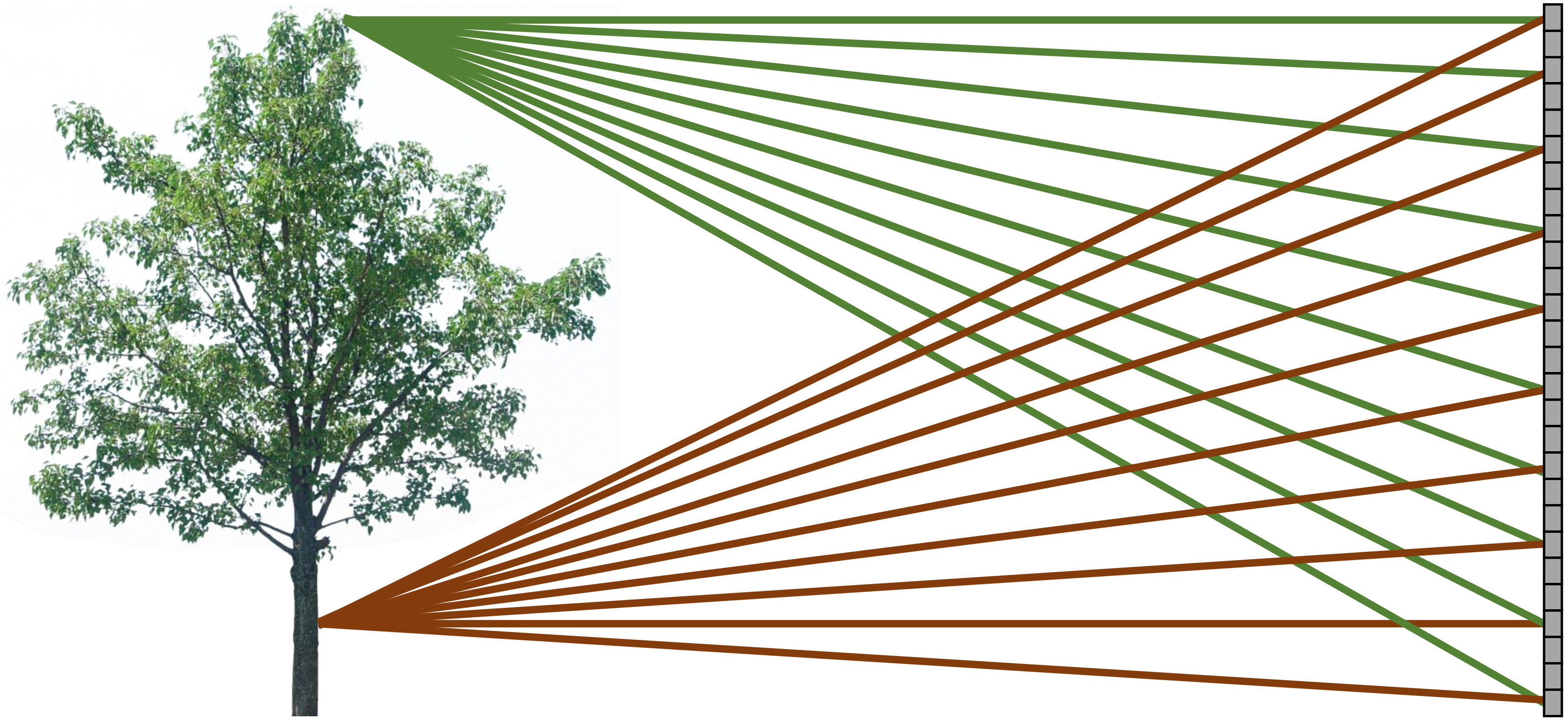
real-world
object



digital sensor
(CCD or CMOS)

Bare-sensor imaging

real-world
object



digital sensor
(CCD or CMOS)

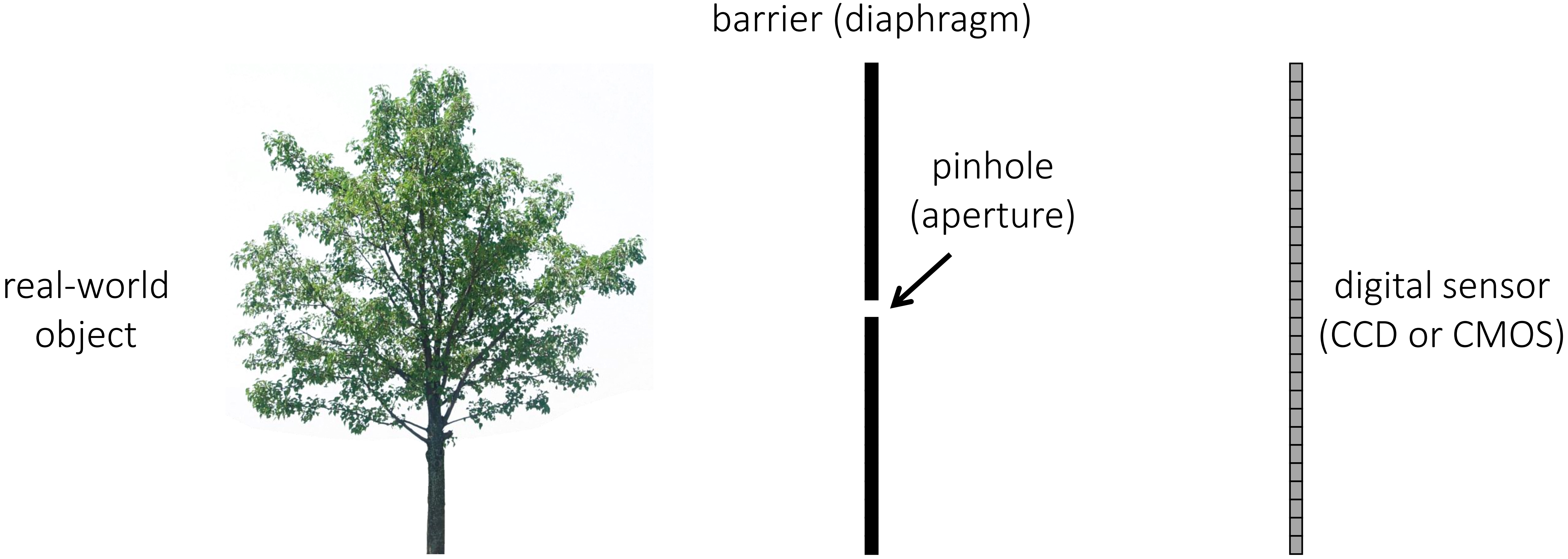
All scene points contribute to all sensor pixels

What does the
image on the
sensor look like?

Bare-sensor imaging

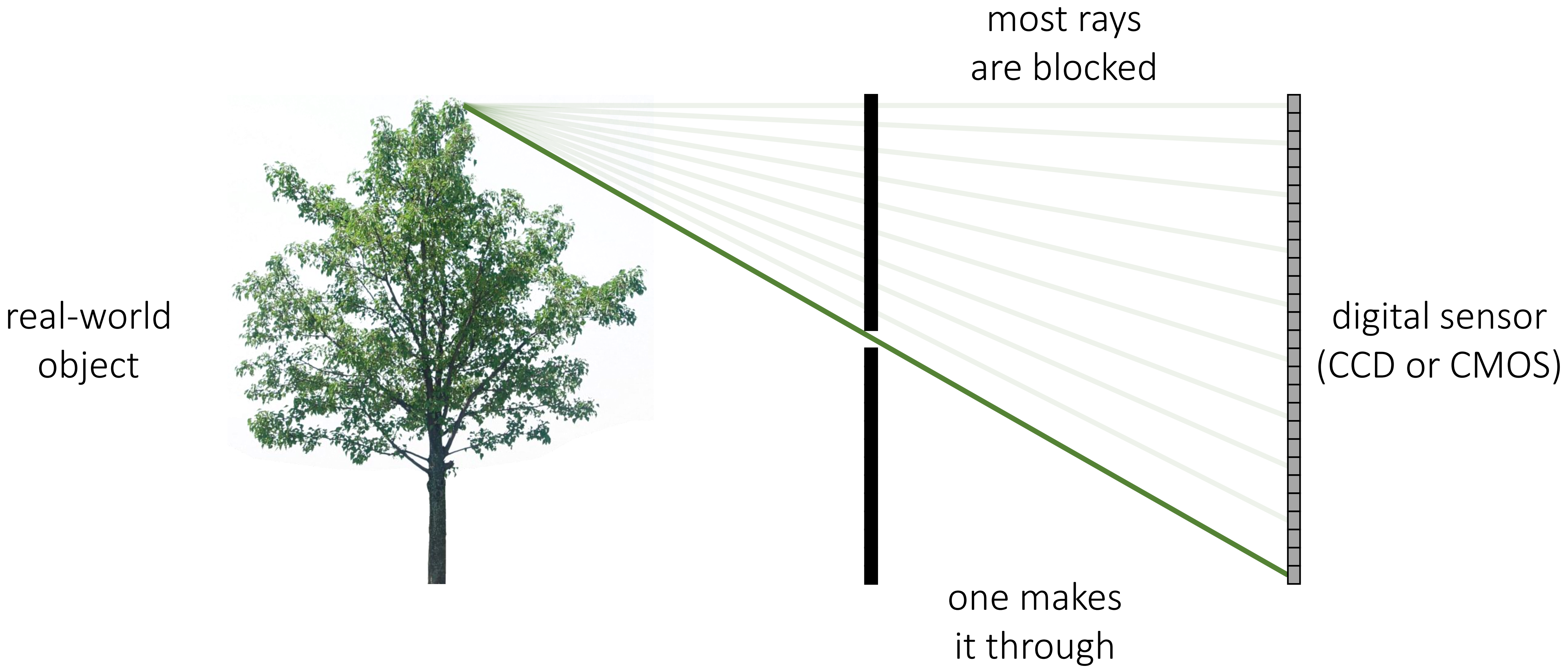
All scene points contribute to all sensor pixels

Let’s add something to this scene

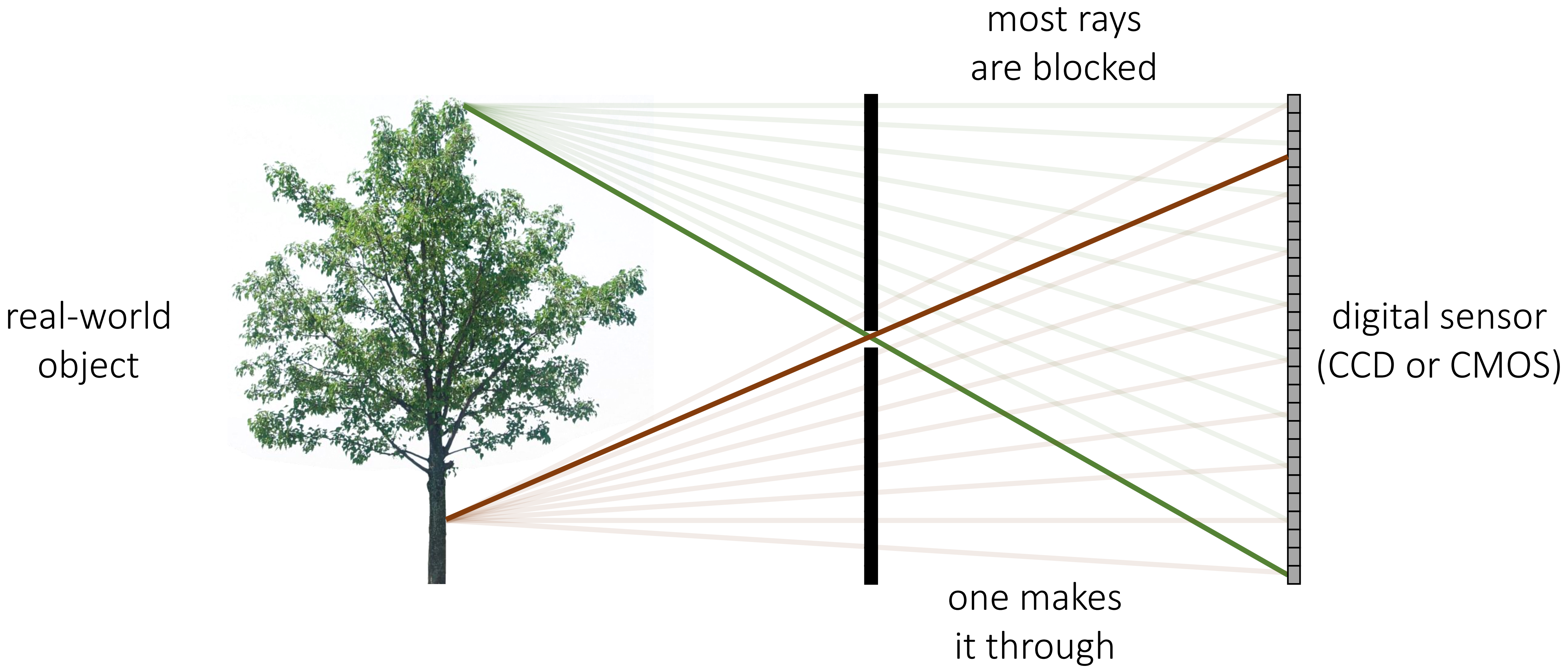


What would an image taken like this look like?

Pinhole imaging

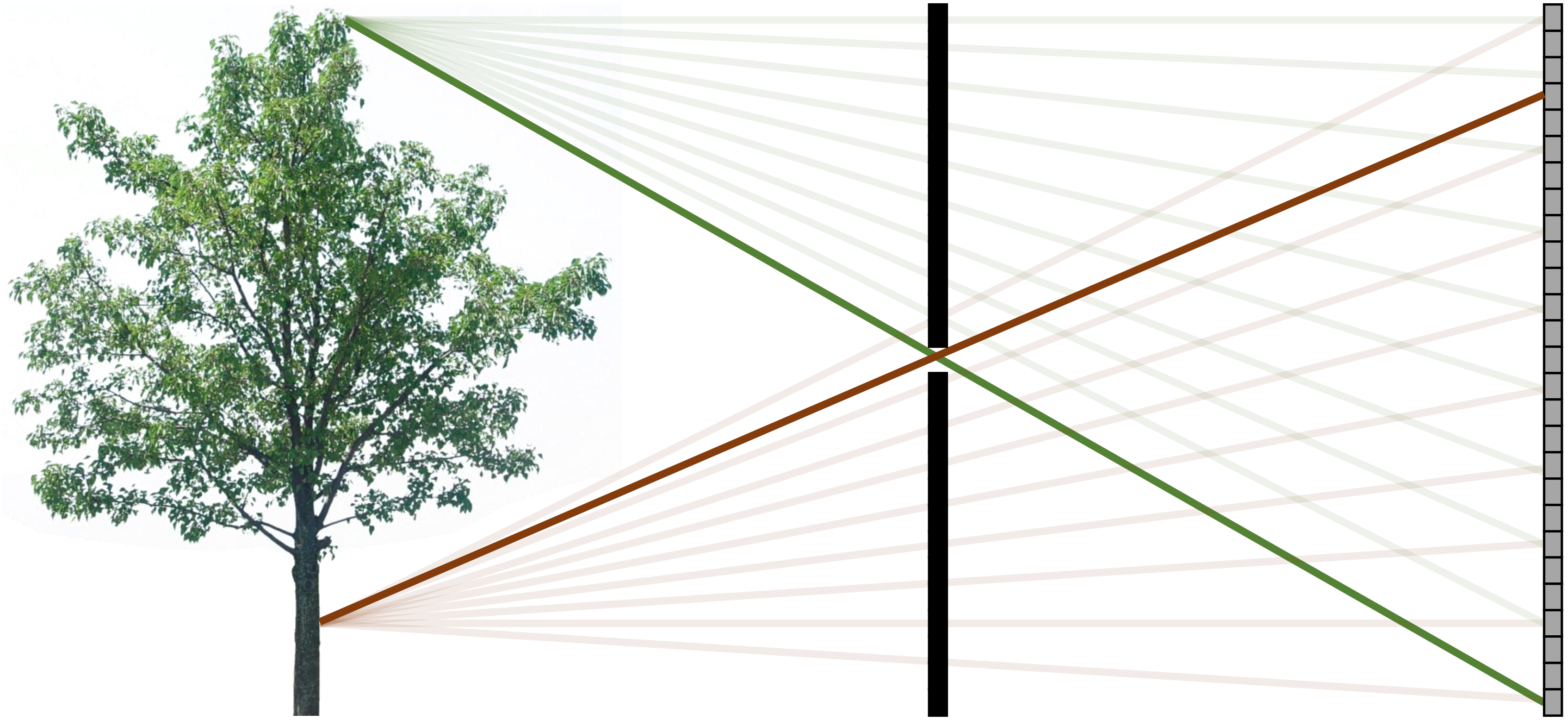


Pinhole imaging



Pinhole imaging

real-world object

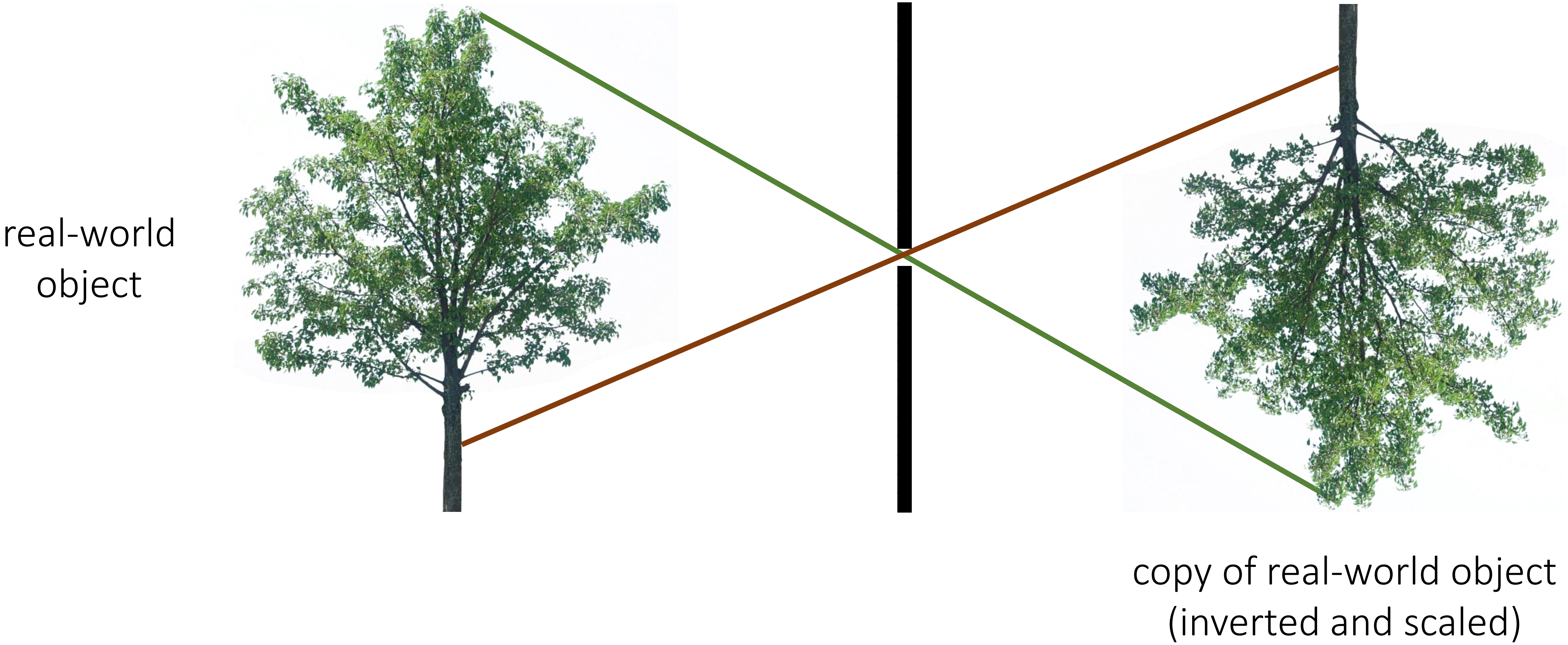


digital sensor (CCD or CMOS)

Each scene point contributes to only one sensor pixel

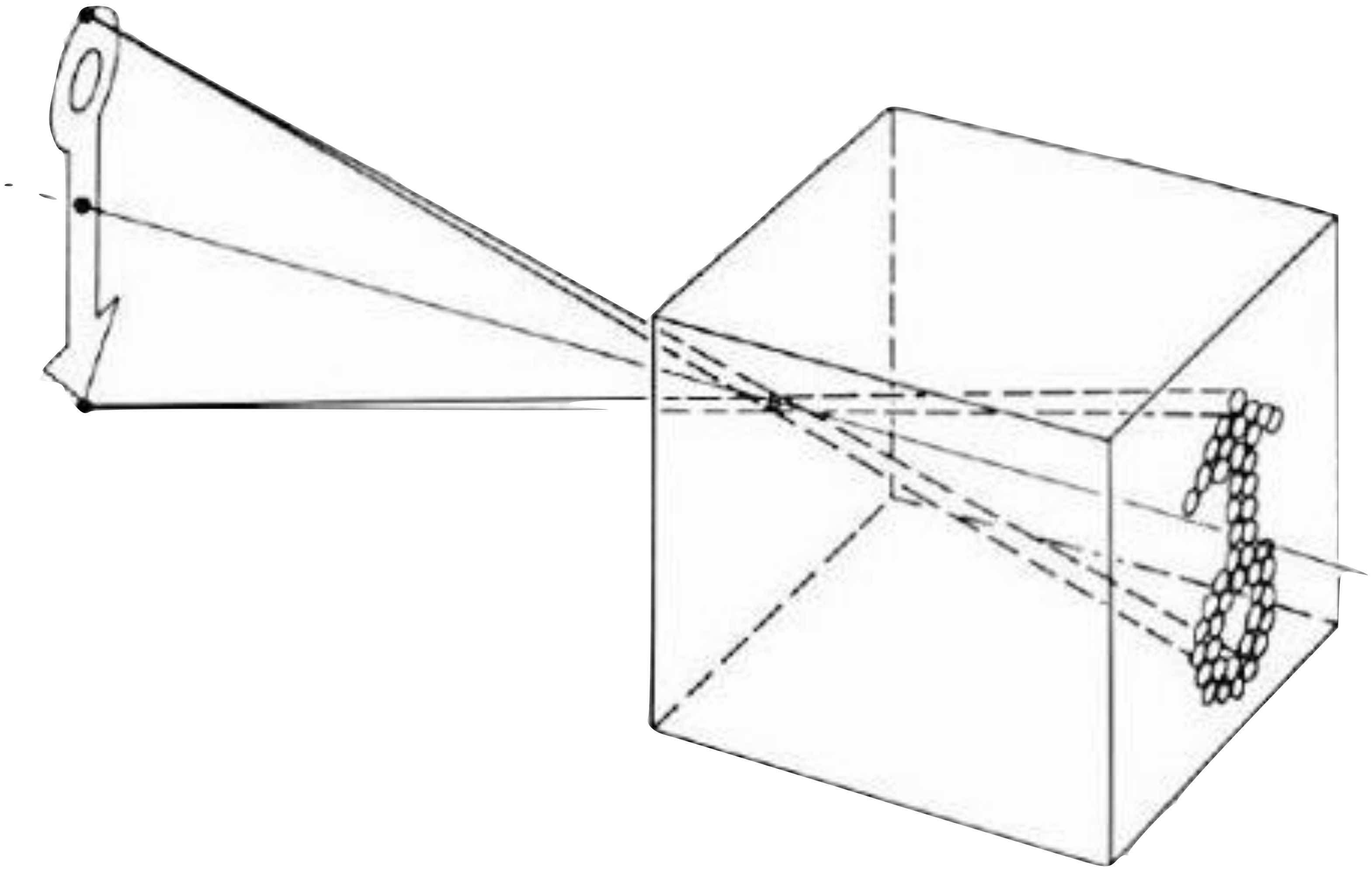
What does the image on the sensor look like?

