

# Image Warping and Morphing

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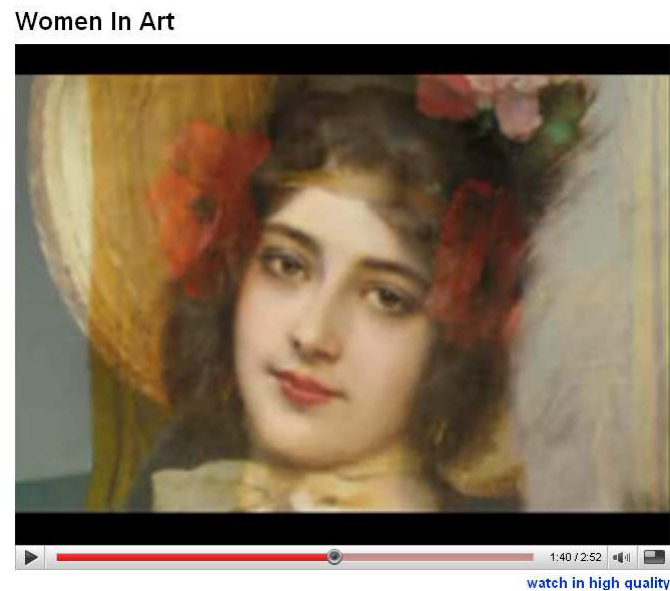


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15-463: Computational Photography  
Alexei Efros, CMU, Fall 2011

# Women in Art video

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[http://youtube.com/watch?v=nUDIoN-\\_Hxs](http://youtube.com/watch?v=nUDIoN-_Hxs)

# Image Warping in Biology

D'Arcy Thompson

<http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html>

[http://en.wikipedia.org/wiki/D'Arcy\\_Thompson](http://en.wikipedia.org/wiki/D'Arcy_Thompson)

Importance of shape and structure in evolution

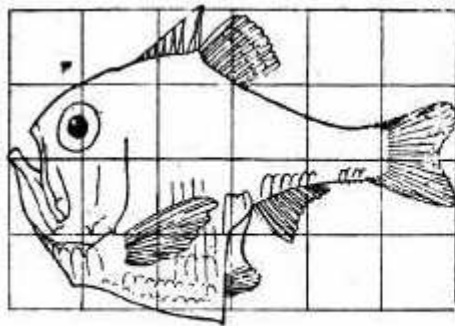
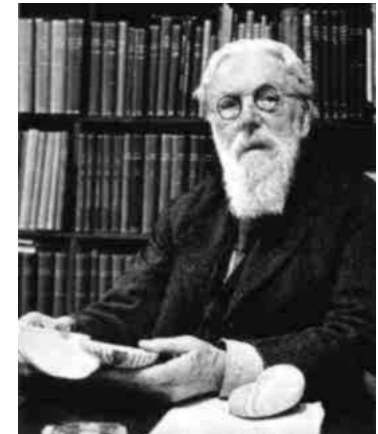
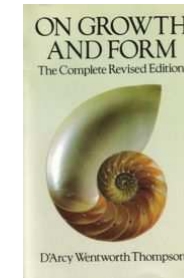


Fig. 517. *Argyropelecus Olfersi*.

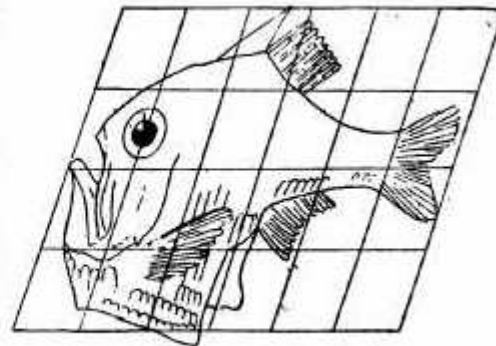
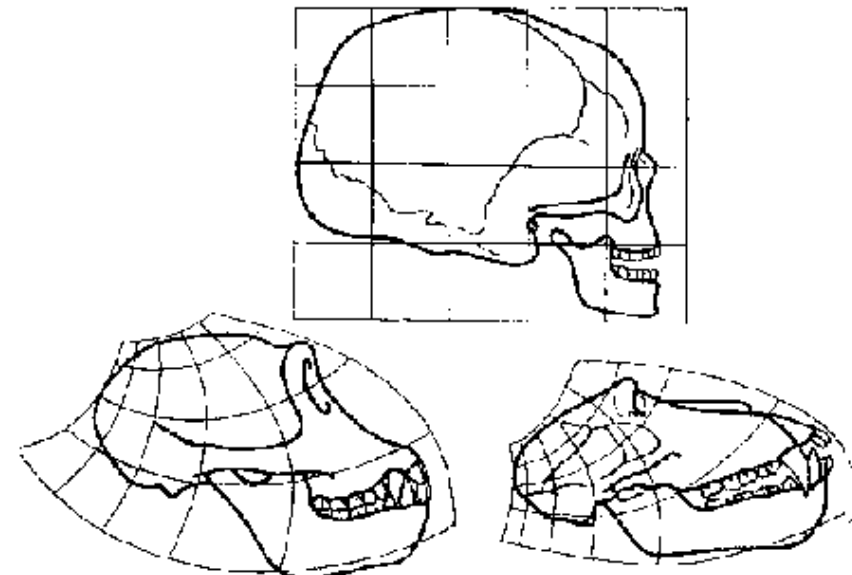


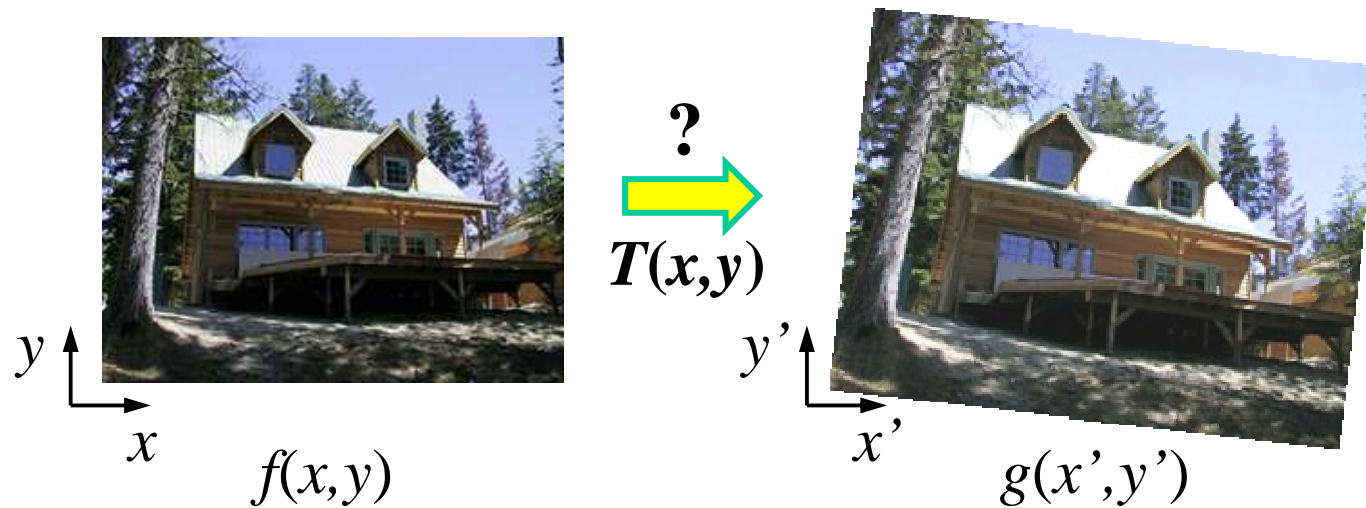
Fig. 518. *Sternoptyx diaphana*.