Pinhole imaging



Pinhole camera

Pinhole camera a.k.a. camera obscura

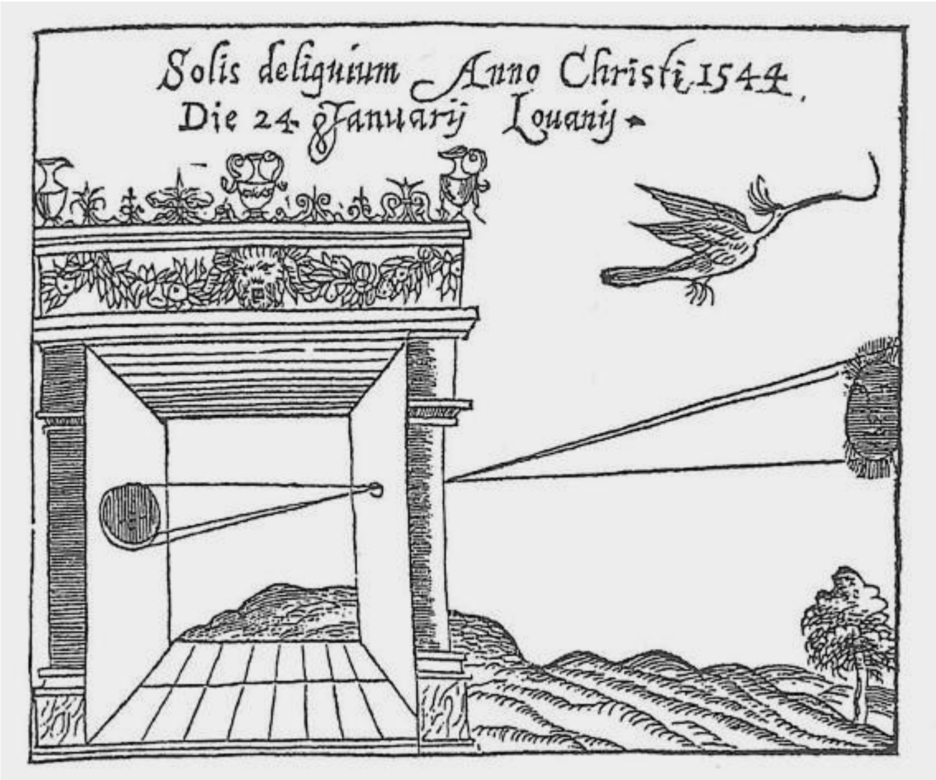


Pinhole camera a.k.a. camera obscura

First mention ...

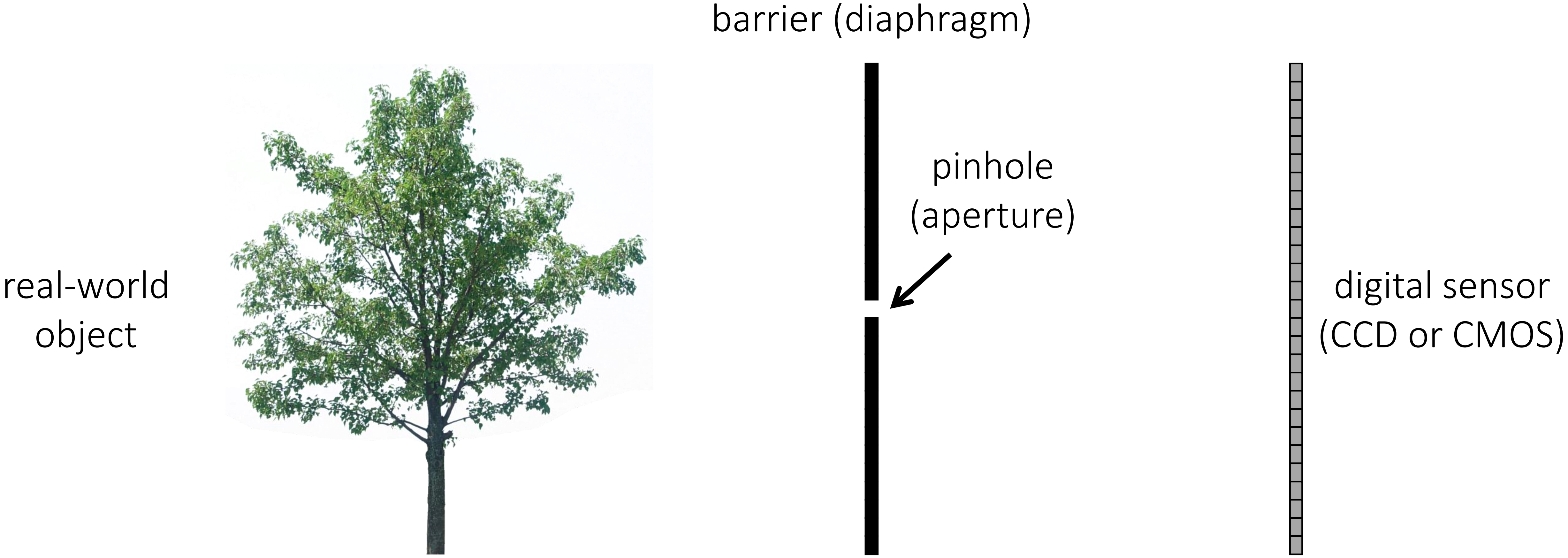
Chinese philosopher Mozi
(470 to 390 BC)

First camera ...

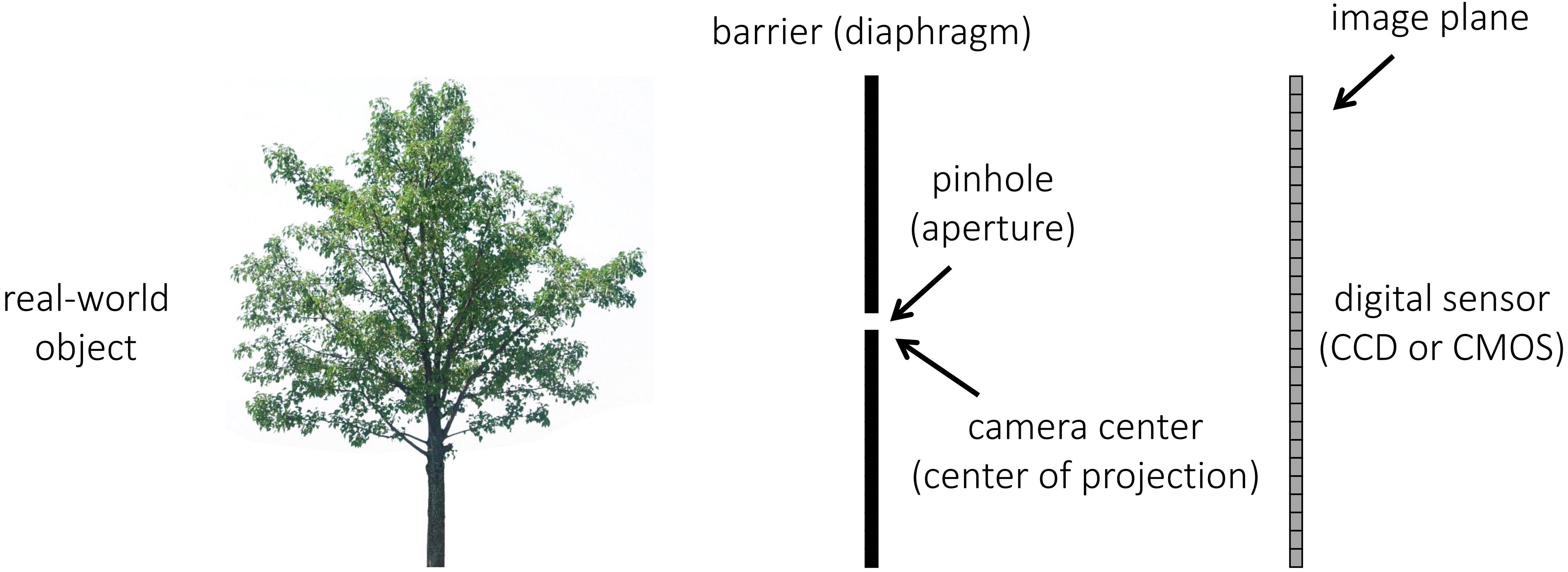


Greek philosopher Aristotle
(384 to 322 BC)

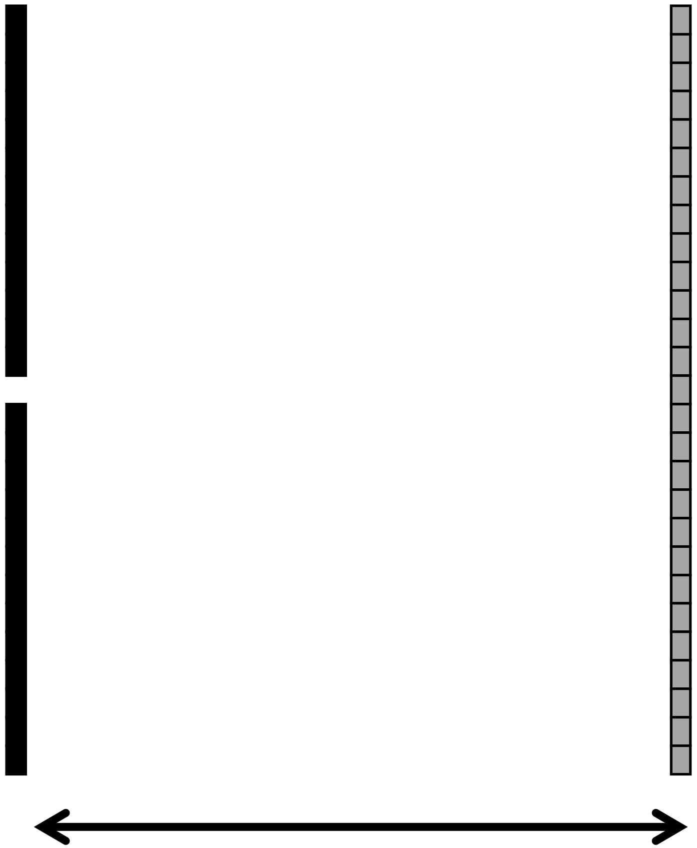
Pinhole camera terms



Pinhole camera terms



Focal length



Focal length

What happens as we change the focal length?

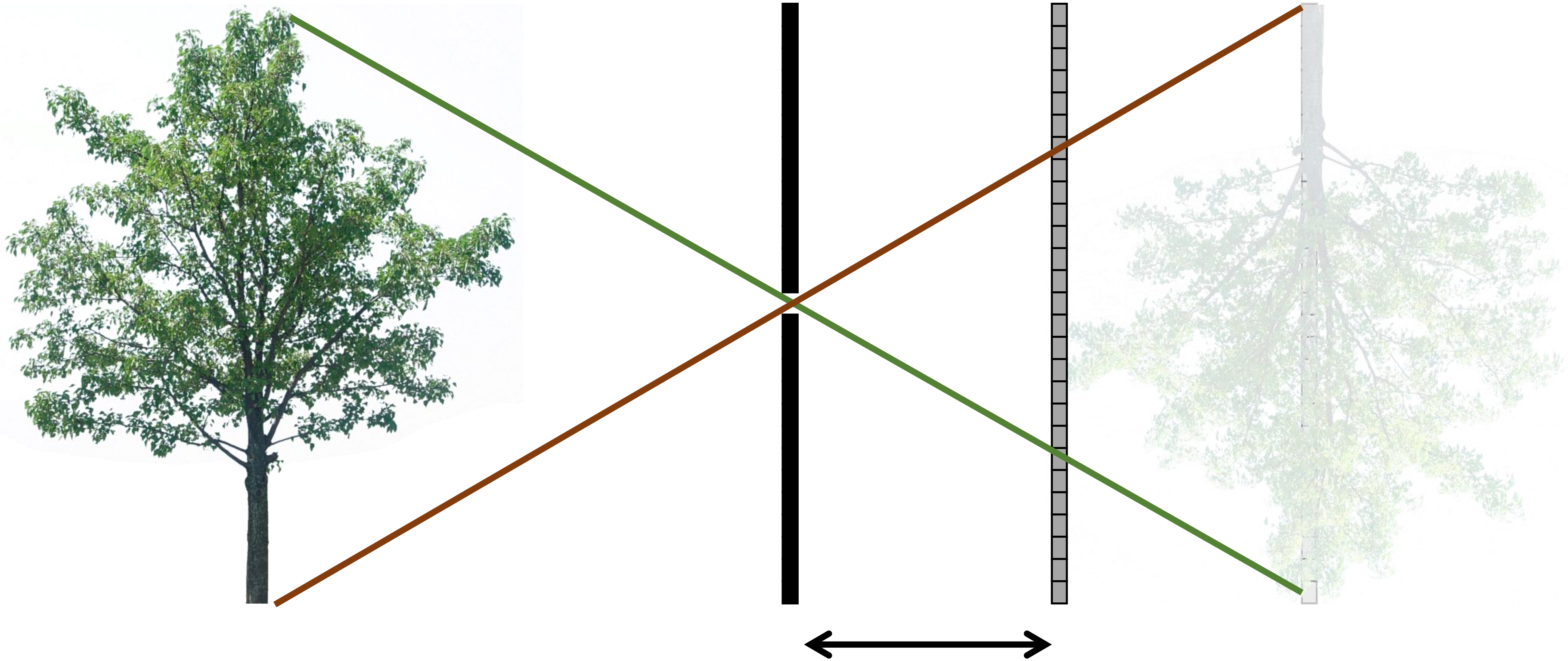
real-world object

focal length 0.5 f

Focal length

What happens as we change the focal length?

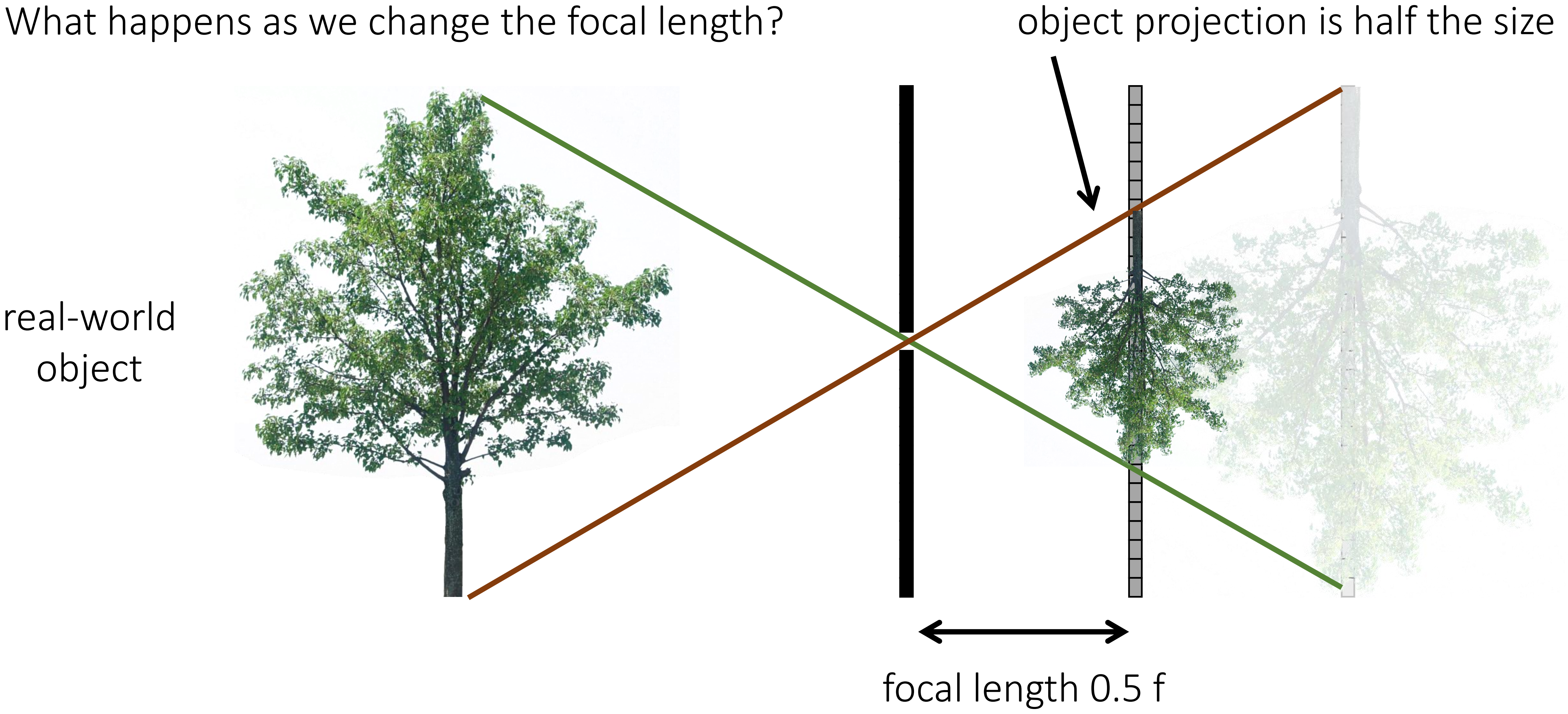
real-world
object



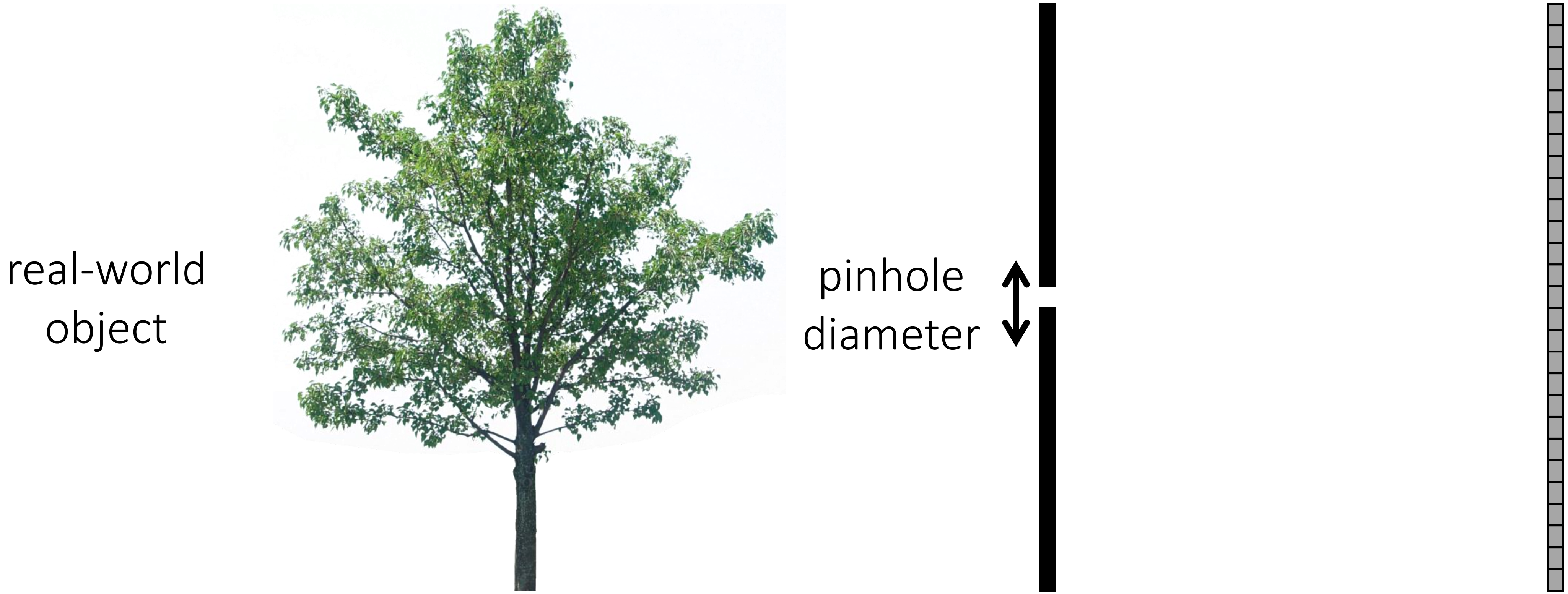
focal length 0.5 f

Focal length

What happens as we change the focal length?