Skulls of a human, a chimpanzee and a baboon and transformations between them

# Recovering Transformations

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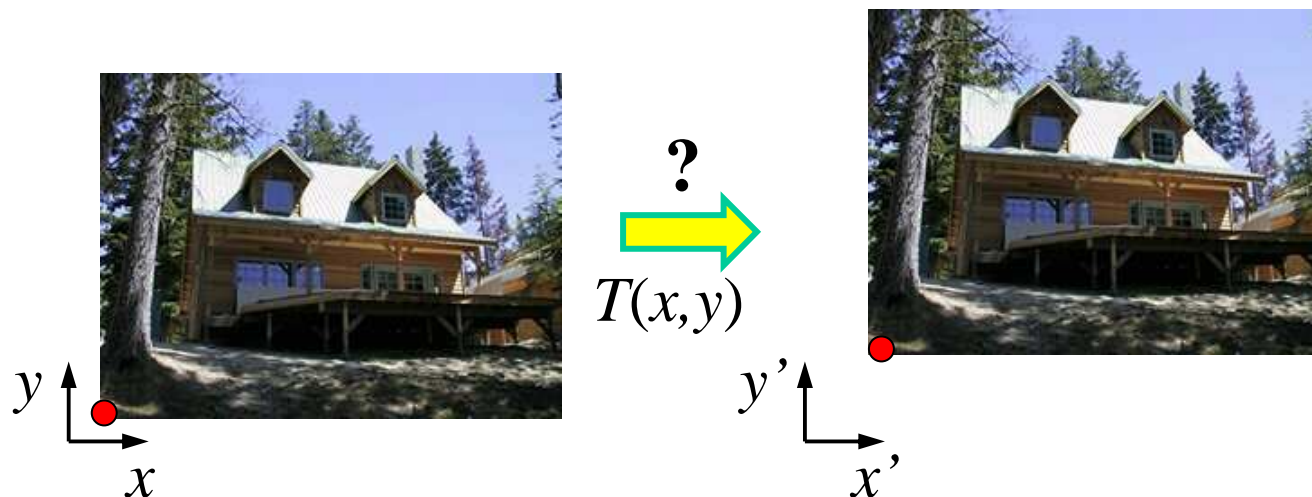


What if we know  $f$  and  $g$  and want to recover the transform  $T$ ?

- e.g. better align images from Project 1
- willing to let user provide correspondences
  - How many do we need?

# Translation: # correspondences?

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How many correspondences needed for translation?

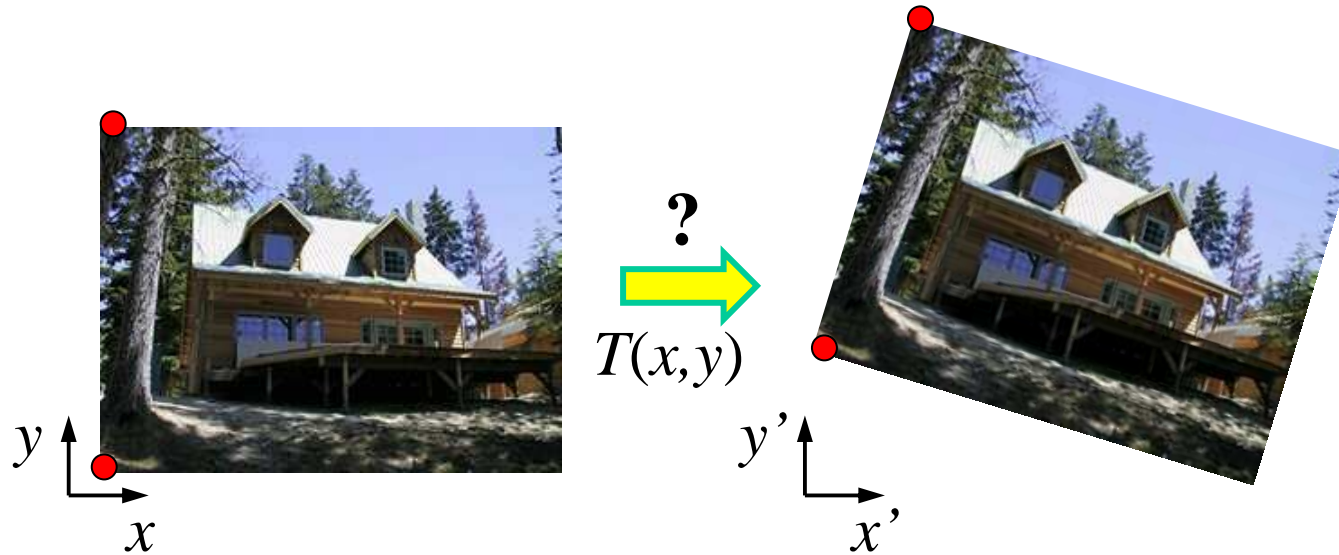
How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

# Euclidian: # correspondences?

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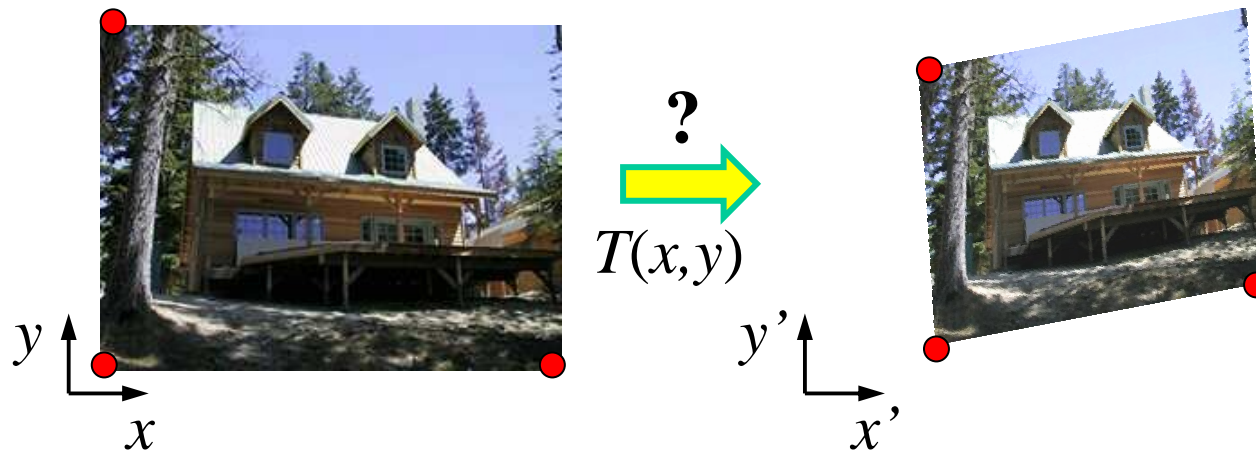


How many correspondences needed for translation+rotation?

How many DOF?

# Affine: # correspondences?

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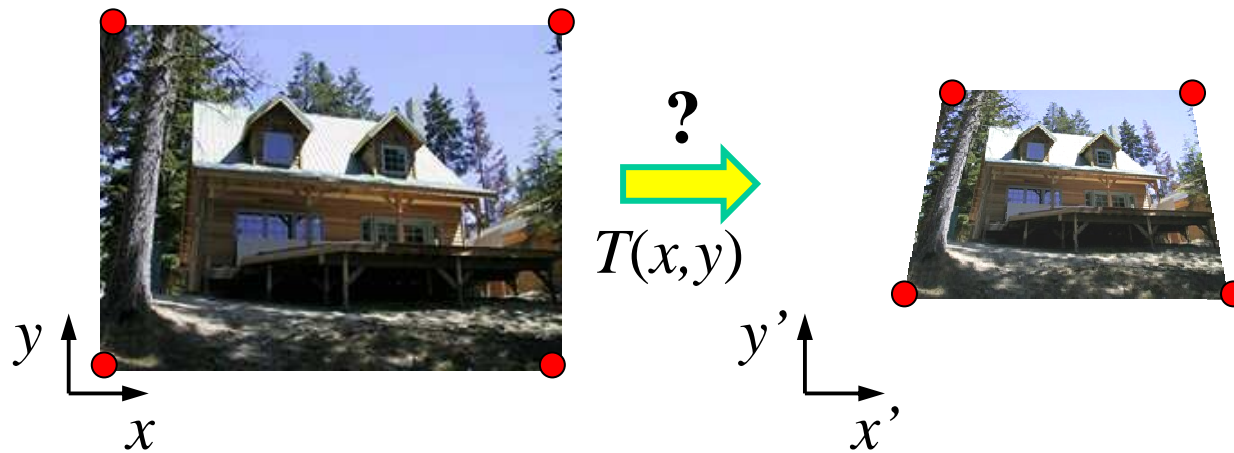


How many correspondences needed for affine?