Pinhole size

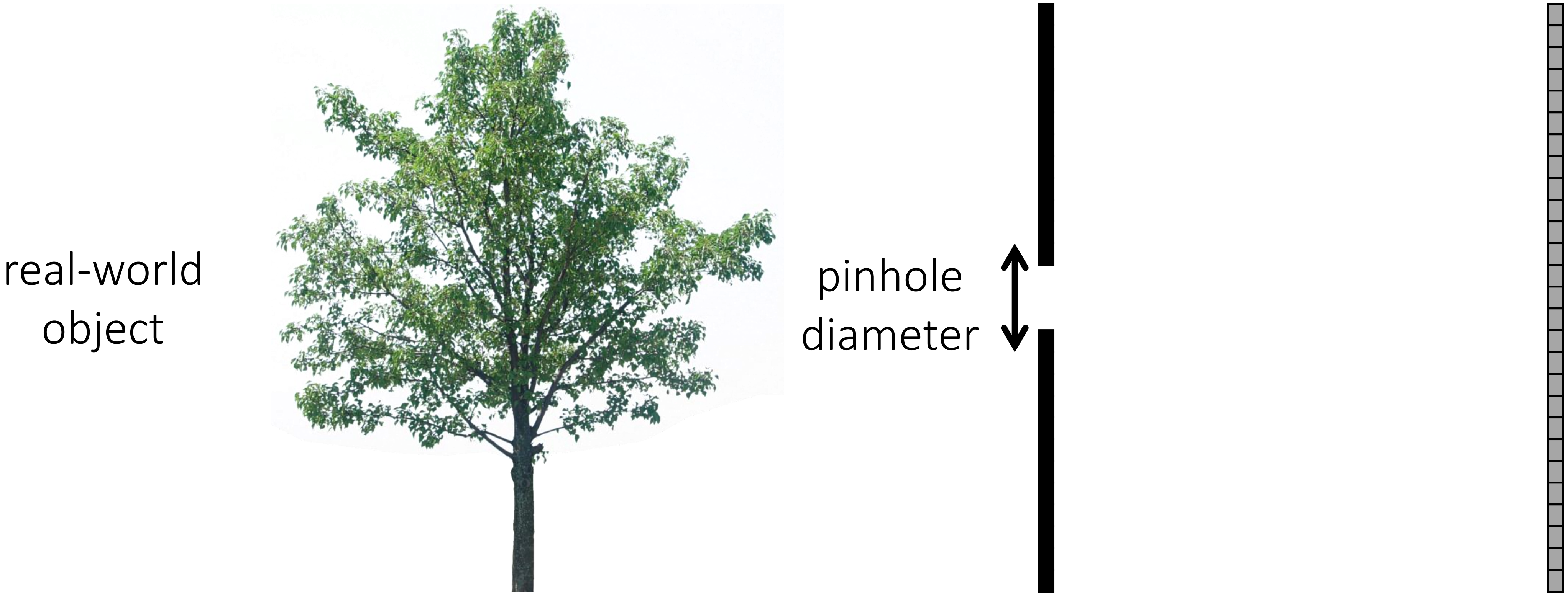


Ideal pinhole has infinitesimally small size

- In practice that is impossible.

Pinhole size

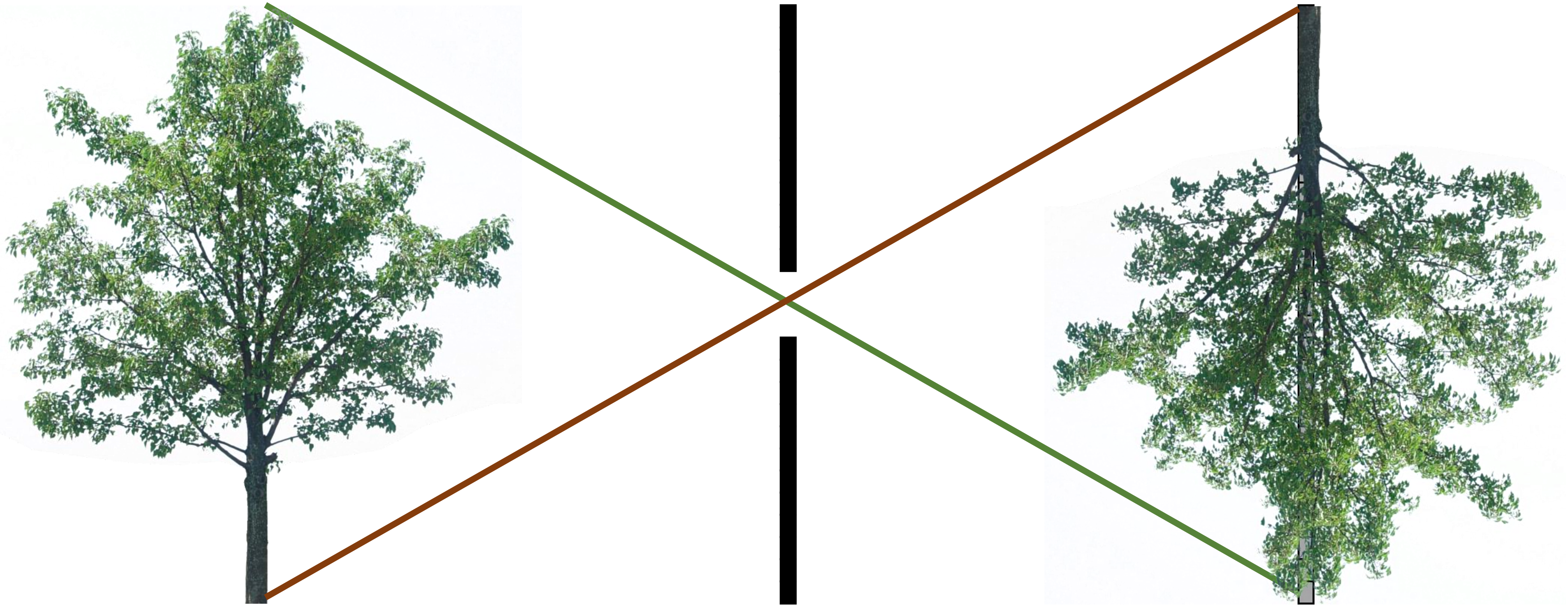
What happens as we change the pinhole diameter?



Pinhole size

What happens as we change the pinhole diameter?

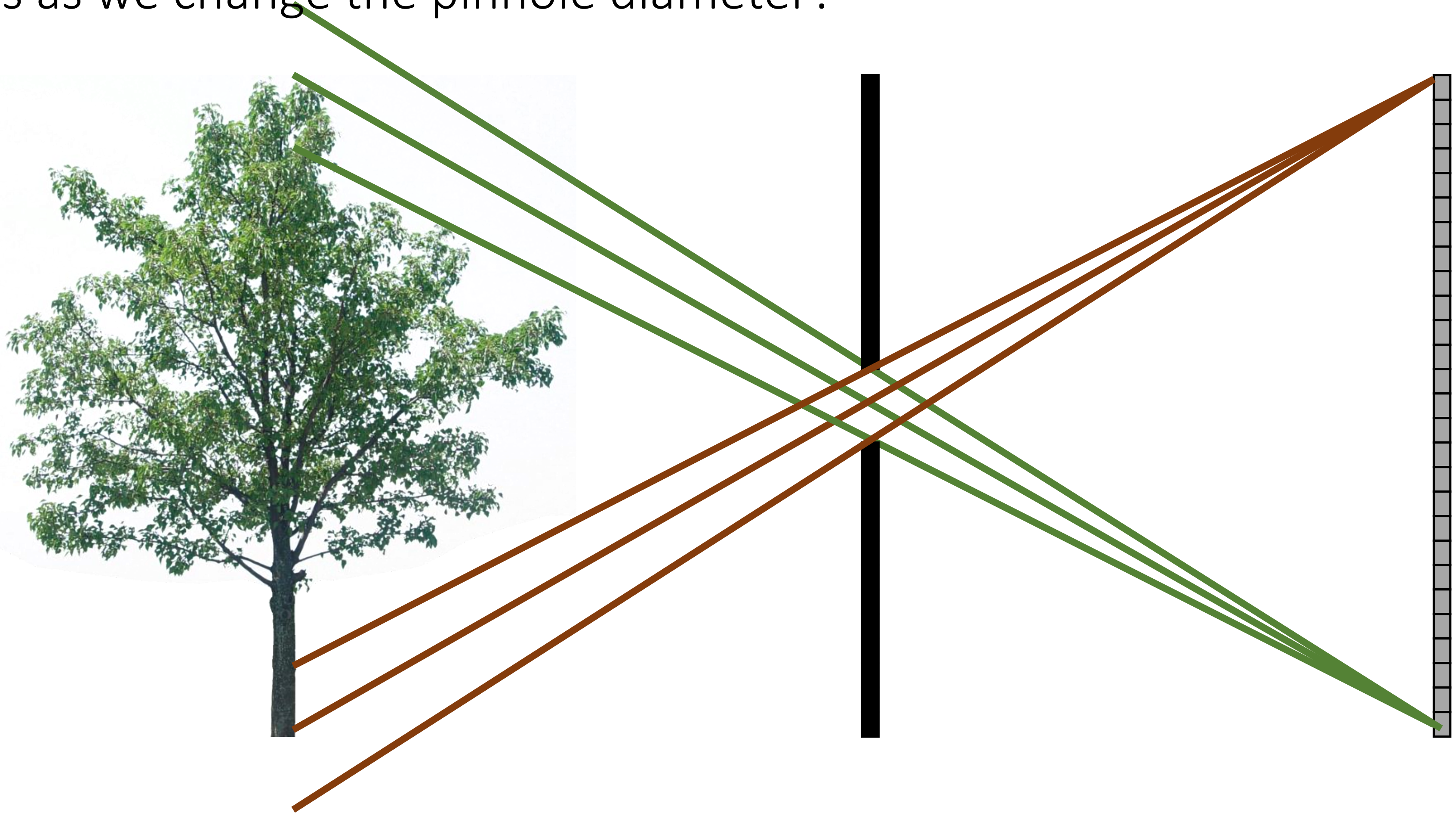
real-world object



Pinhole size

What happens as we change the pinhole diameter?

real-world object

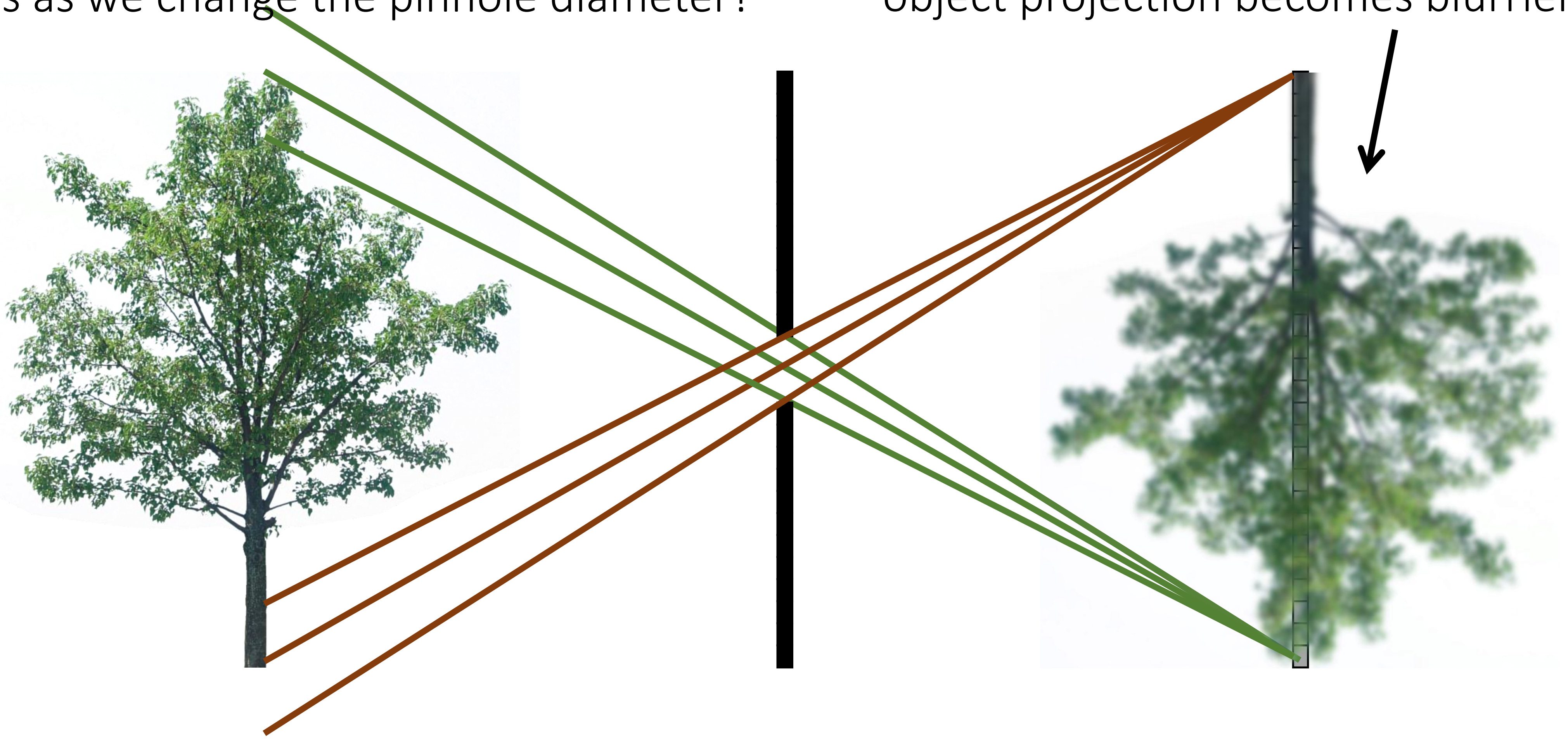


Pinhole size

What happens as we change the pinhole diameter?

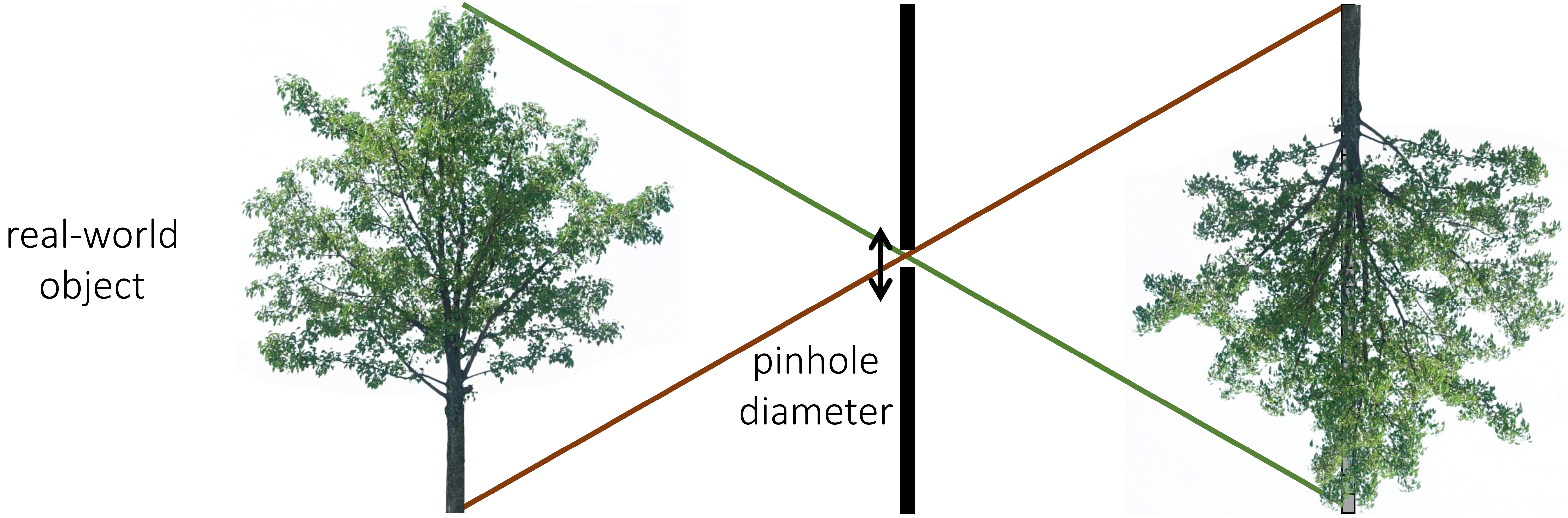
object projection becomes blurrier

real-world
object



Pinhole size

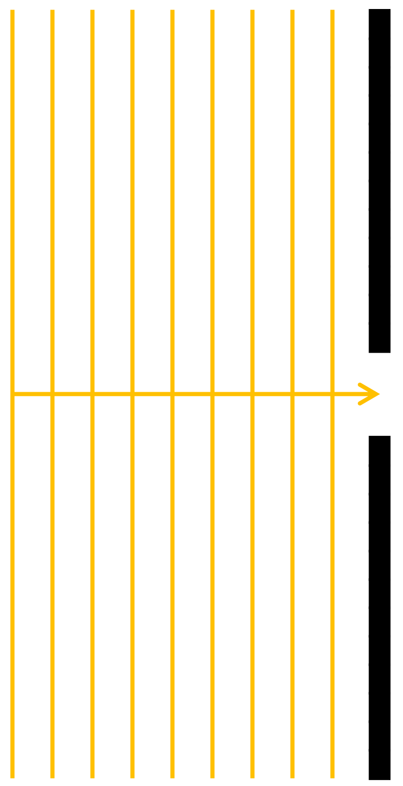
What happens as we change the pinhole diameter?



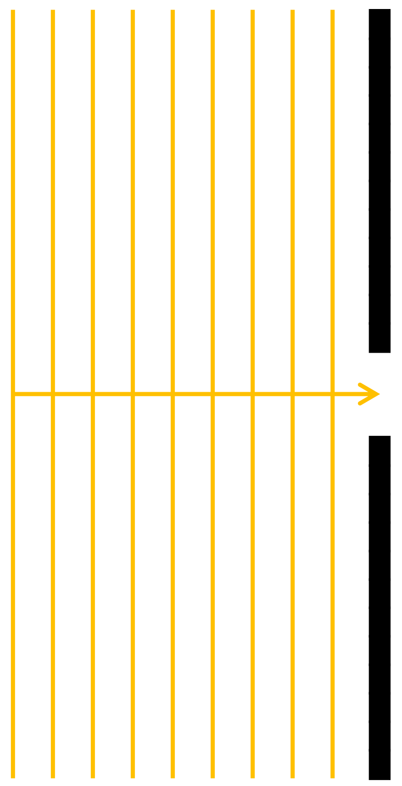
Will the image keep getting sharper the smaller we make the pinhole?

Diffraction limit

A consequence of the wave nature of light



What do geometric optics predict will happen?



What do wave optics predict will happen?