How many DOF?

# Projective: # correspondences?

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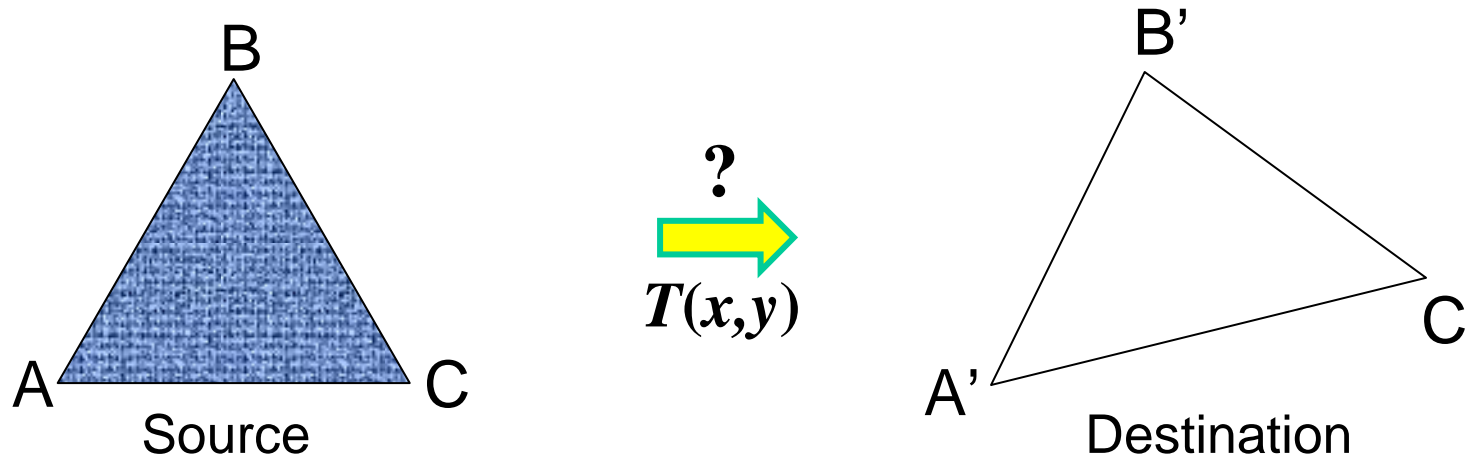


How many correspondences needed for projective?

How many DOF?

# Example: warping triangles

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Given two triangles: ABC and A'B'C' in 2D (12 numbers)  
Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T?

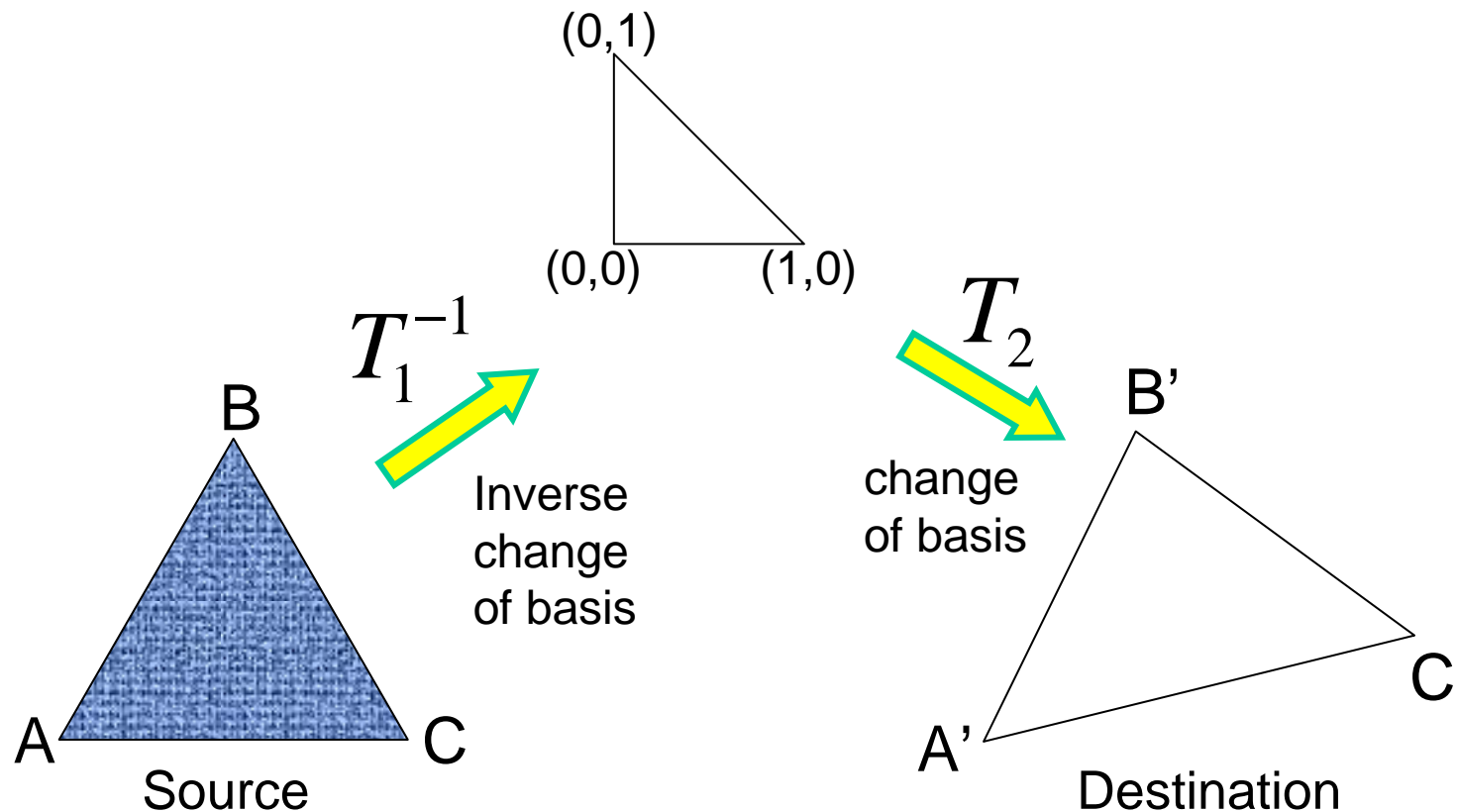
How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways:  
Algebraic and  
geometric

# warping triangles (Barycentric Coordinates)

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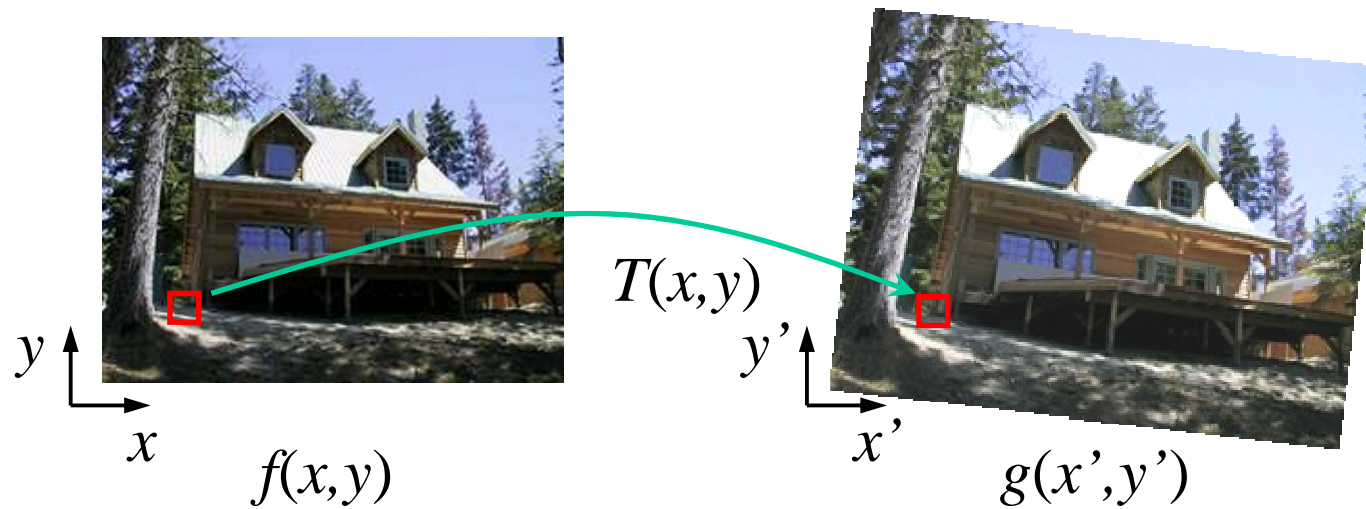


Don't forget to move the origin too!

Very useful for Project 3... (hint, hint, nudge, nudge)

# Image warping

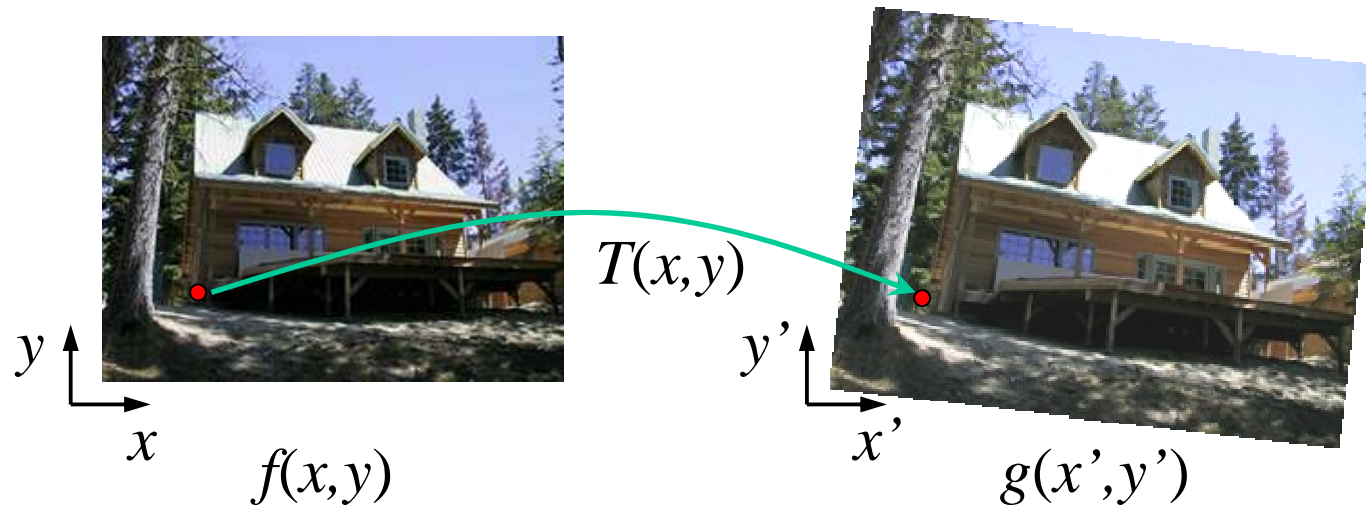
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Given a coordinate transform  $(x',y') = T(x,y)$  and a source image  $f(x,y)$ , how do we compute a transformed image  $g(x',y') = f(T(x,y))$ ?

# Forward warping

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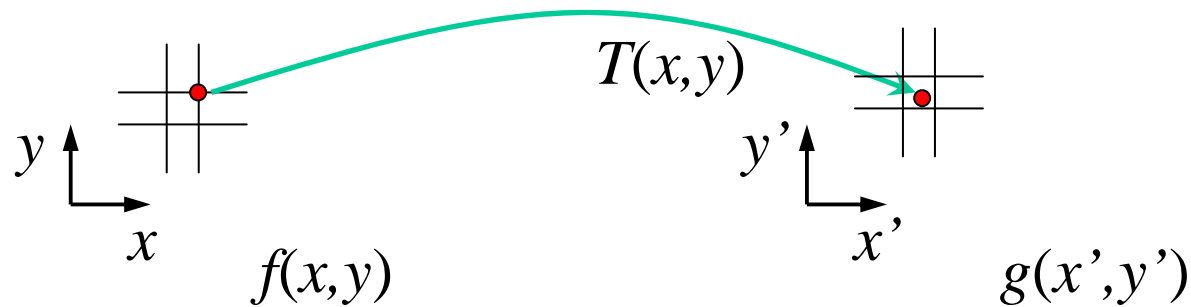
Send each pixel  $f(x,y)$  to its corresponding location

$(x',y') = T(x,y)$  in the second image

Q: what if pixel lands “between” two pixels?

# Forward warping

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Send each pixel  $f(x, y)$  to its corresponding location  
 $(x', y') = T(x, y)$  in the second image

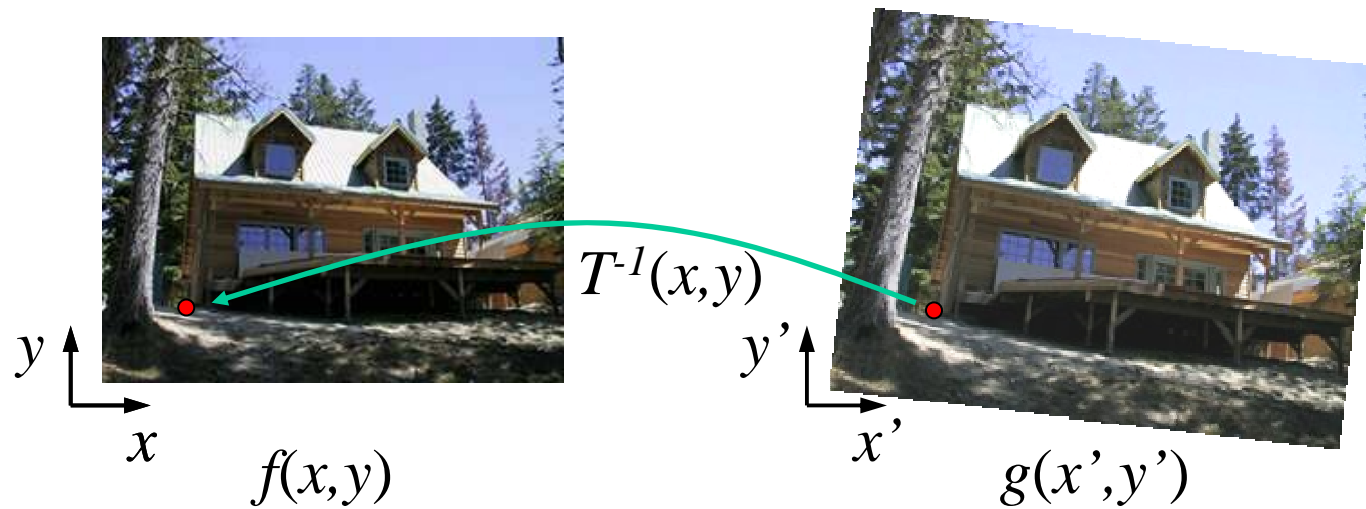
Q: what if pixel lands “between” two pixels?