Diffraction limit

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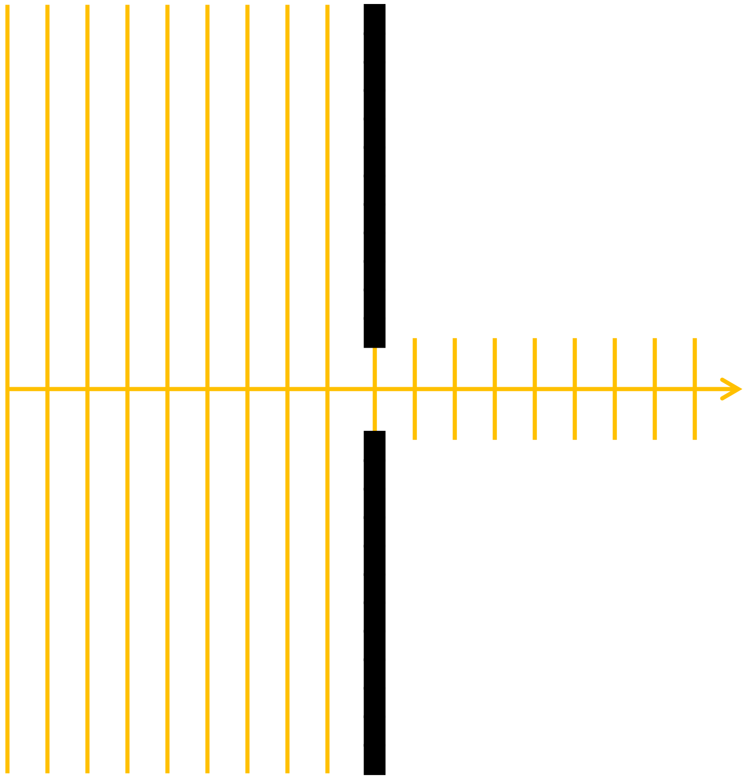

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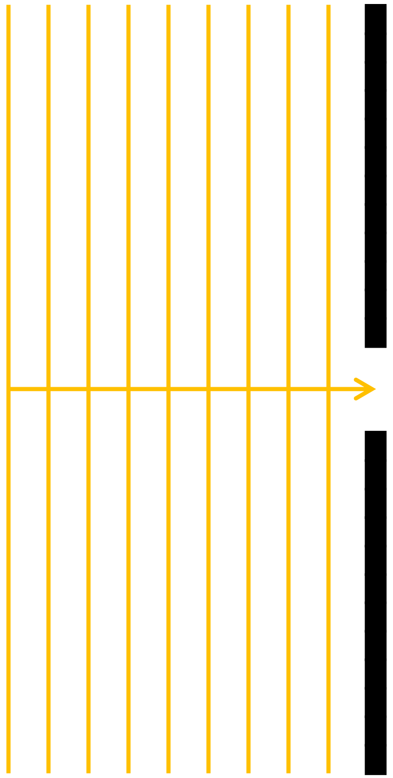

What do wave optics predict will happen?

Diffraction limit

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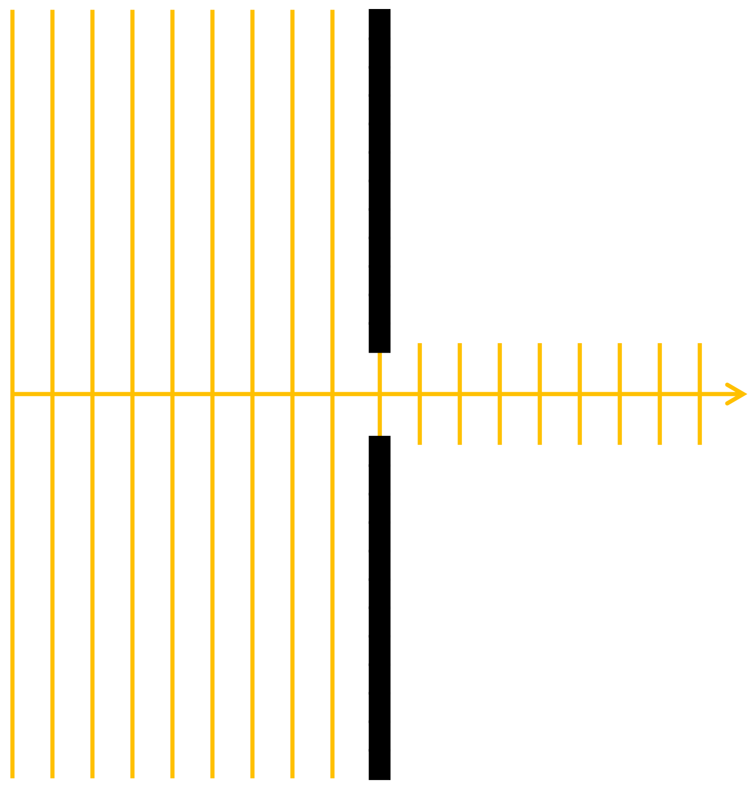

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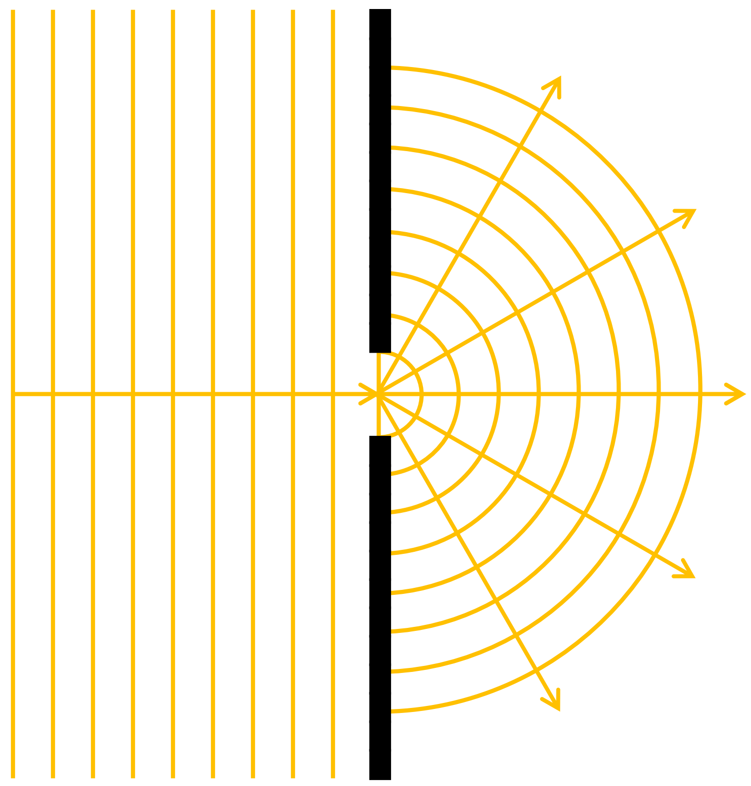
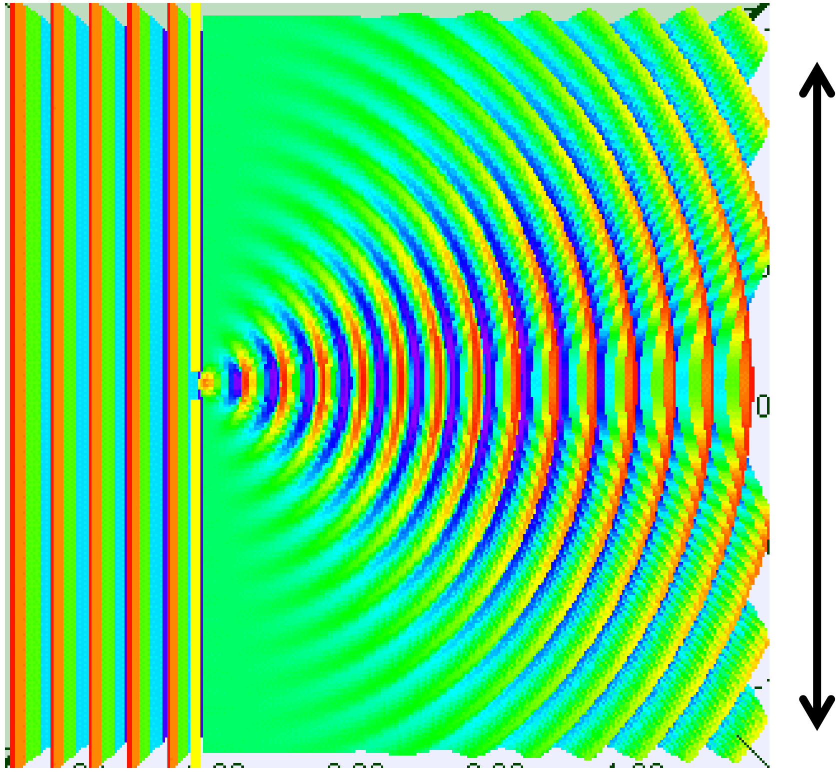


What do wave optics predict will happen?

Diffraction limit

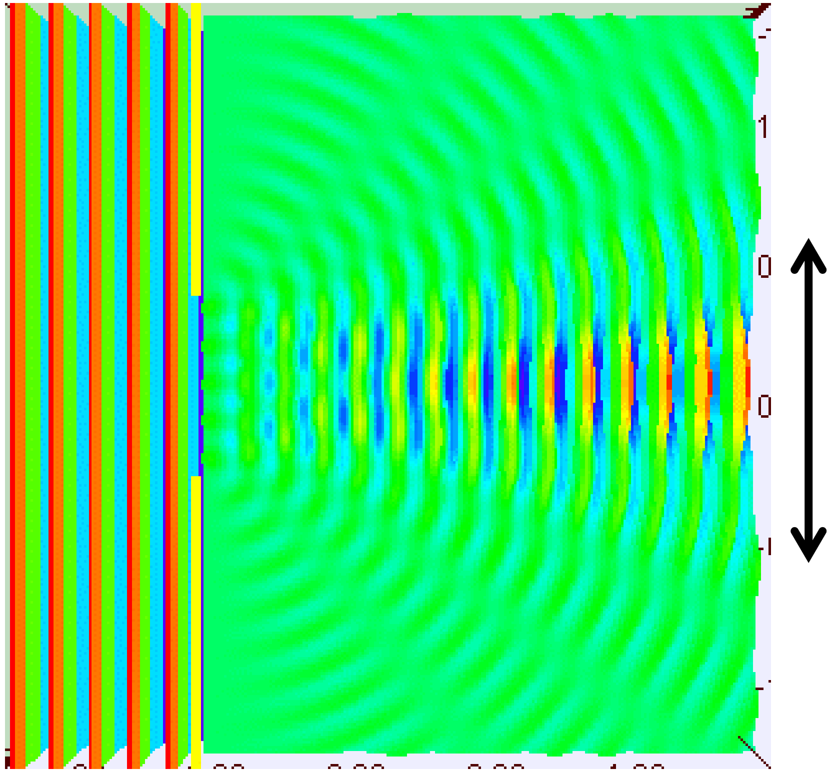
Diffraction pattern = Fourier transform of the pinhole.

- Smaller pinhole means bigger Fourier spectrum.
- Smaller pinhole means more diffraction.



wide diffraction pattern

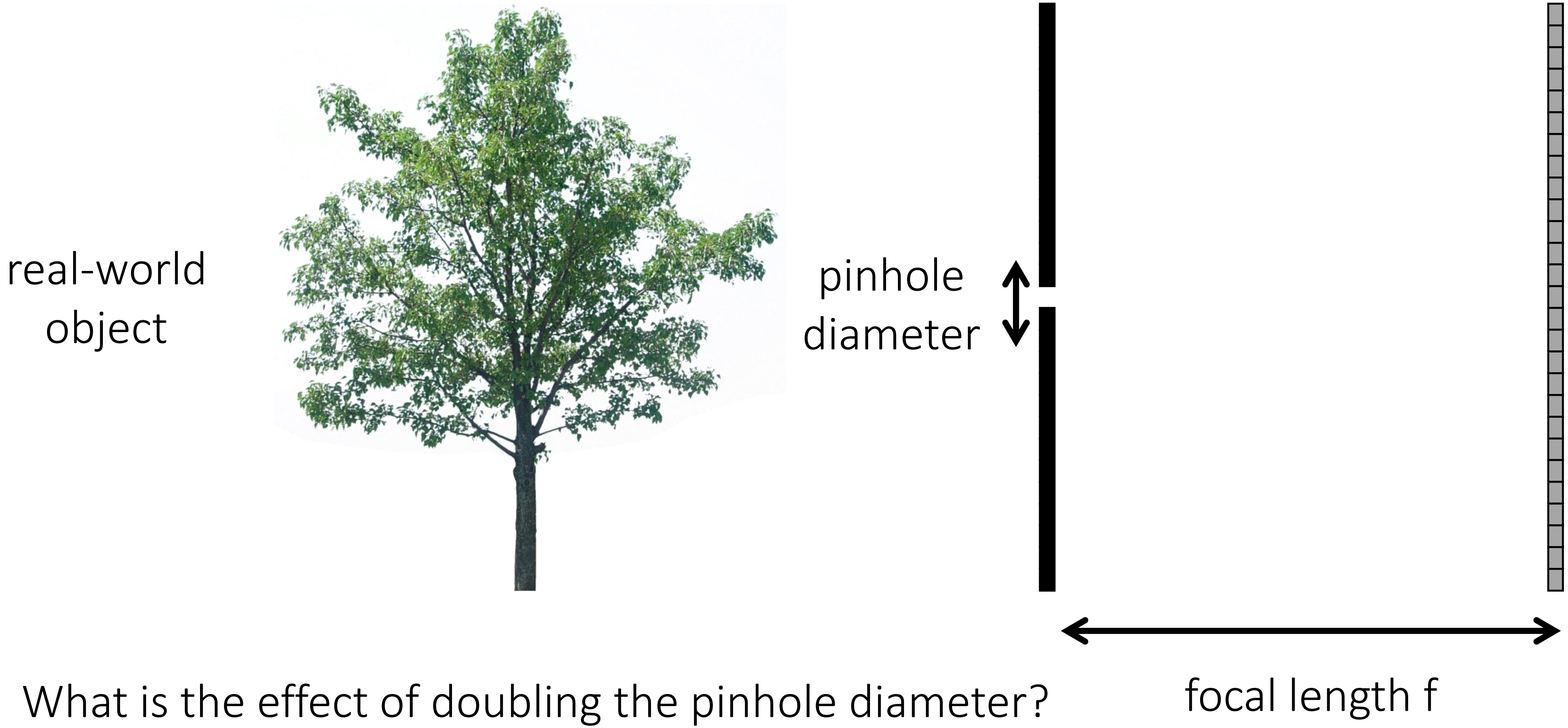
small pinhole



narrow diffraction pattern

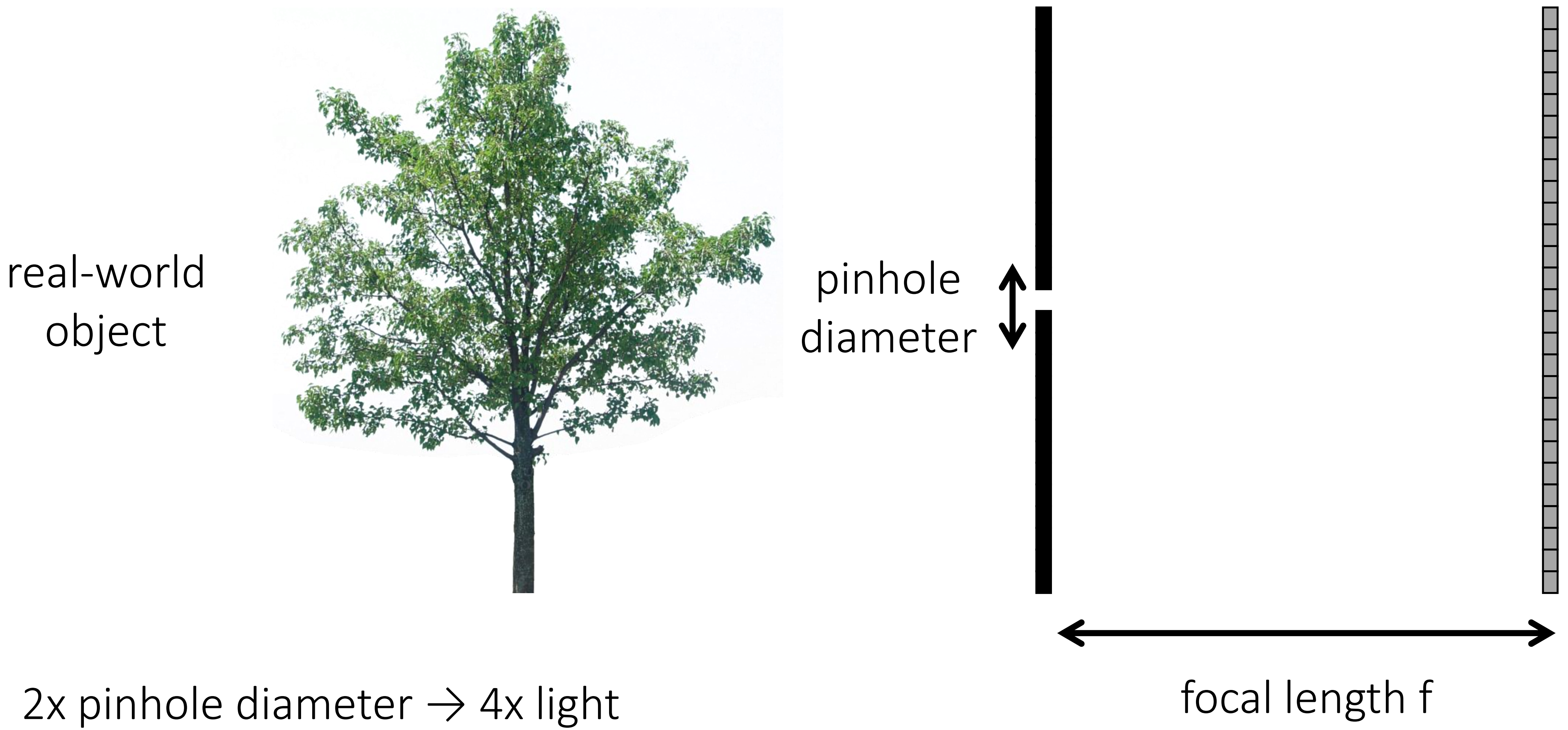
large pinhole

What about light efficiency?



- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?

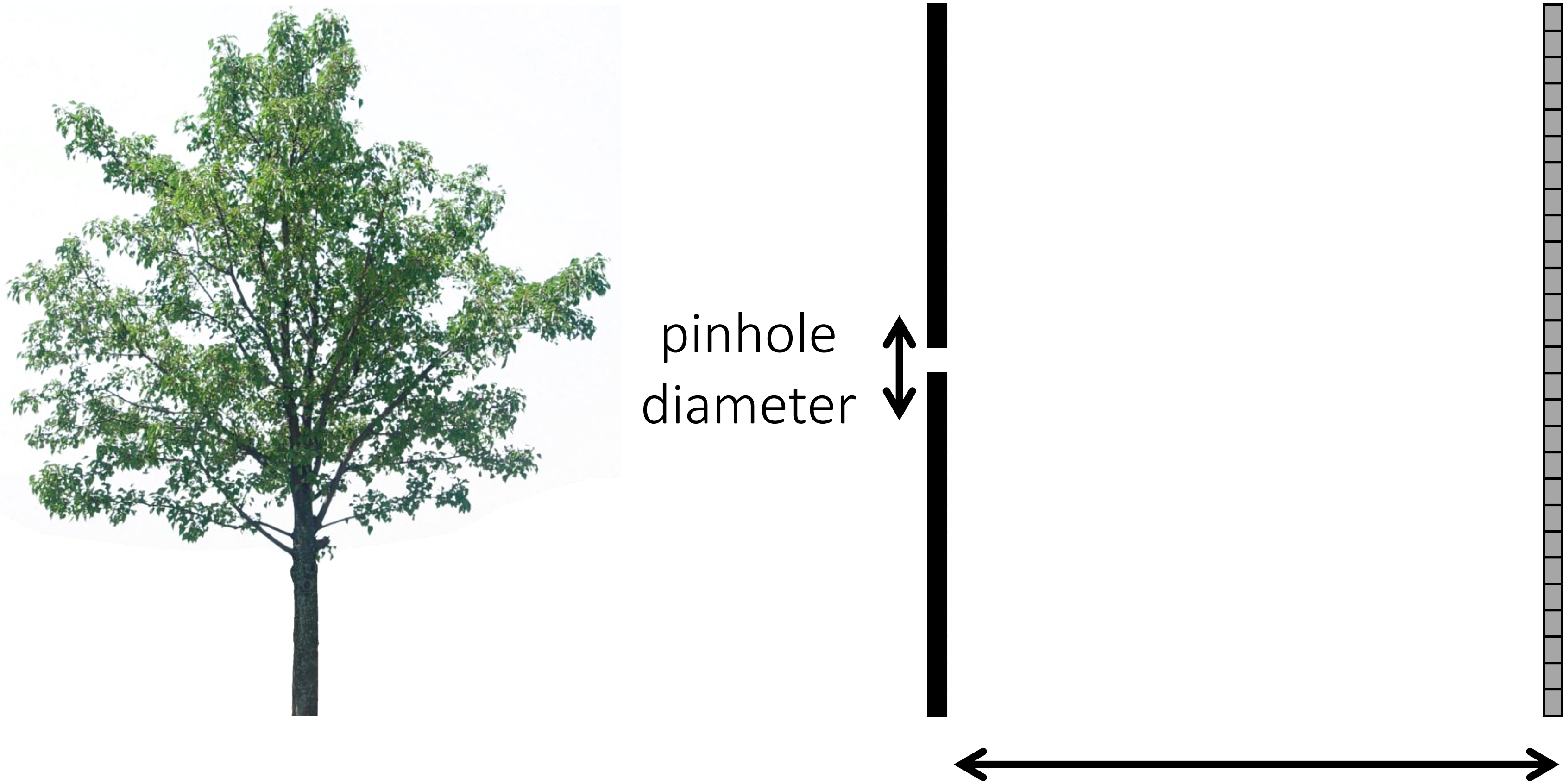
What about light efficiency?