A: distribute color among neighboring pixels  $(x', y')$

- Known as “splatting”
- Check out `griddata` in Matlab

# Inverse warping

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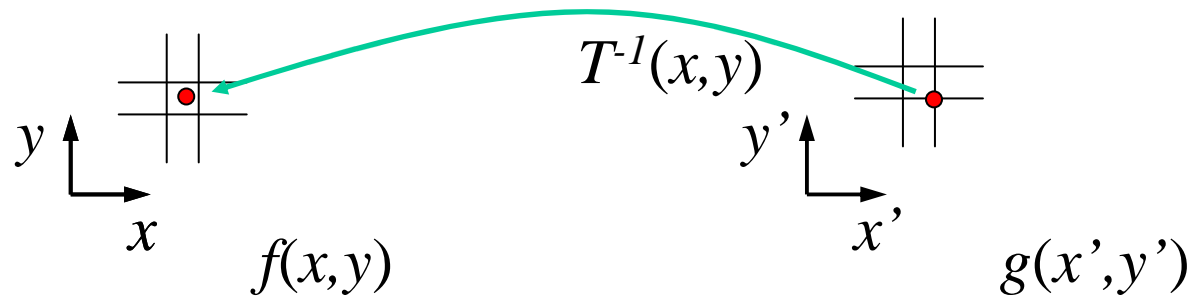


Get each pixel  $g(x',y')$  from its corresponding location  
 $(x,y) = T^{-1}(x',y')$  in the first image

Q: what if pixel comes from “between” two pixels?

# Inverse warping

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Get each pixel  $g(x', y')$  from its corresponding location  
 $(x, y) = T^{-1}(x', y')$  in the first image

Q: what if pixel comes from “between” two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab

# Forward vs. inverse warping

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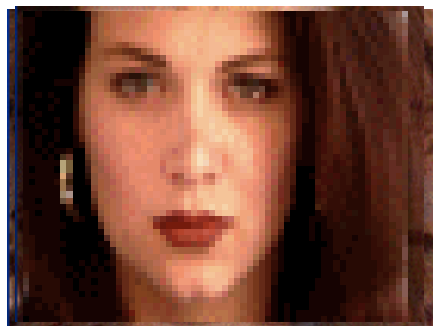
Q: which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

# Morphing = Object Averaging

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The aim is to find “an average” between two objects

- Not an average of two images of objects...
- ...but an image of the average object!
- How can we make a smooth transition in time?
  - Do a “weighted average” over time  $t$

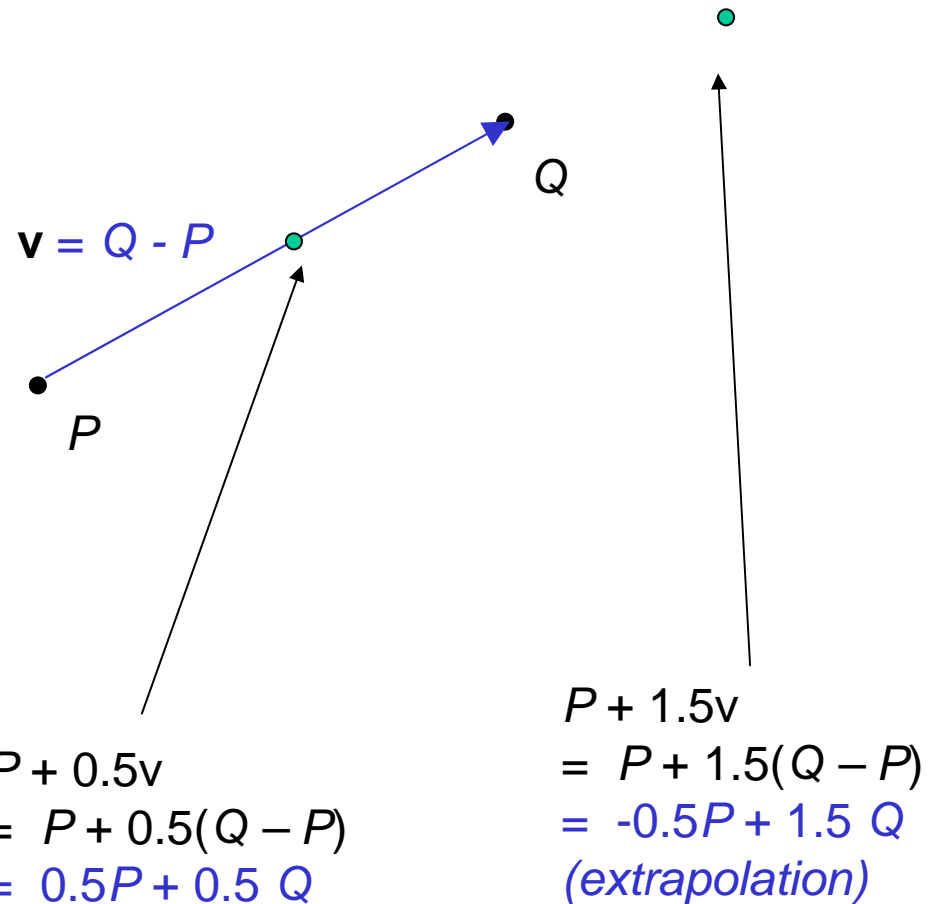
How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
  - Usually required user/artist input

# Averaging Points

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What's the average  
of P and Q?



Linear Interpolation  
(Affine Combination):  
New point  $aP + bQ$ ,  
defined only when  $a+b = 1$   
So  $aP+bQ = aP+(1-a)Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)... etc.

# Idea #1: Cross-Dissolve

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Interpolate whole images:

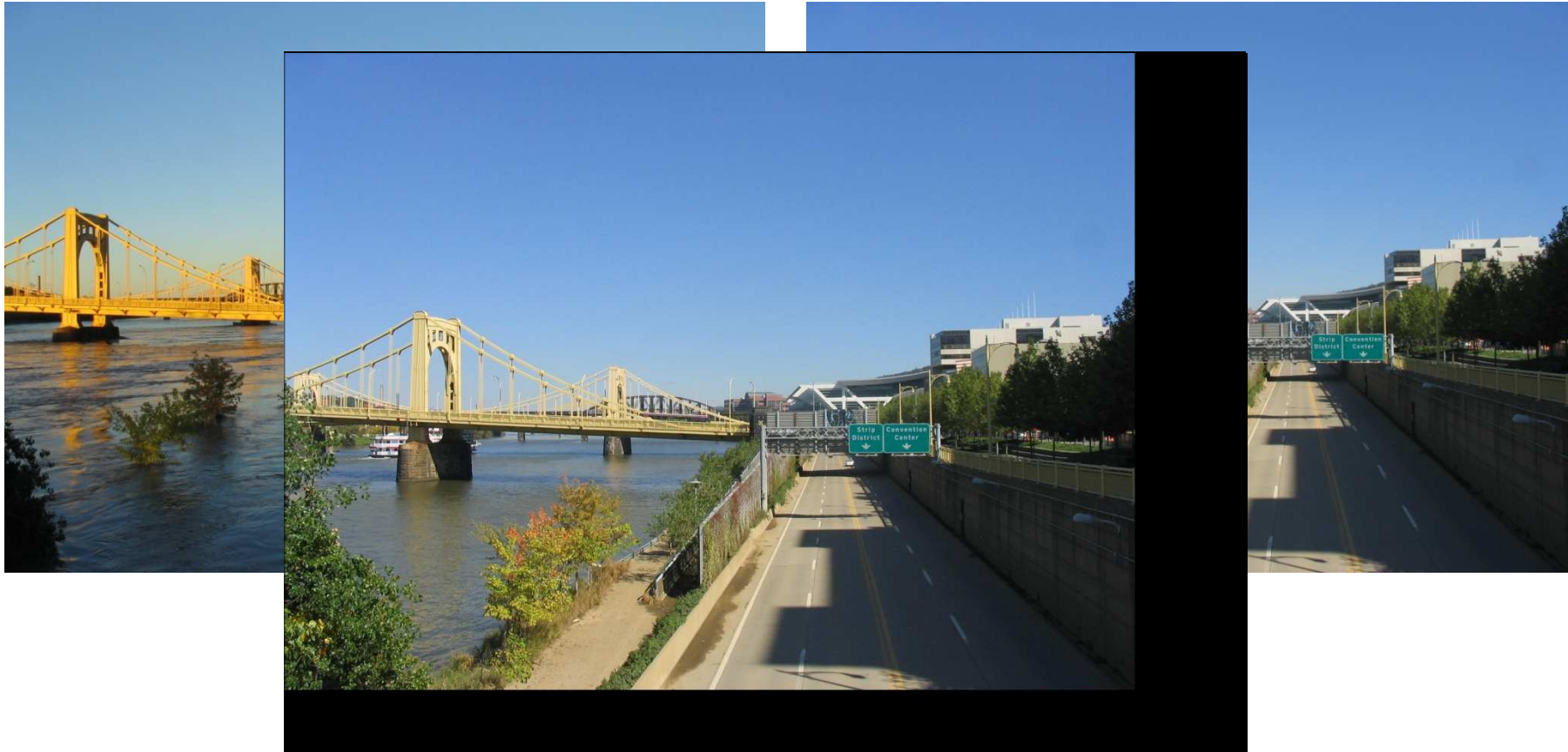
$$\text{Image}_{\text{halfway}} = (1-t) \cdot \text{Image}_1 + t \cdot \text{Image}_2$$