- 2x pinhole diameter → 4x light
- 2x focal length → ¼x light

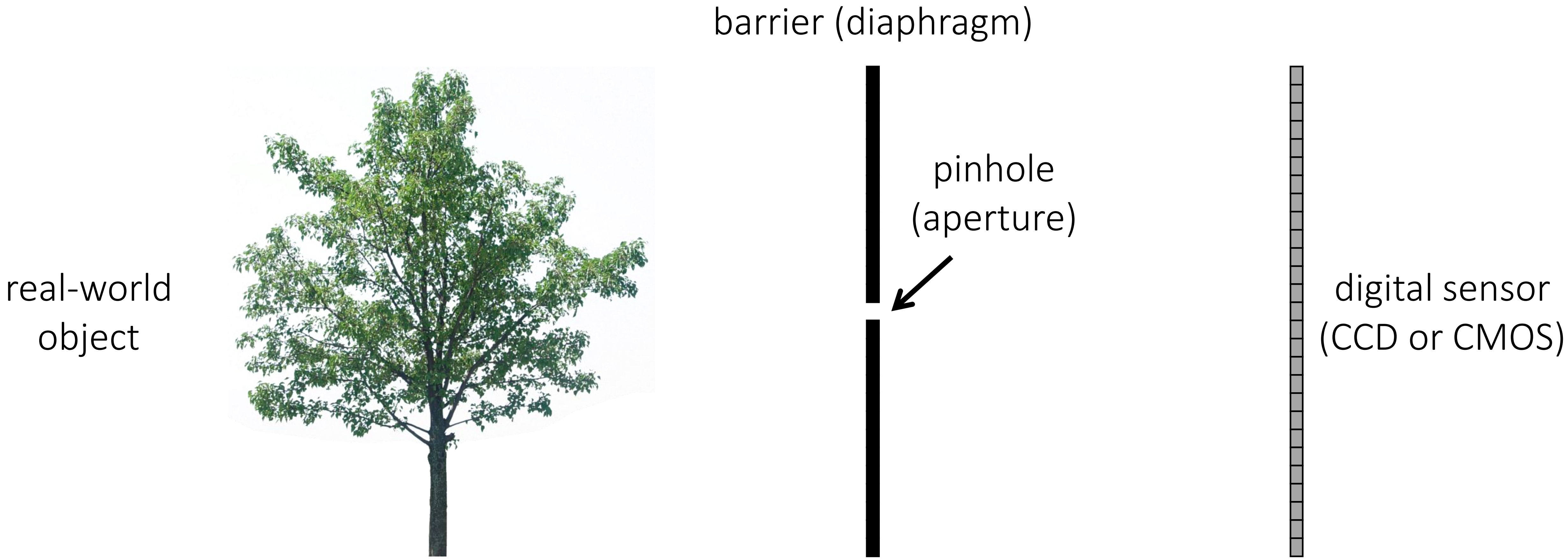
Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2



The “f-number” is the ratio: focal length / pinhole diameter

Can we do better than pinhole imaging?



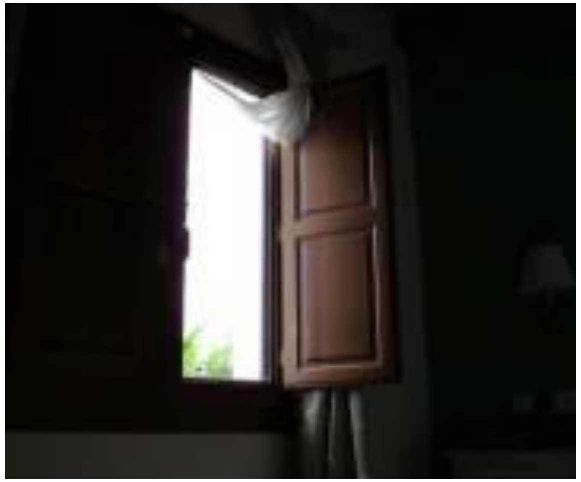
Accidental pinholes

What does this image say about the world outside?

Accidental pinhole camera

Antonio Torralba, William T. Freeman
Computer Science and Artificial Intelligence Laboratory (CSAIL)
MIT
torralba@mit.edu, billf@mit.edu

Accidental pinhole camera



window is an aperture

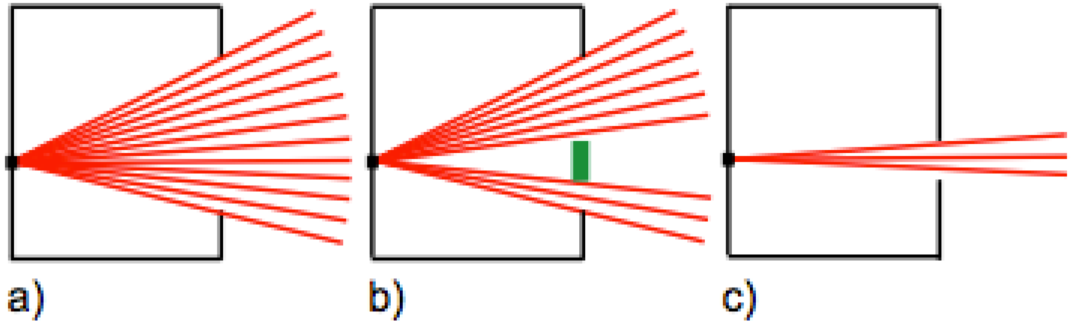
projected pattern on the wall

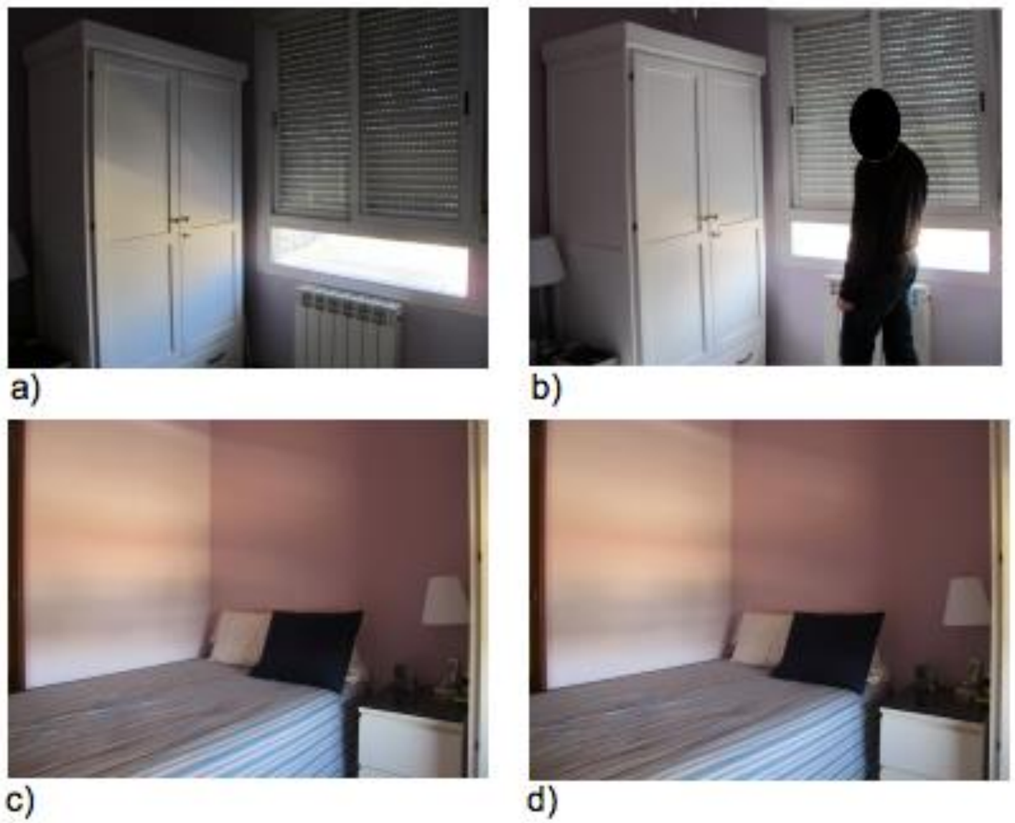
upside down

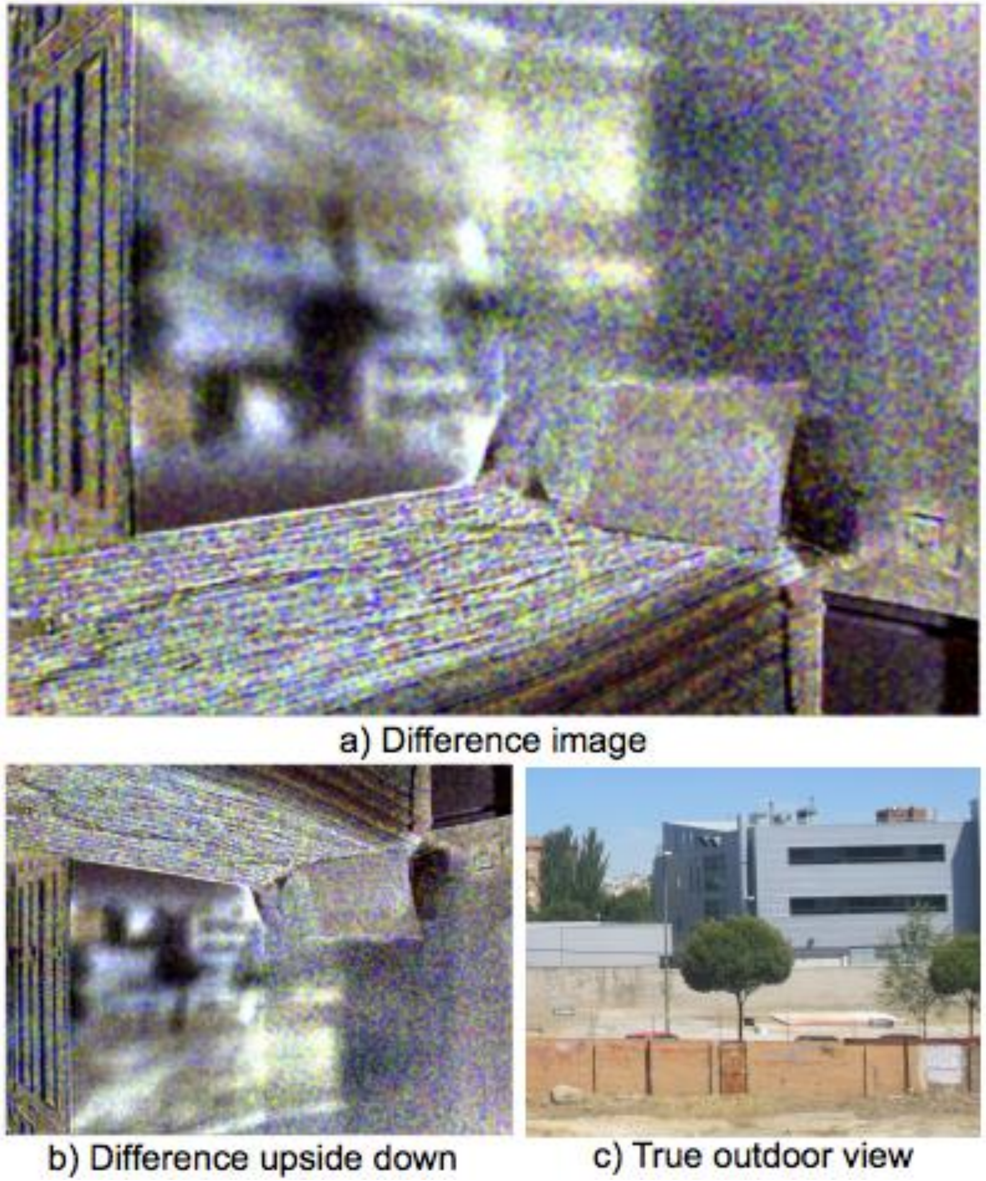
window with smaller gap

view outside window

Accidental pinspeck camera







Camera matrix

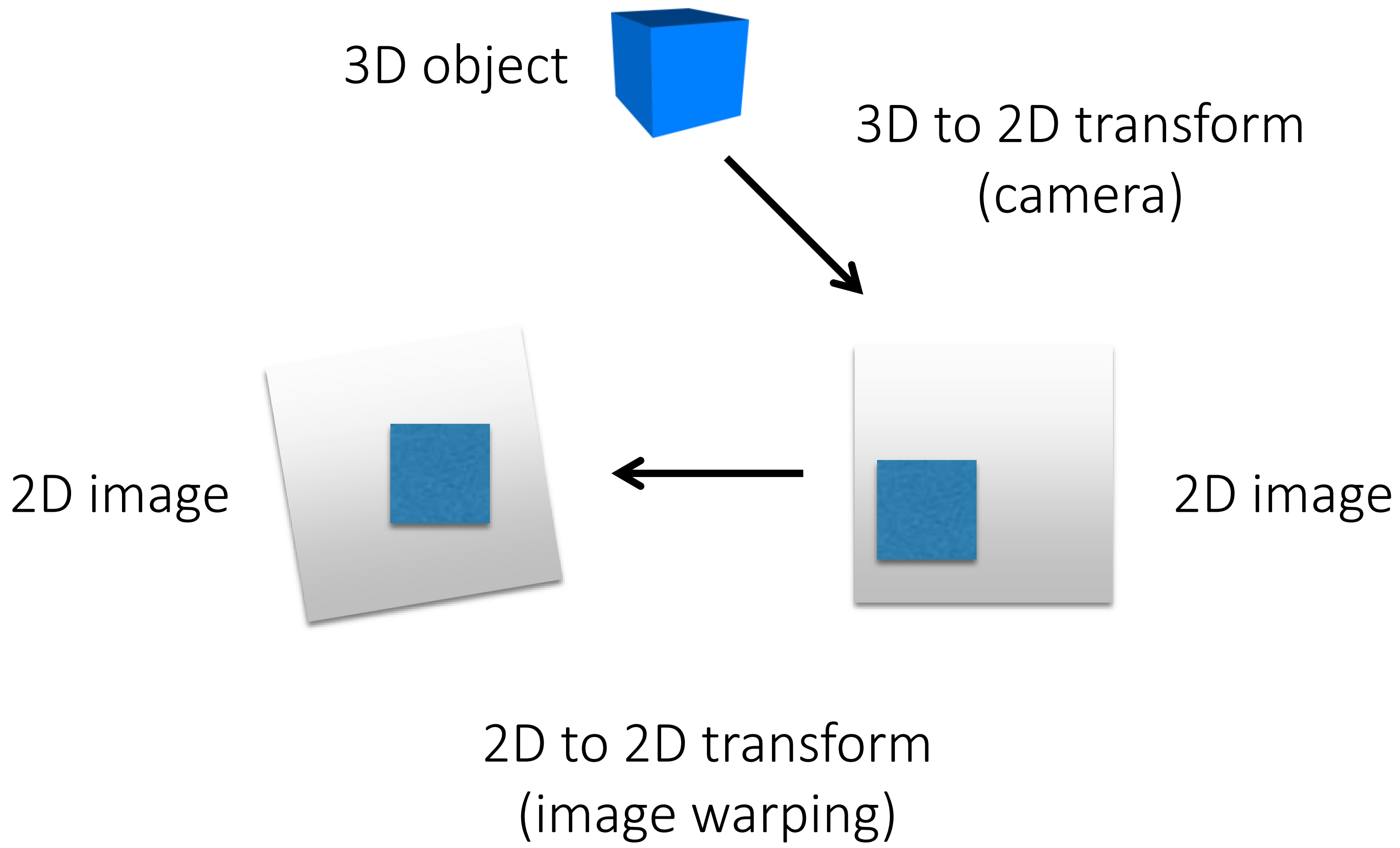
The camera as a coordinate transformation

A camera is a mapping from:

the 3D world

to:

a 2D image



The camera as a coordinate transformation

A camera is a mapping from:

the 3D world

to:

a 2D image

homogeneous coordinates

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

2D image point

camera matrix

3D world point

What are the dimensions of each variable?

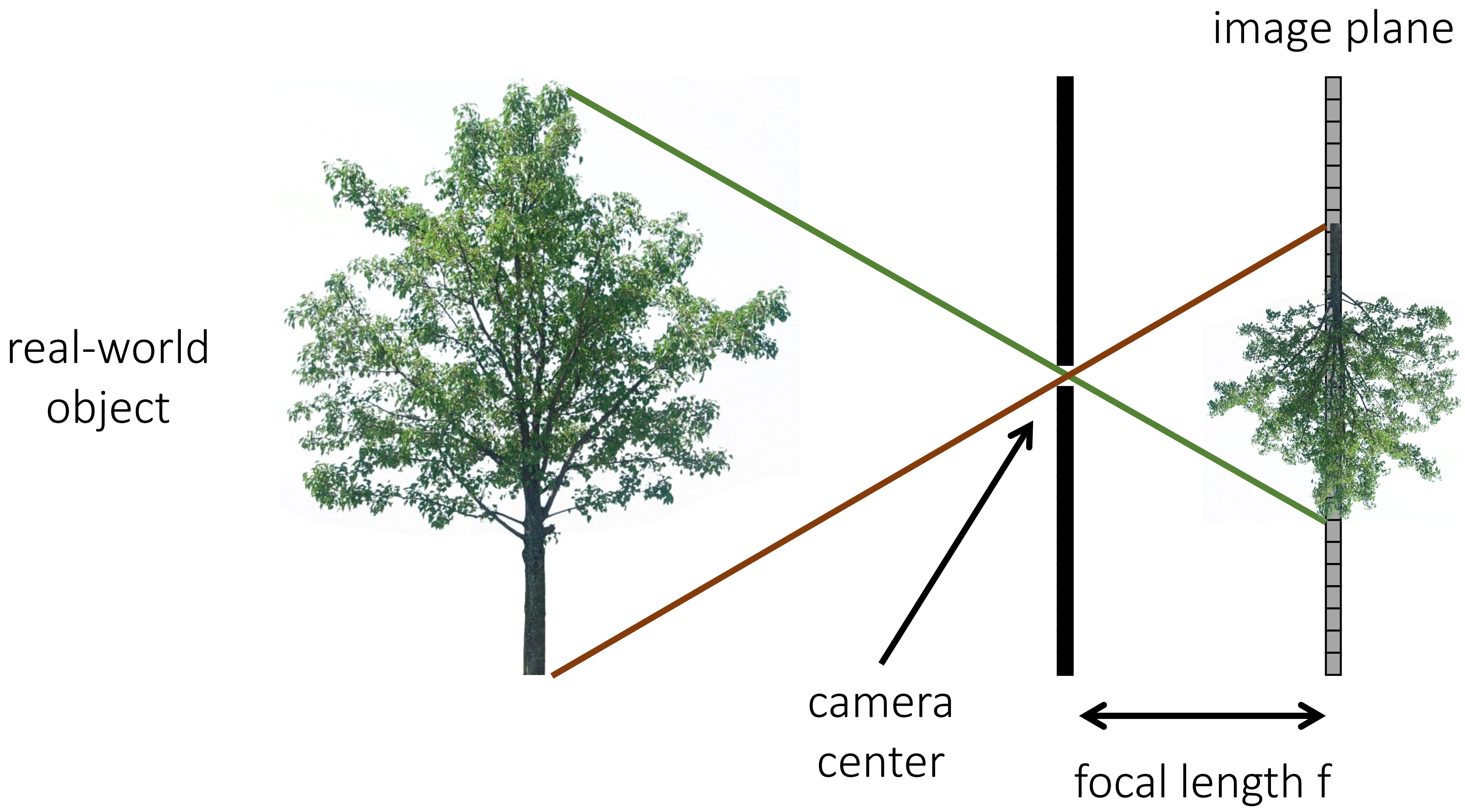
The camera as a coordinate transformation

$$\boldsymbol{x} = \mathbf{PX}$$

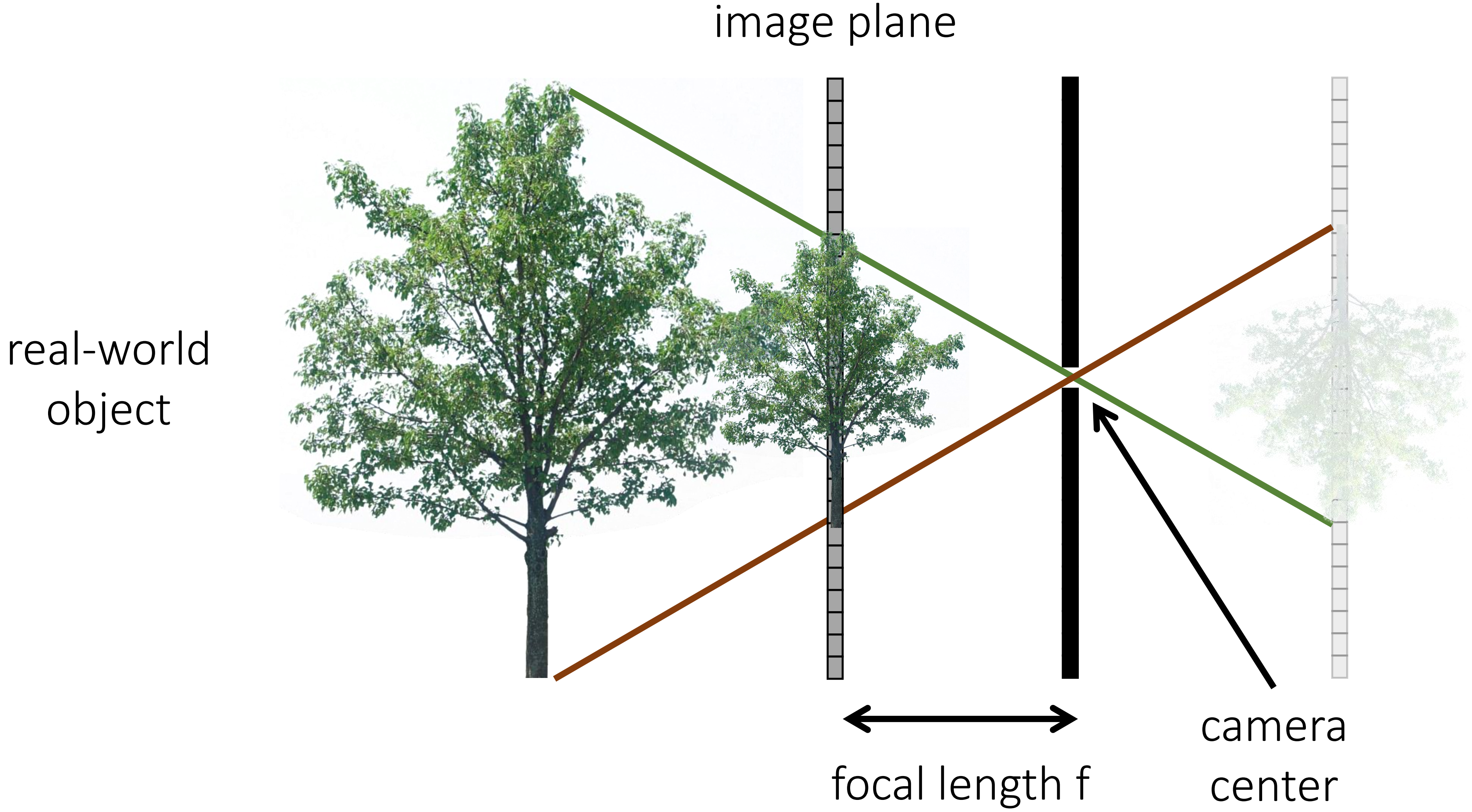
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

homogeneous image coordinates	camera matrix	homogeneous world coordinates
3 x 1	3 x 4	4 x 1

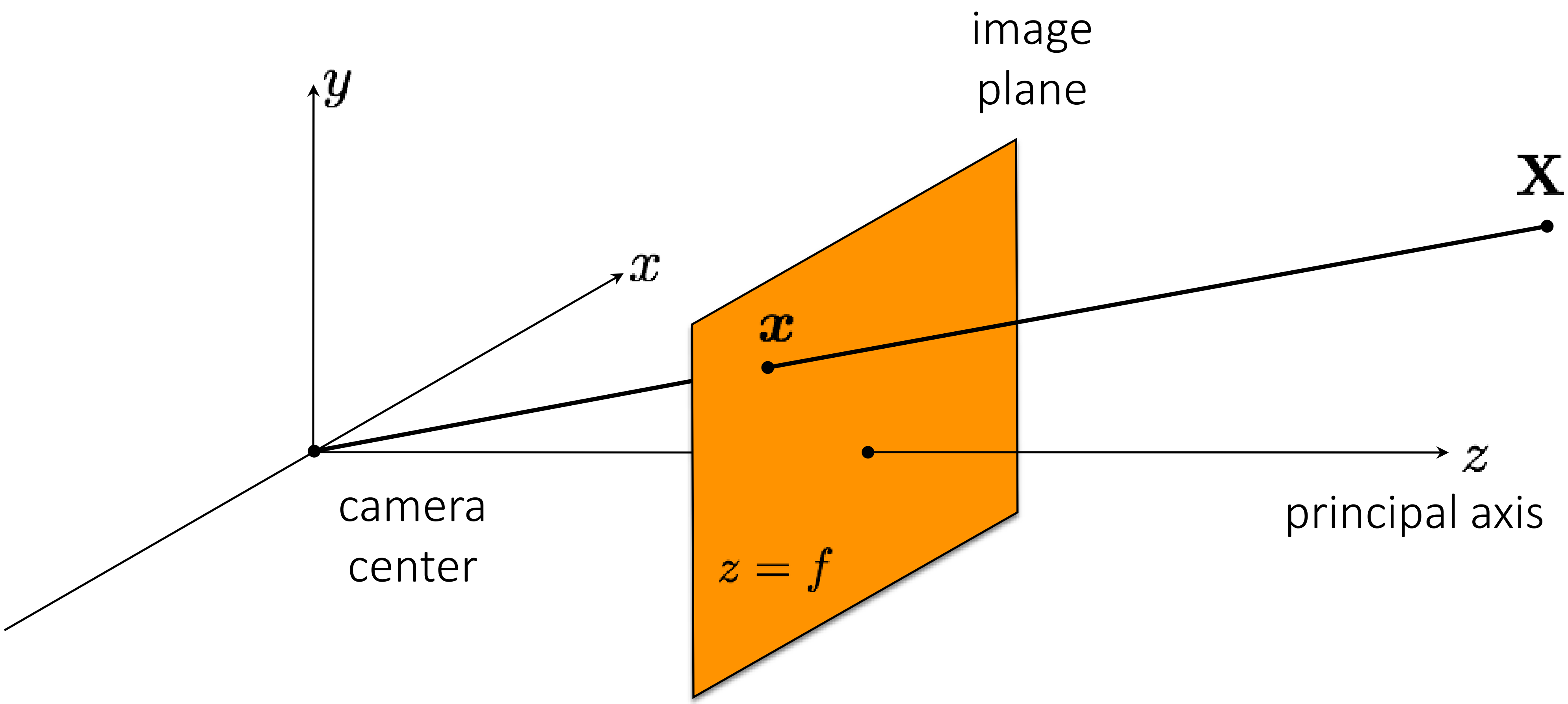
The pinhole camera



The (rearranged) pinhole camera

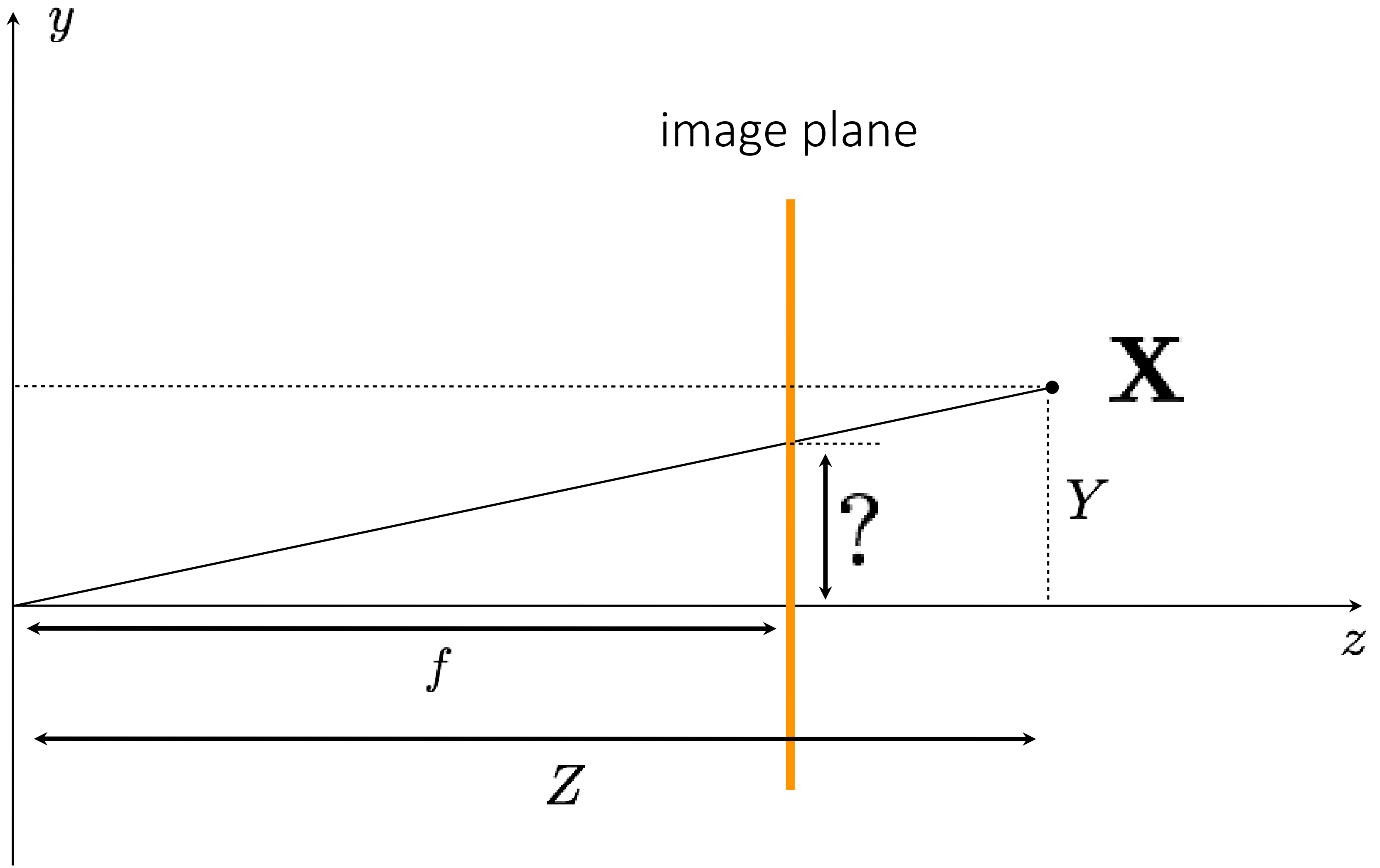


The (rearranged) pinhole camera



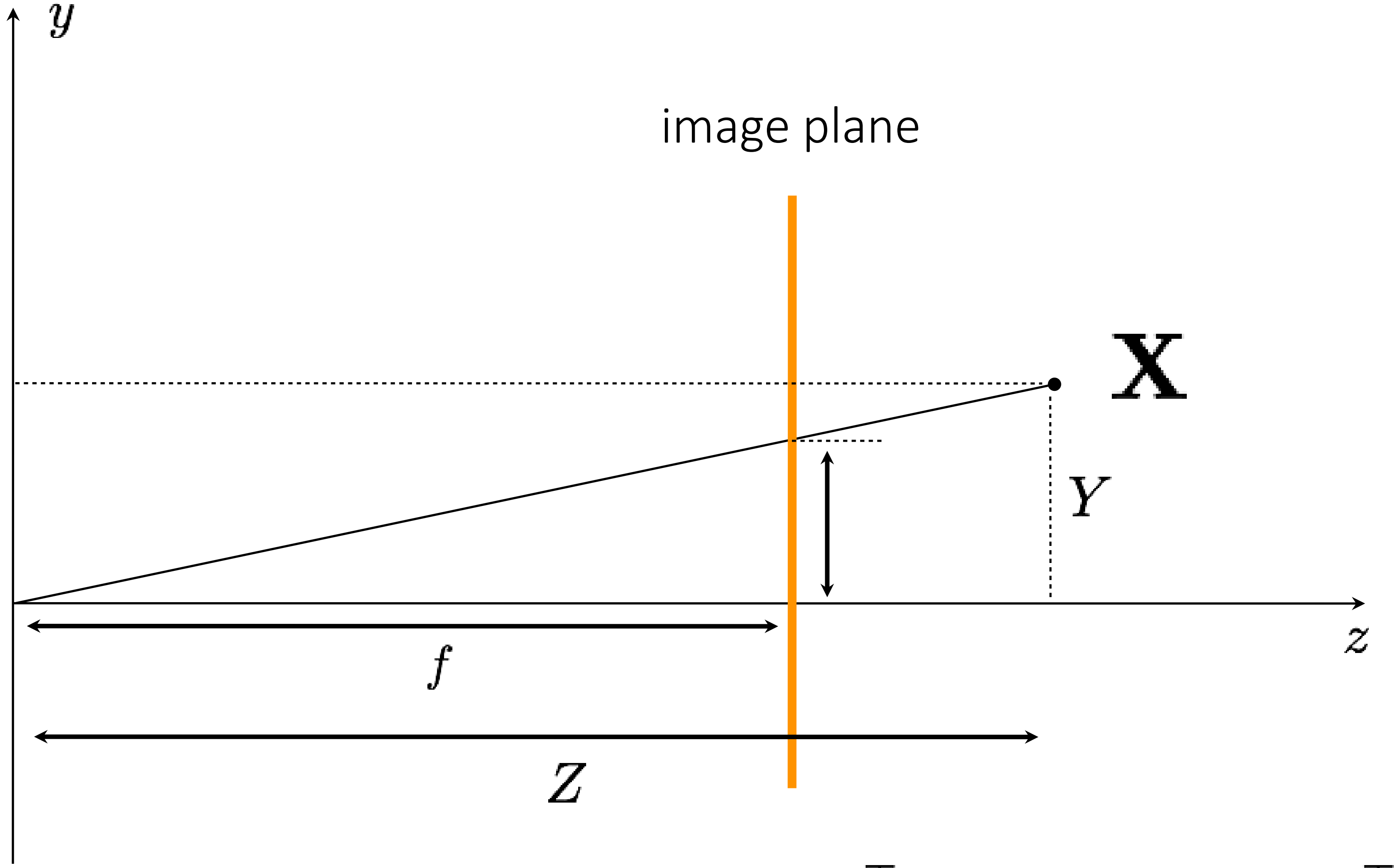
What is the equation for image coordinate **x** in terms of **X**?

The 2D view of the (rearranged) pinhole camera



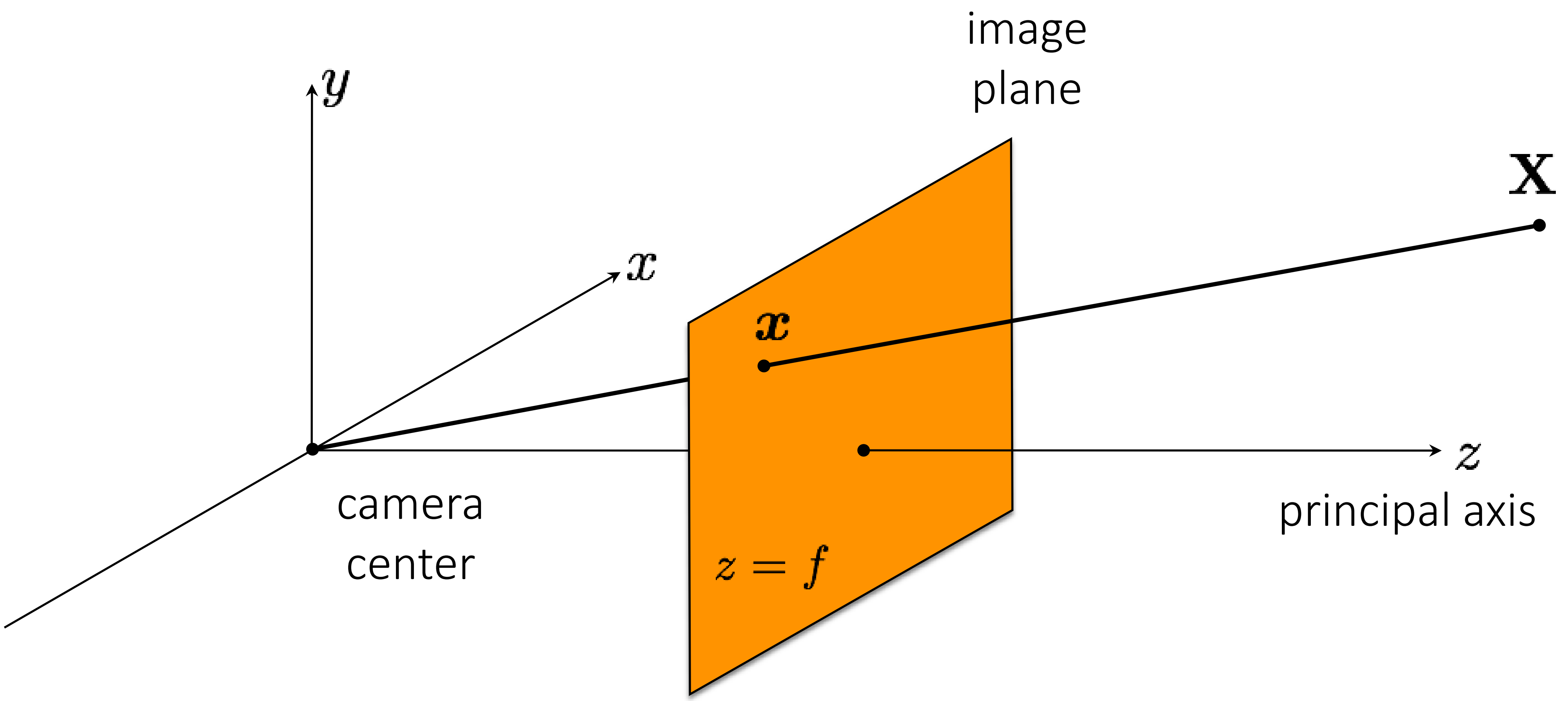
What is the equation for image coordinate **x** in terms of **X**?