This is called **cross-dissolve** in film industry

But what if the images are not aligned?

## Idea #2: Align, then cross-dissolve

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Align first, then cross-dissolve

- Alignment using global warp – picture still valid

# Dog Averaging

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What to do?

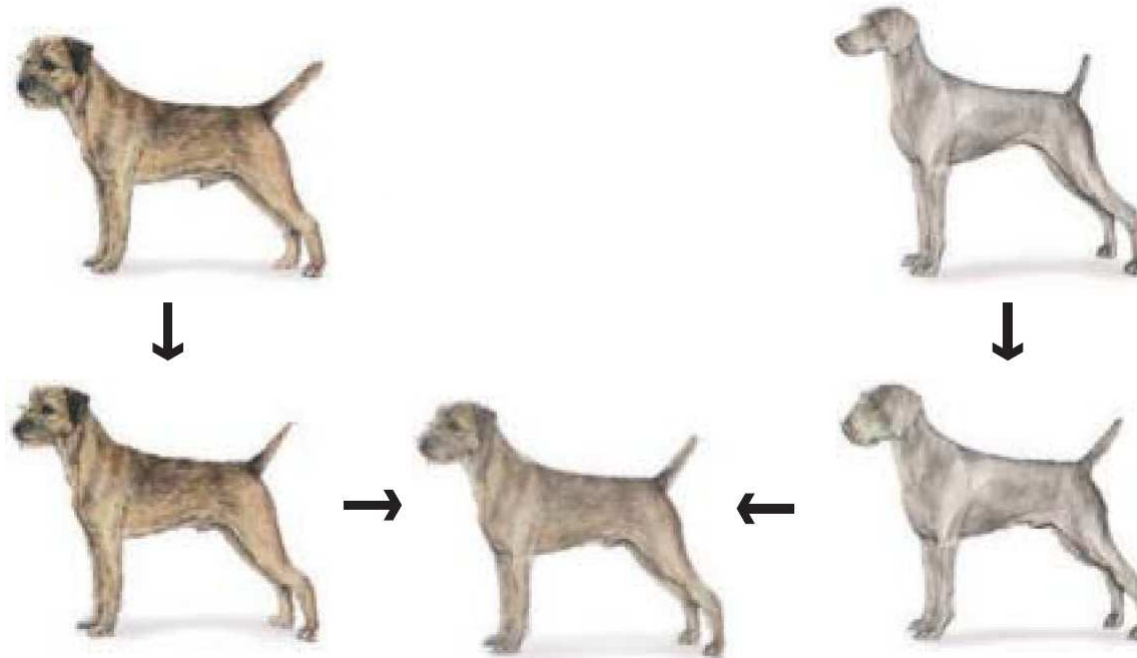
- Cross-dissolve doesn't work
- Global alignment doesn't work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

- Nose to nose, tail to tail, etc.
- This is a local (non-parametric) warp

# Idea #3: Local warp, then cross-dissolve

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Morphing procedure:

*for every  $t$ ,*

1. Find the average shape (the “mean dog” 😊)
  - local warping
2. Find the average color
  - Cross-dissolve the warped images

# Local (non-parametric) Image Warping

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Need to specify a more detailed warp function

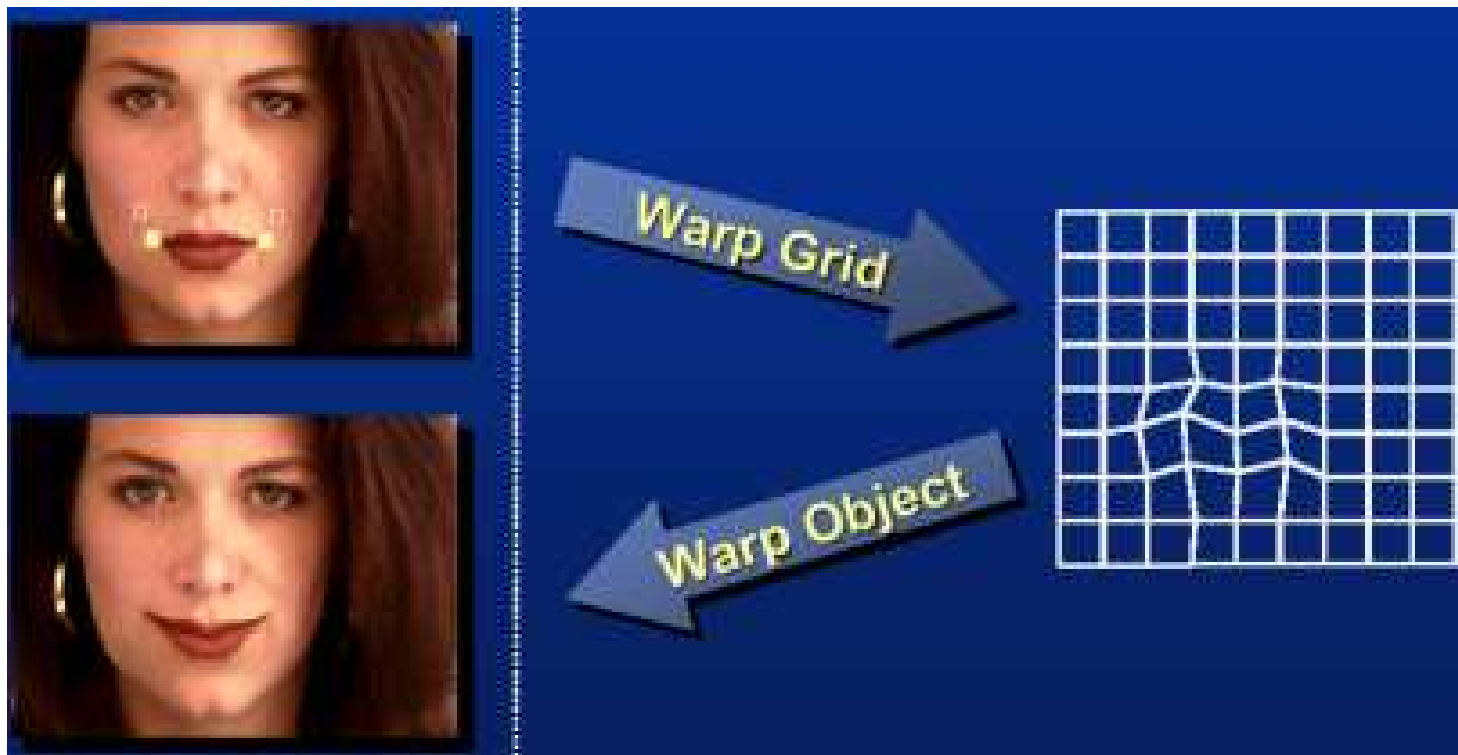
- Global warps were functions of a few (2,4,8) parameters
- Non-parametric warps  $u(x,y)$  and  $v(x,y)$  can be defined independently for every single location  $x,y$ !
- Once we know vector field  $u,v$  we can easily warp each pixel (use backward warping with interpolation)

# Image Warping – non-parametric

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Move control points to specify a spline warp

Spline produces a smooth vector field



# Warp specification - dense

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How can we specify the warp?

Specify corresponding *spline control points*

- *interpolate* to a complete warping function



But we want to specify only a few points, not a grid

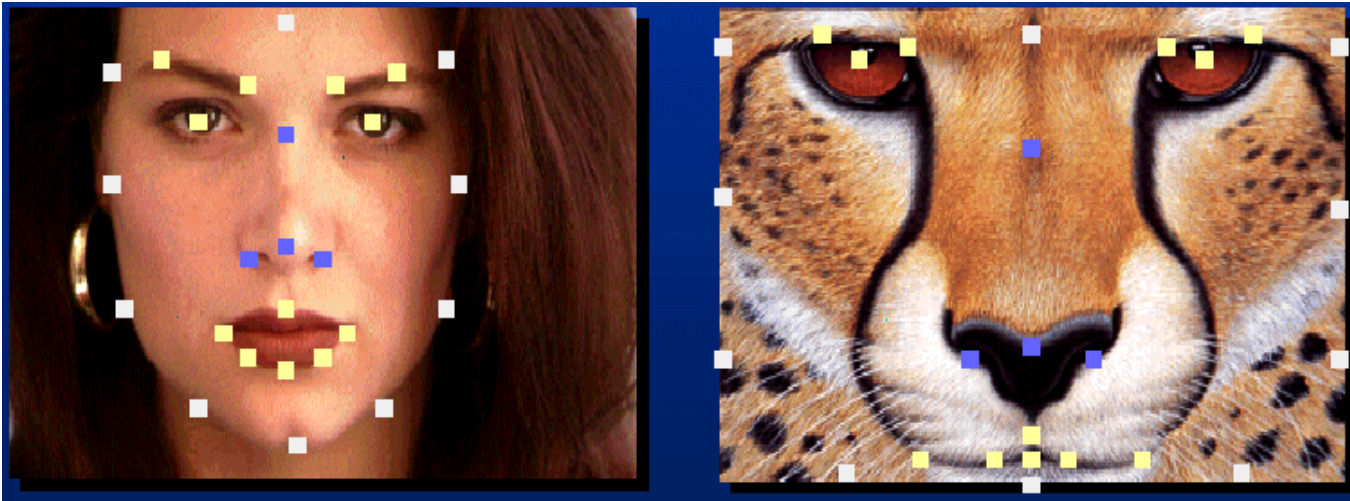
# Warp specification - sparse

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How can we specify the warp?

Specify corresponding *points*

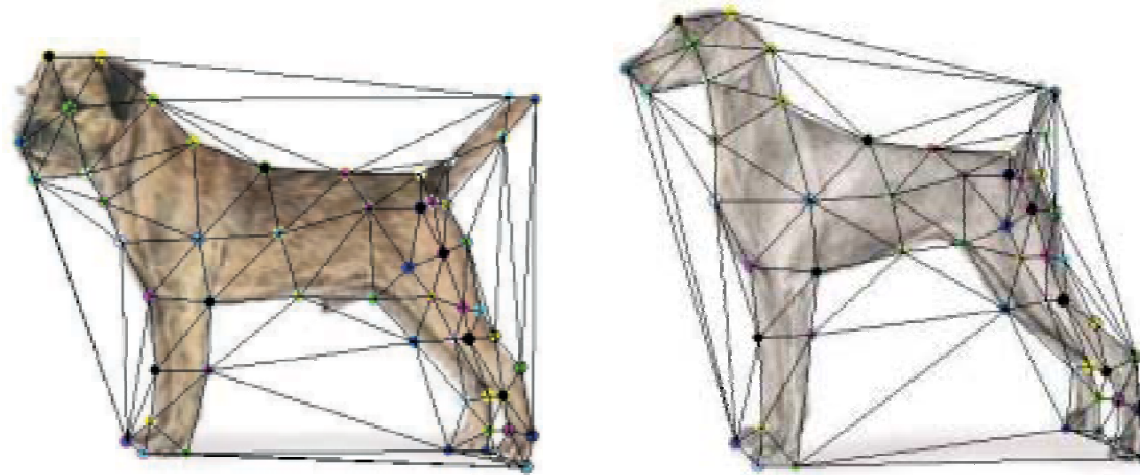
- *interpolate* to a complete warping function
- How do we do it?



How do we go from feature points to pixels?

# Triangular Mesh

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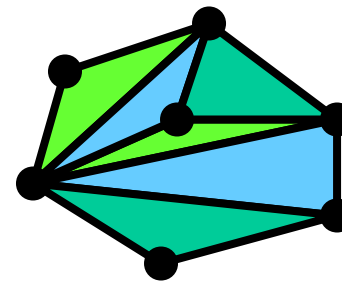
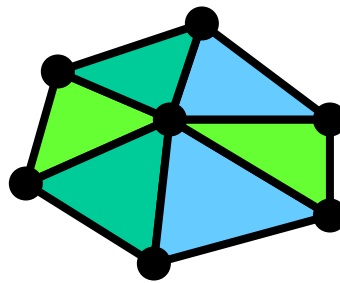
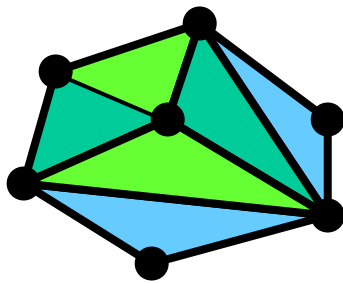
1. Input correspondences at key feature points
2. Define a triangular mesh over the points
  - Same mesh in both images!
  - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
  - How do we warp a triangle?
  - 3 points = affine warp!
  - Just like texture mapping

# Triangulations

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A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

There are an exponential number of triangulations of a point set.

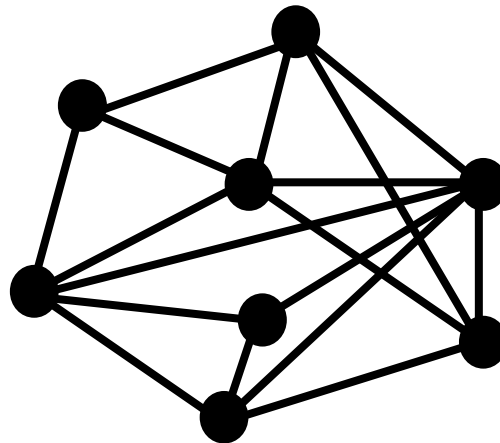


# An $O(n^3)$ Triangulation Algorithm

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Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



# “Quality” Triangulations

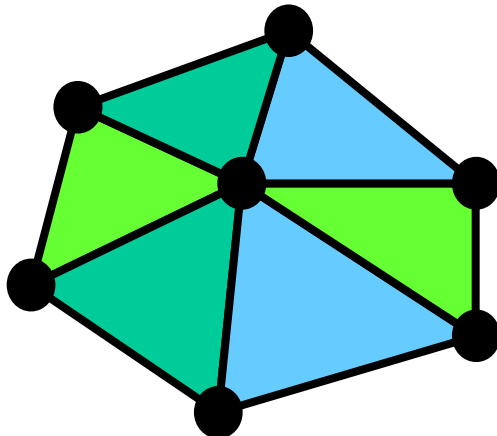
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Let  $\alpha(T) = (\alpha_1, \alpha_2, \dots, \alpha_{3t})$  be the vector of angles in the triangulation  $T$  in increasing order.