The 2D view of the (rearranged) pinhole camera



$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

The (rearranged) pinhole camera



What is the camera matrix **P** for a pinhole camera?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

The pinhole camera matrix

Relationship from similar triangles:

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

General camera model:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What does the pinhole camera projection look like?

$$\mathbf{P} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

The pinhole camera matrix

Relationship from similar triangles:

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

General camera model:

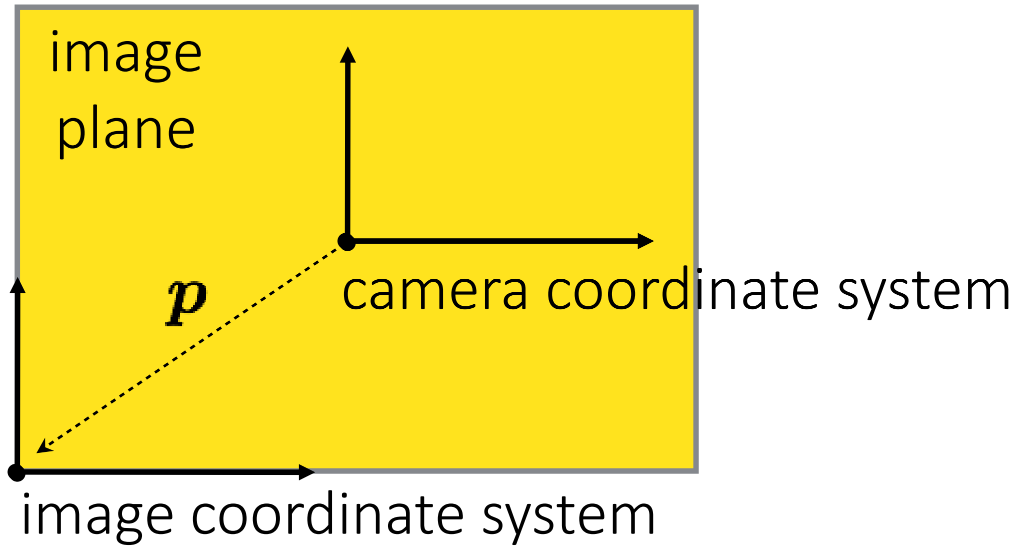
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What does the pinhole camera projection look like?

$$\mathbf{P} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Generalizing the camera matrix

Camera origin and image origin might be different

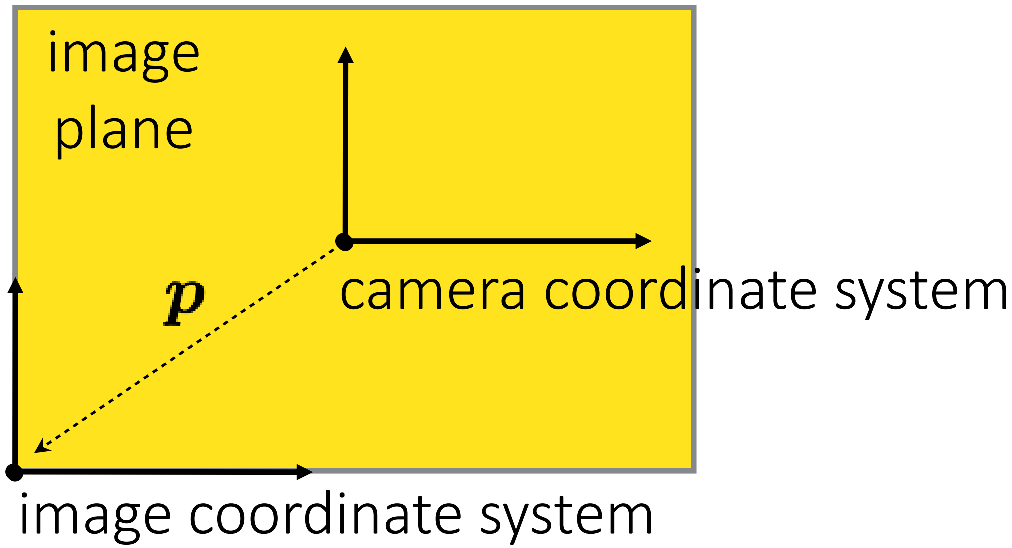


How does the camera matrix change?

$$\mathbf{P} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Generalizing the camera matrix

Camera origin and image origin might be different



How does the camera matrix change?

$$\mathbf{P} = \left[\begin{array}{cccc} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

Camera matrix decomposition

We can decompose the camera matrix like this:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

intrinsic (3 x 3)   extrinsic (3 x 4)

$$\mathbf{P} = \mathbf{K}[\mathbf{I}|\mathbf{0}]$$

$$\mathbf{K} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$ calibration matrix

Extrinsic camera parameters

We can decompose the camera matrix like this:

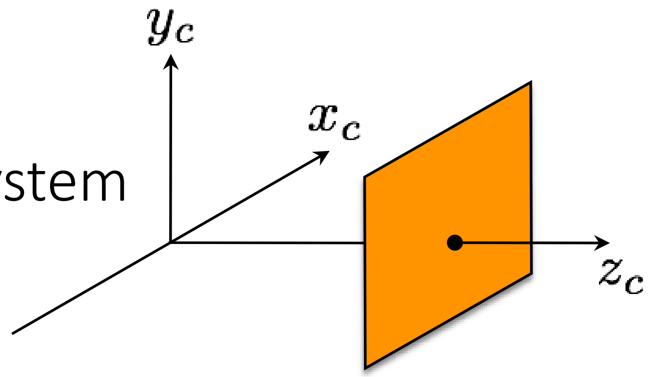
$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow$$

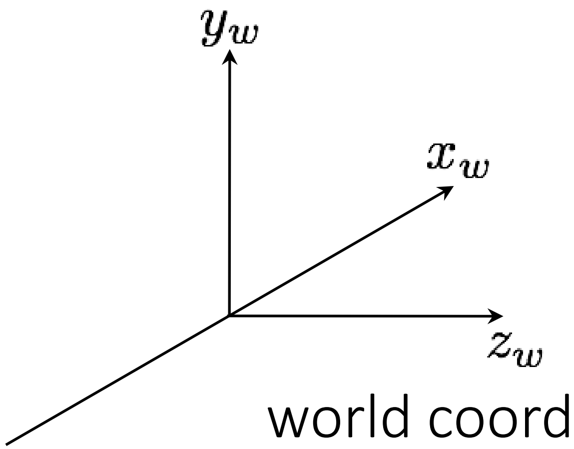
assumes camera and world share the same coordinate system

intrinsic (3 x 3) extrinsic (3 x 4)

What if world and camera coordinate systems are different?

camera coordinate system





world coordinate system

Extrinsic camera parameters

We can decompose the camera matrix like this:

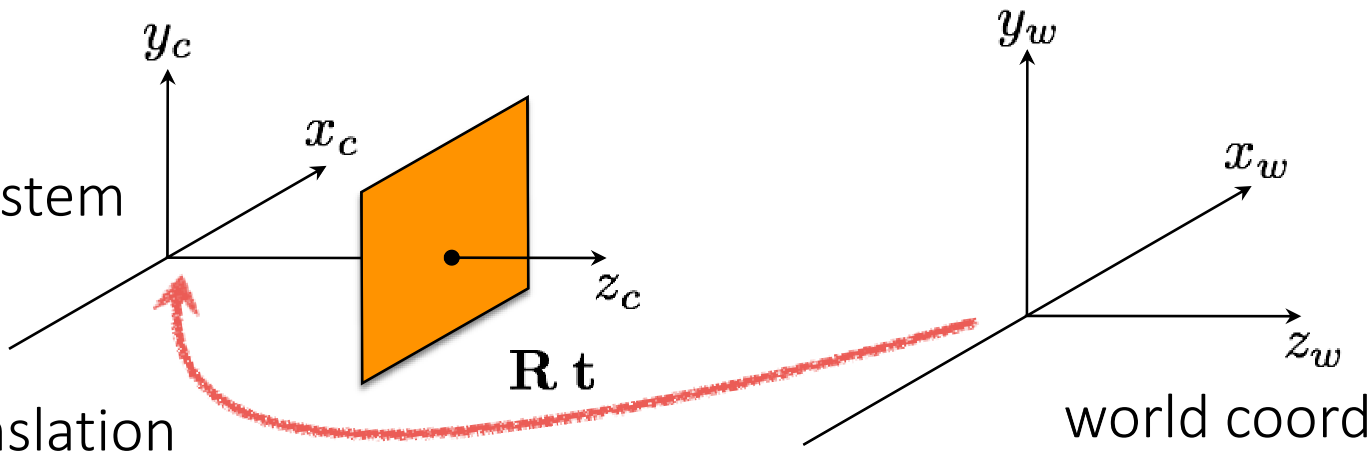
$$\mathbf{P} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow$$

assumes camera and world share the same coordinate system

intrinsic (3 x 3) extrinsic (3 x 4)

What if world and camera coordinate systems are different?

camera coordinate system



3D rotation and translation

world coordinate system

Extrinsic camera parameters

We can decompose the camera matrix like this:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

← assumes camera and world share the same coordinate system

intrinsic (3 x 3) extrinsic (3 x 4)

What if world and camera coordinate systems are different?

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Extrinsic camera parameters

We can decompose the camera matrix like this:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c:c} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0} & 1 \end{array} \right]$$

intrinsic (3 x 3) extrinsic (3 x 4)

What if world and camera coordinate systems are different?

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

General pinhole camera matrix

We can decompose the camera matrix like this:

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}| - \mathbf{C}]$$

(translate first then rotate)

Another way to write the mapping:

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

where $\mathbf{t} = -\mathbf{RC}$

(rotate first then translate)

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I} | -\mathbf{C}]$$

? ? ? ?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

?

?

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

3x3
3D rotation

?

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

3x3
3D rotation

3x3
identity

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

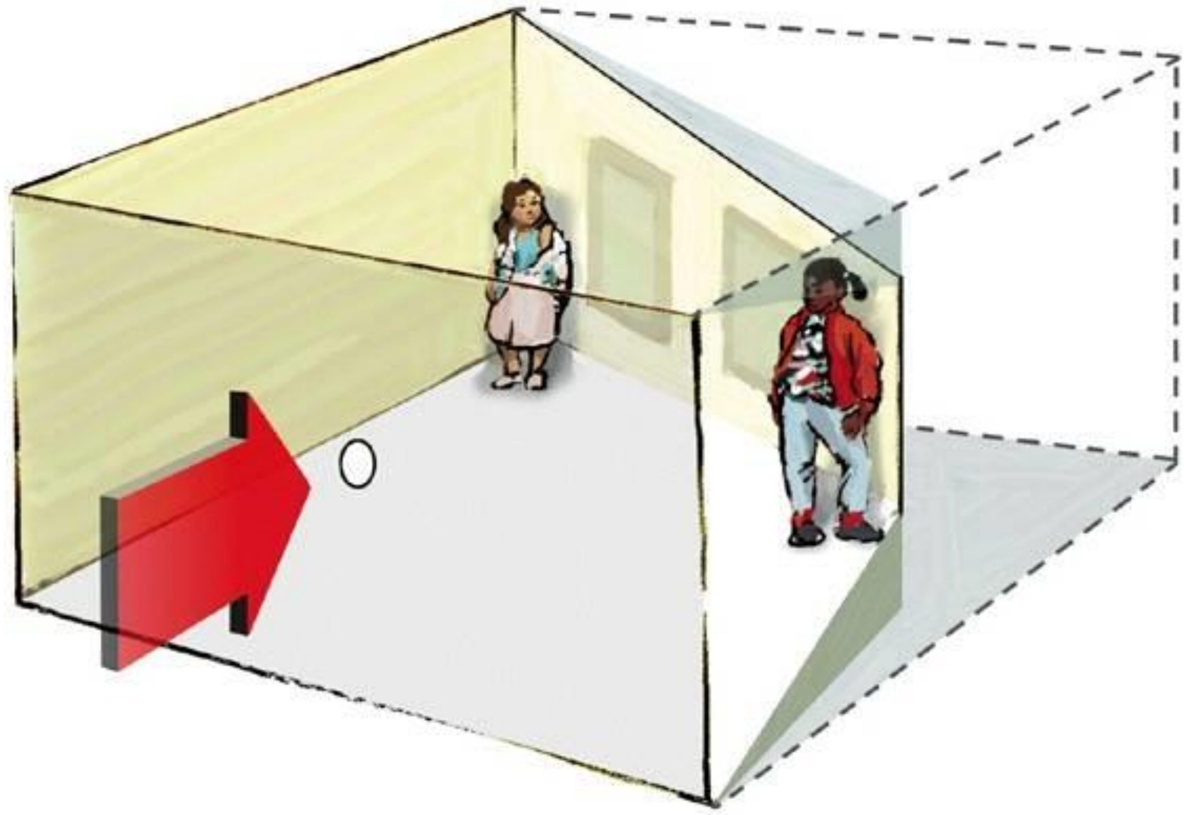
3x3	3x3	3x3	3x1
intrinsics	3D rotation	identity	3D translation

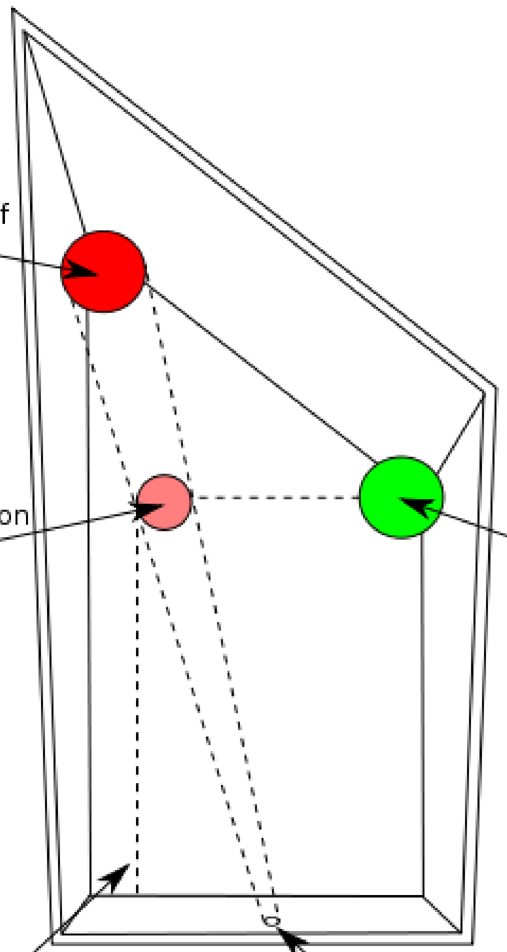
Perspective

Forced perspective

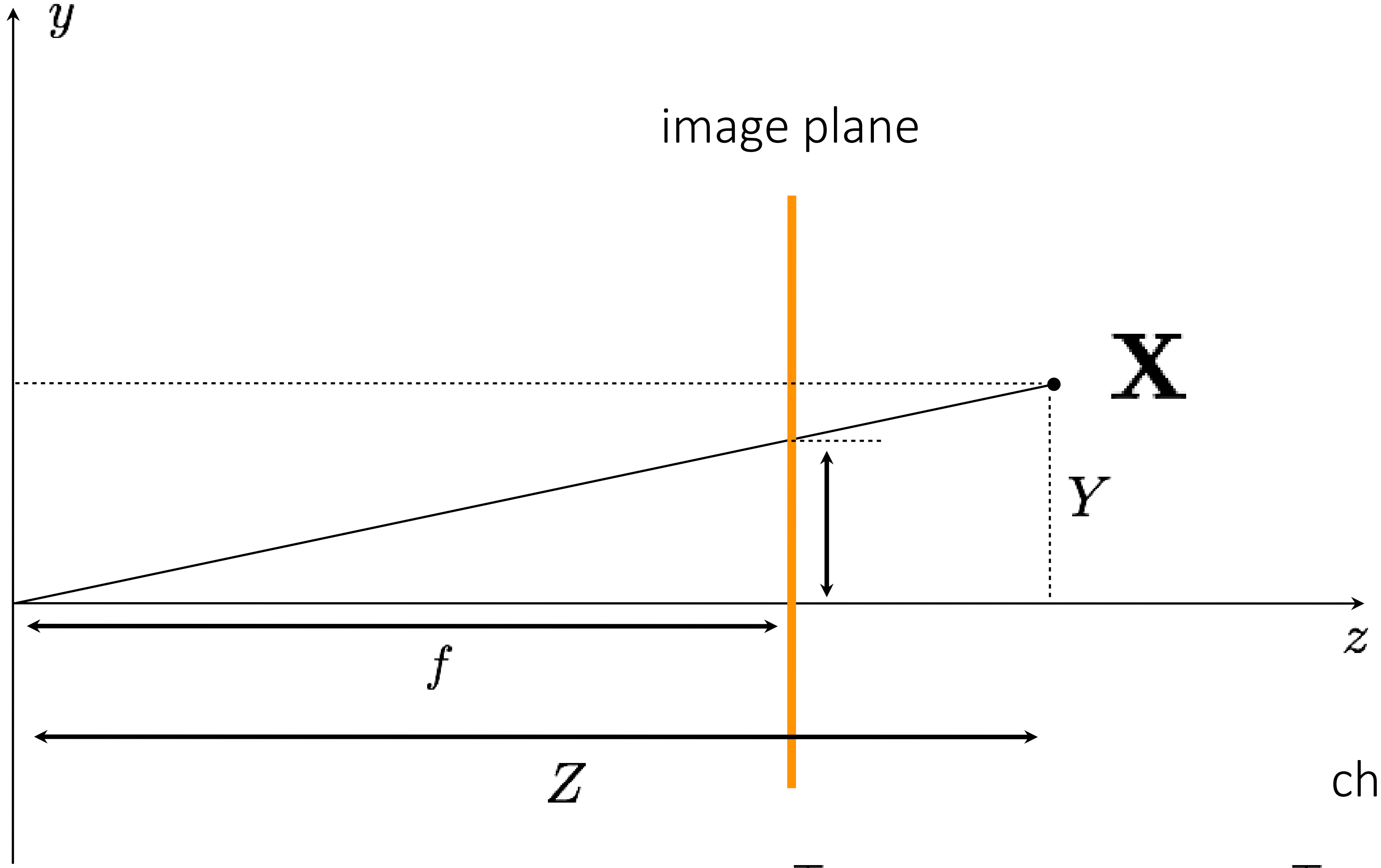
The Ames room illusion

The Ames room illusion





The 2D view of the (rearranged) pinhole camera



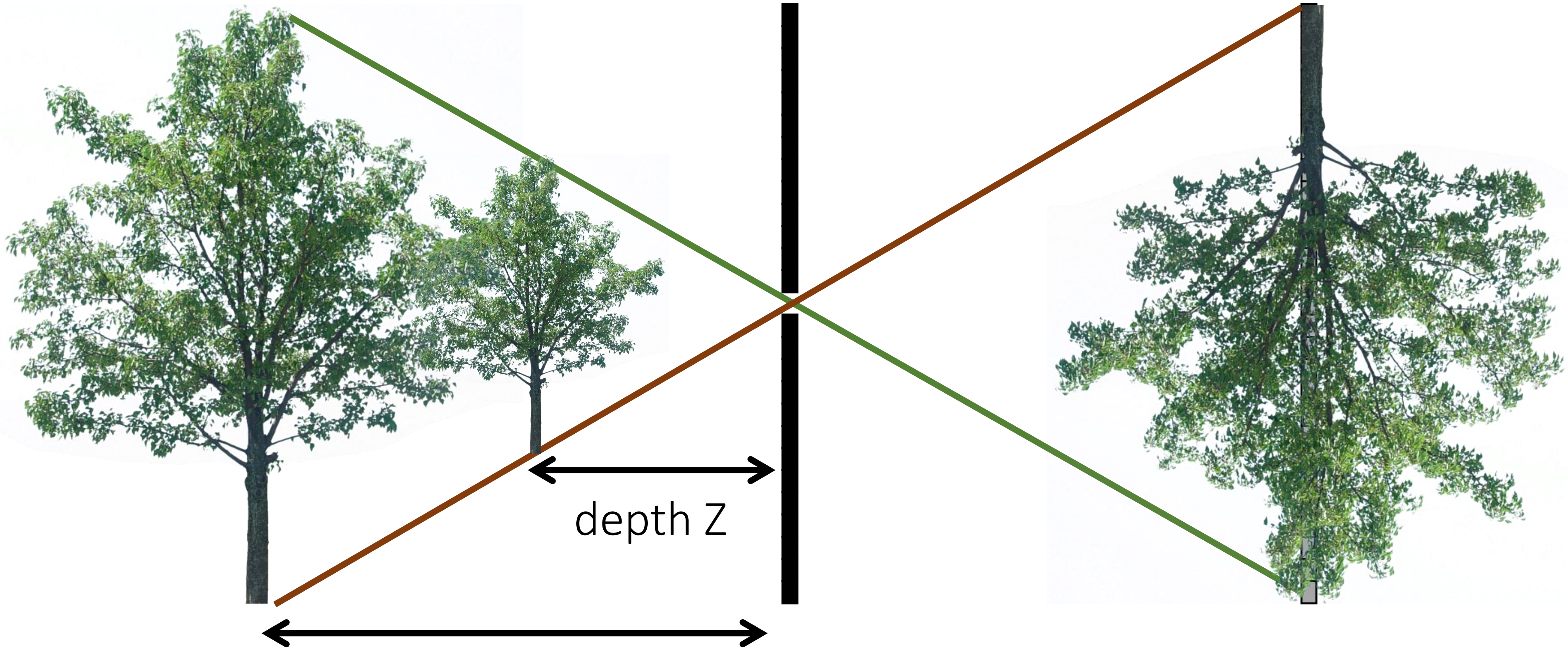
$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

magnification changes with depth

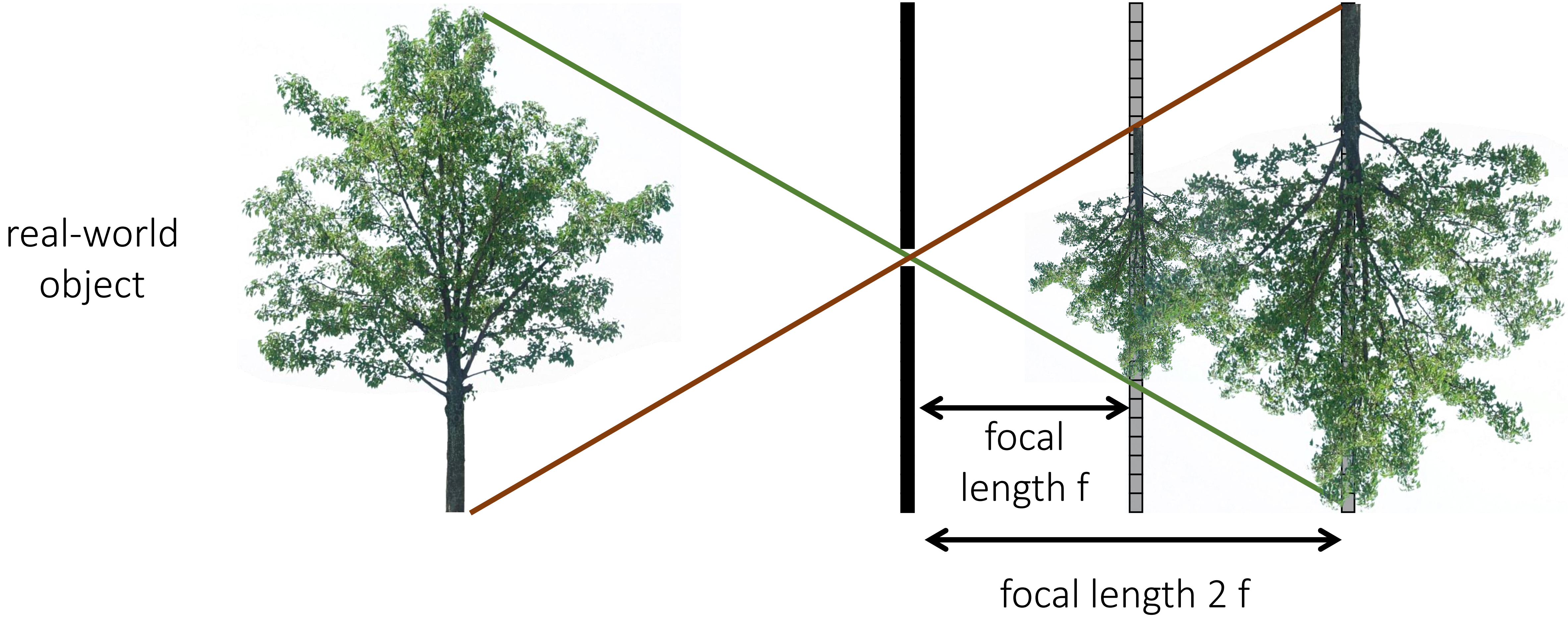
Magnification depends on depth

What happens as we change the focal length?

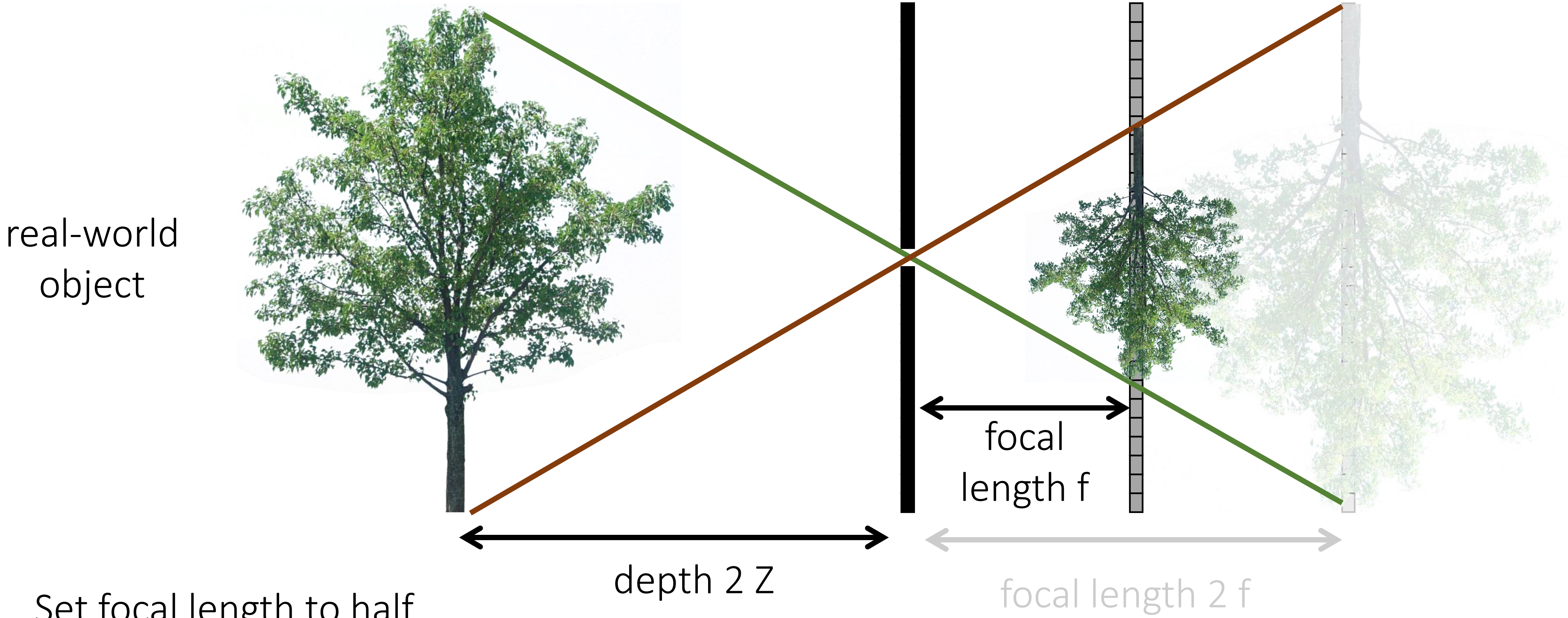
real-world object



Magnification depends on focal length



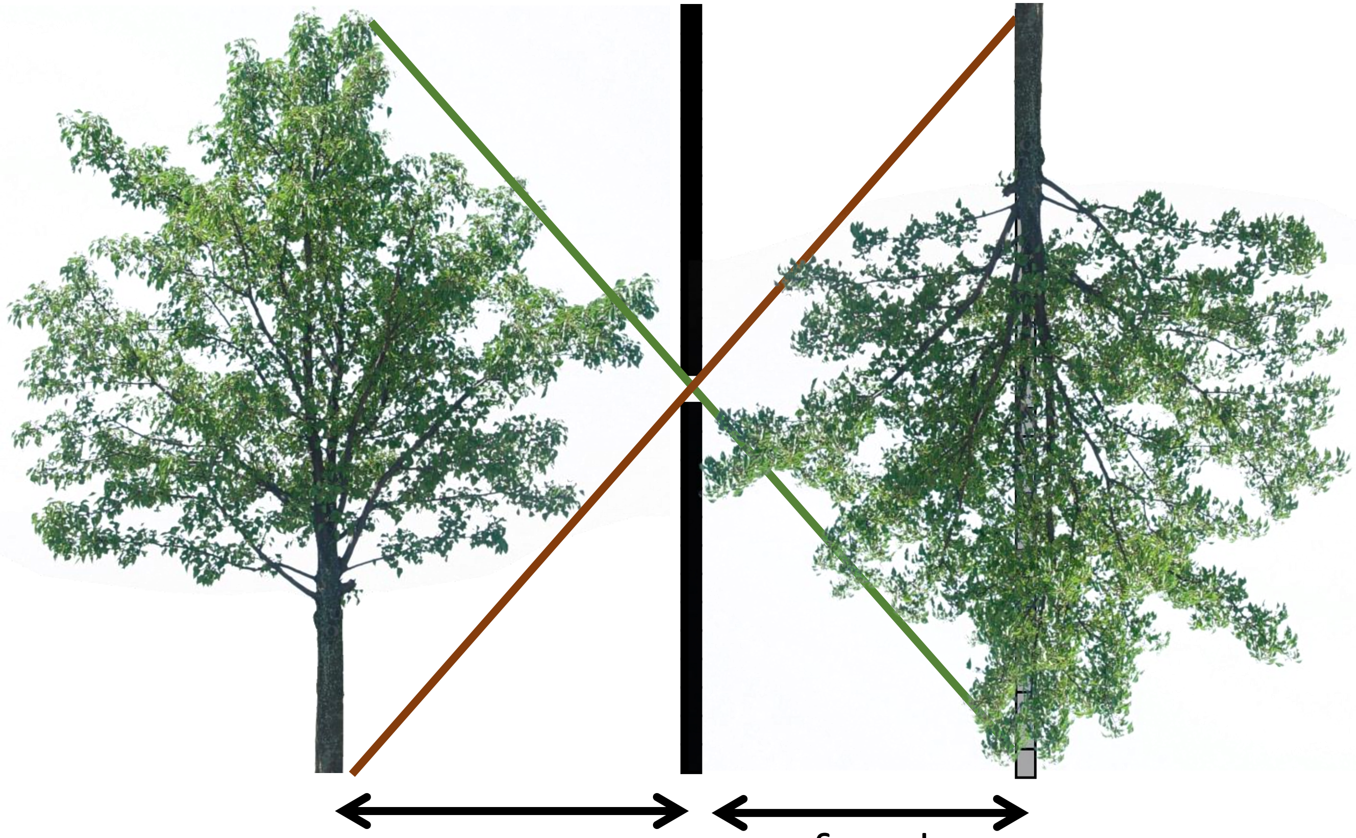
What if...



1. Set focal length to half

What if...

real-world object



Is this the same image as the one I had at focal length 2f and distance 2Z?

1. Set focal length to half
2. Set depth to half

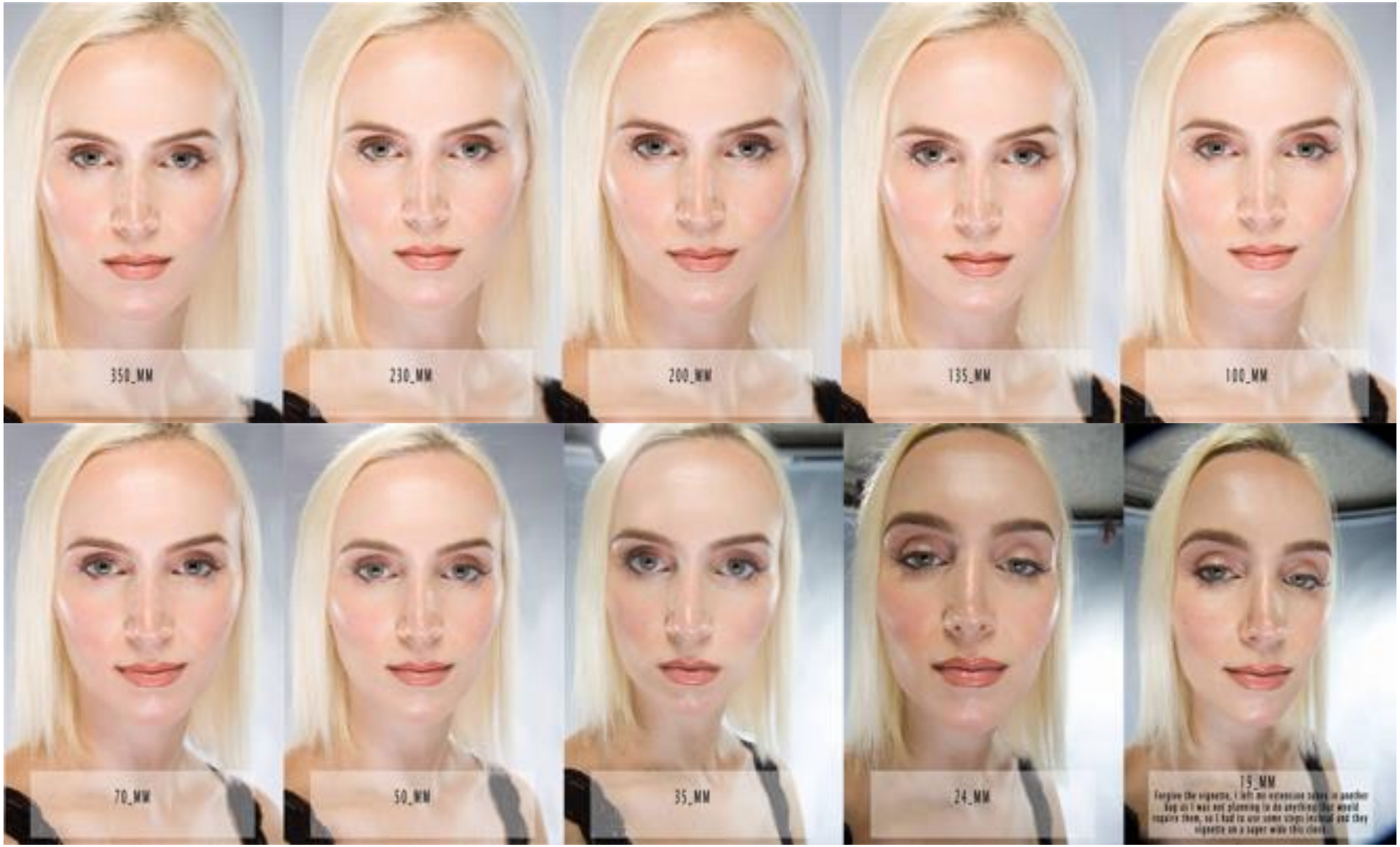
Perspective distortion

long focal length

mid focal length

short focal length

Perspective distortion



What is the best focal length for portraits?

That’s like asking which is better, vi or emacs…

long focal length

mid focal length

short focal length

Vertigo effect

Named after Alfred Hitchcock’s movie

- also known as “dolly zoom”

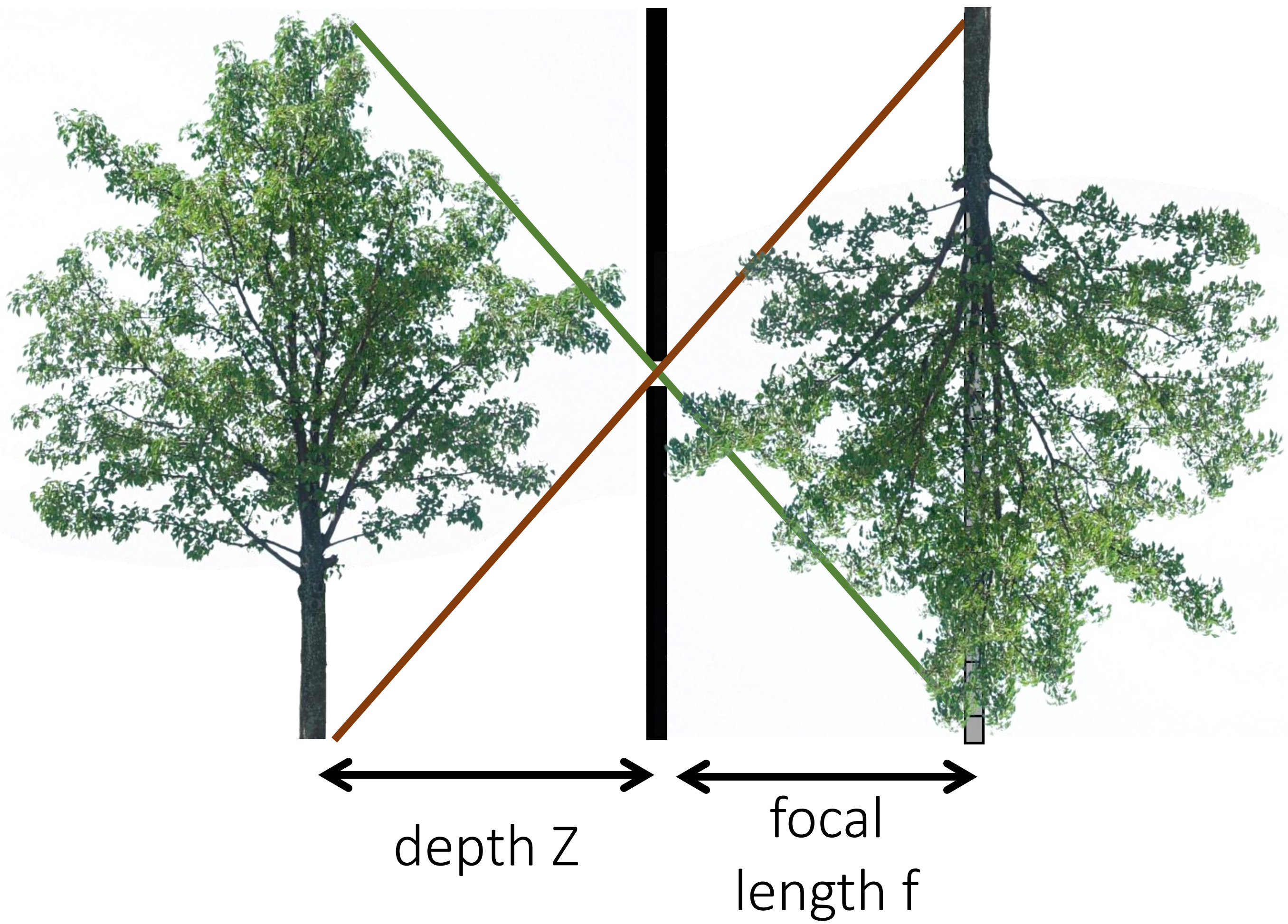
Vertigo effect

How would you create this effect?

Orthographic camera

What if…

real-world object



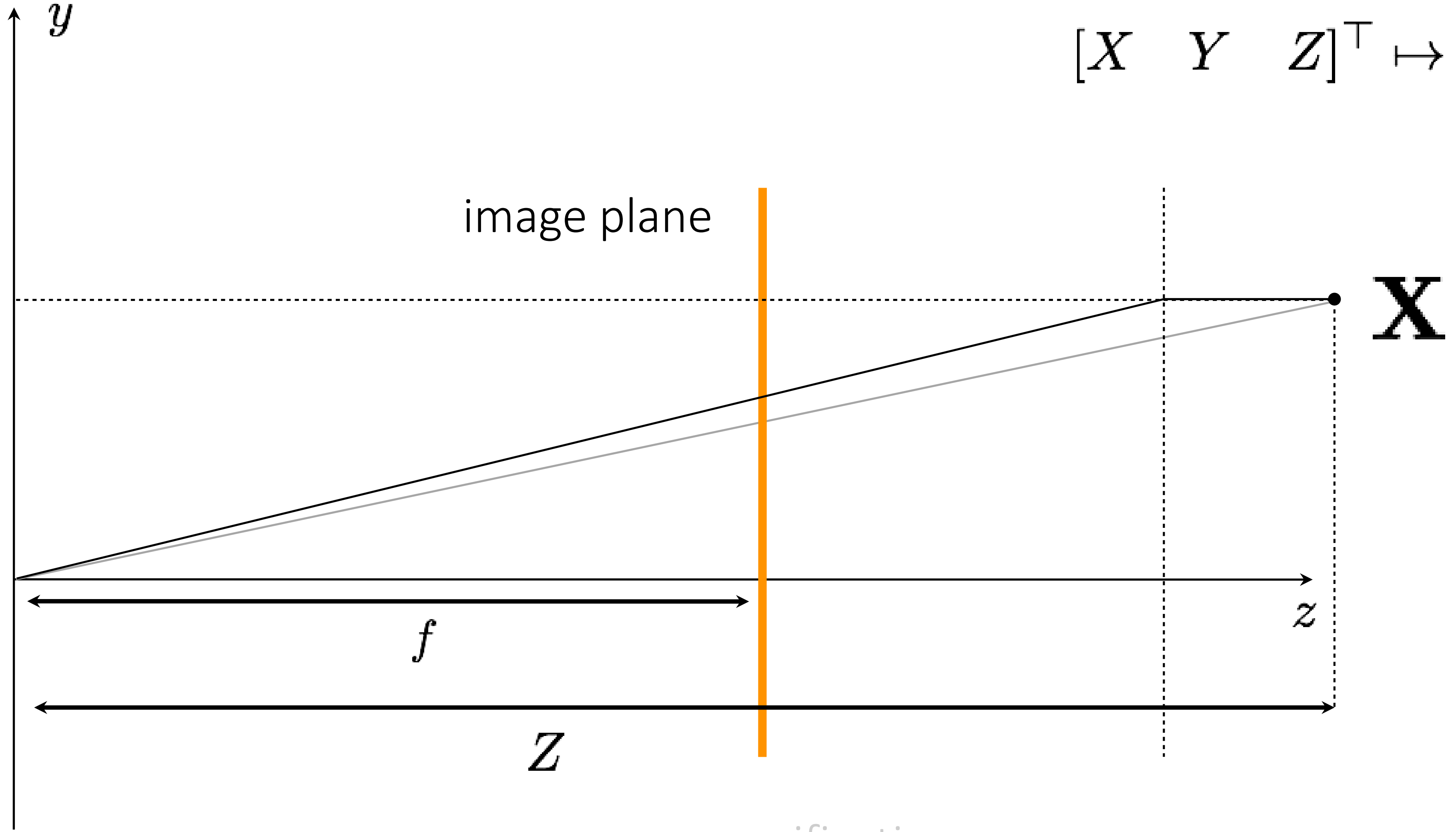
Continue increasing Z and f while maintaining same magnification?

$$f \to \infty \text{ and } \frac{f}{Z} = \text{constant}$$

Orthographic vs pinhole camera

$$[X \quad Y \quad Z]^\top \mapsto [fX \quad fY]^\top$$

magnification does not change with depth



magnification changes with depth

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

Orthographic vs pinhole camera

General pinhole camera:

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}| - \mathbf{C}]$$

General orthographic camera:

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}| - \mathbf{C}]$$

Bottom row is always [0 0 0 1]

We also call these cameras:

Projective camera

Affine camera

What is the rationale behind these names?

References

Basic reading:

- Szeliski textbook, Section 2.1.5.

Additional reading:

- Hartley and Zisserman, “Multiple View Geometry in Computer Vision,” Cambridge University Press 2004.
  chapter 6 of this book is a very thorough treatment of camera models.
- Goodman, “Introduction to Fourier Optics,” W.H. Freeman 2004.
  the standard reference on Fourier optics, chapter 4 covers aperture diffraction.
- Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012.
  the eponymous paper discussed in the slides.