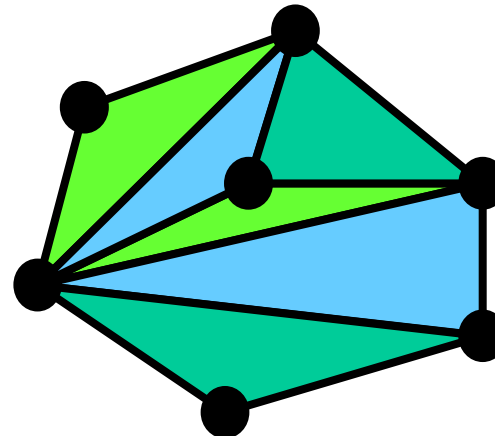
A triangulation  $T_1$  will be “better” than  $T_2$  if  $\alpha(T_1) > \alpha(T_2)$  lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles



good

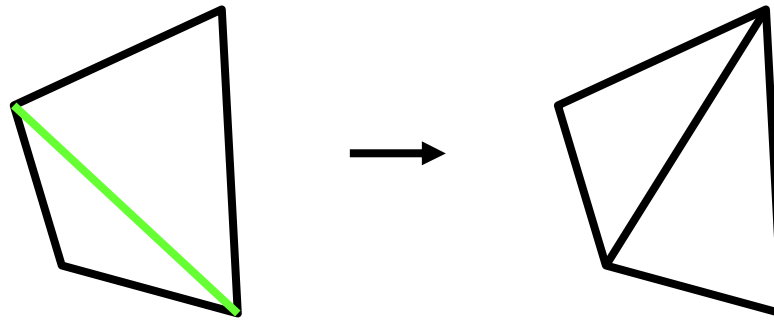


bad

# Improving a Triangulation

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In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.



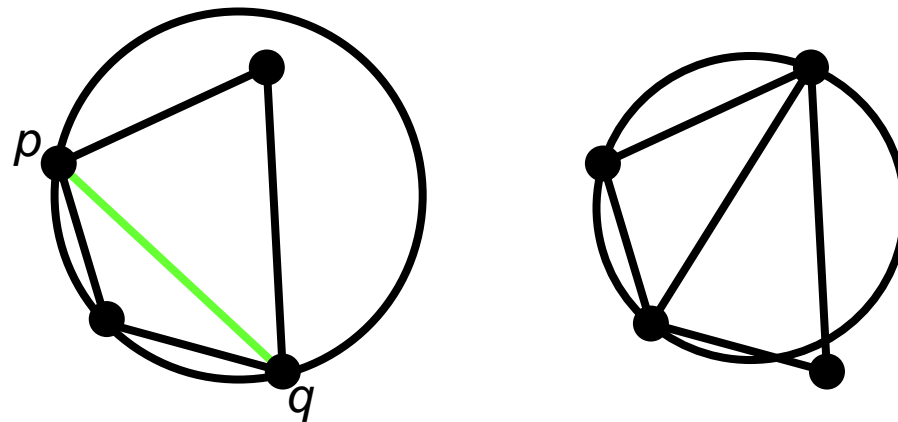
If an edge flip improves the triangulation, the first edge is called *illegal*.

# Illegal Edges

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**Lemma:** An edge  $pq$  is illegal iff one of its opposite vertices is inside the circle defined by the other three vertices.

**Proof:** By Thales' theorem.



**Theorem:** A Delaunay triangulation does not contain illegal edges.

**Corollary:** A triangle is Delaunay iff the circle through its vertices is empty of other sites.

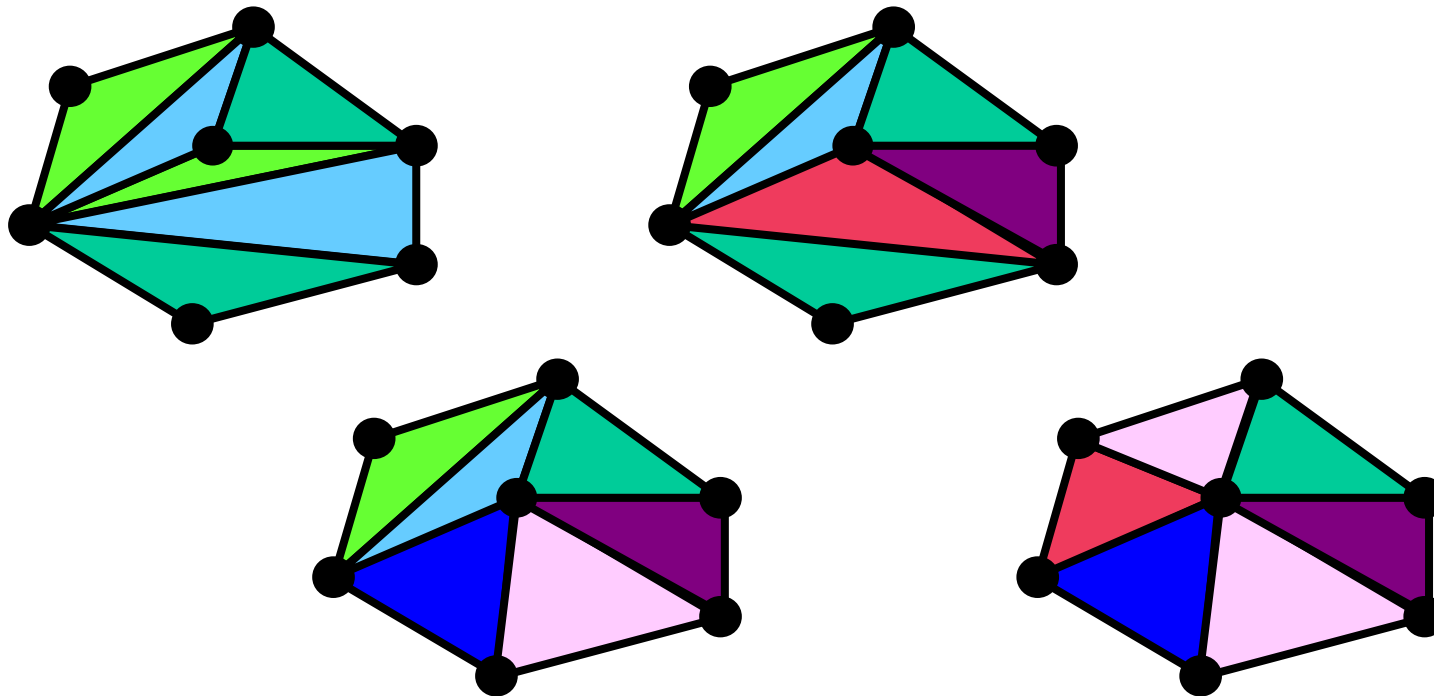
**Corollary:** The Delaunay triangulation is not unique if more than three sites are co-circular.

# Naïve Delaunay Algorithm

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Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

Could take a long time to terminate.



# Delaunay Triangulation by Duality

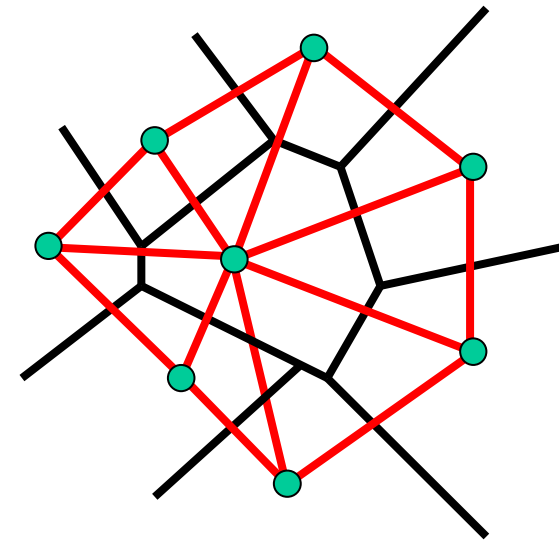
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General position assumption: There are no four co-circular points.

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

**Corollary:** The DT may be constructed in  $O(n \log n)$  time.

This is what Matlab's `delaunay` function uses.

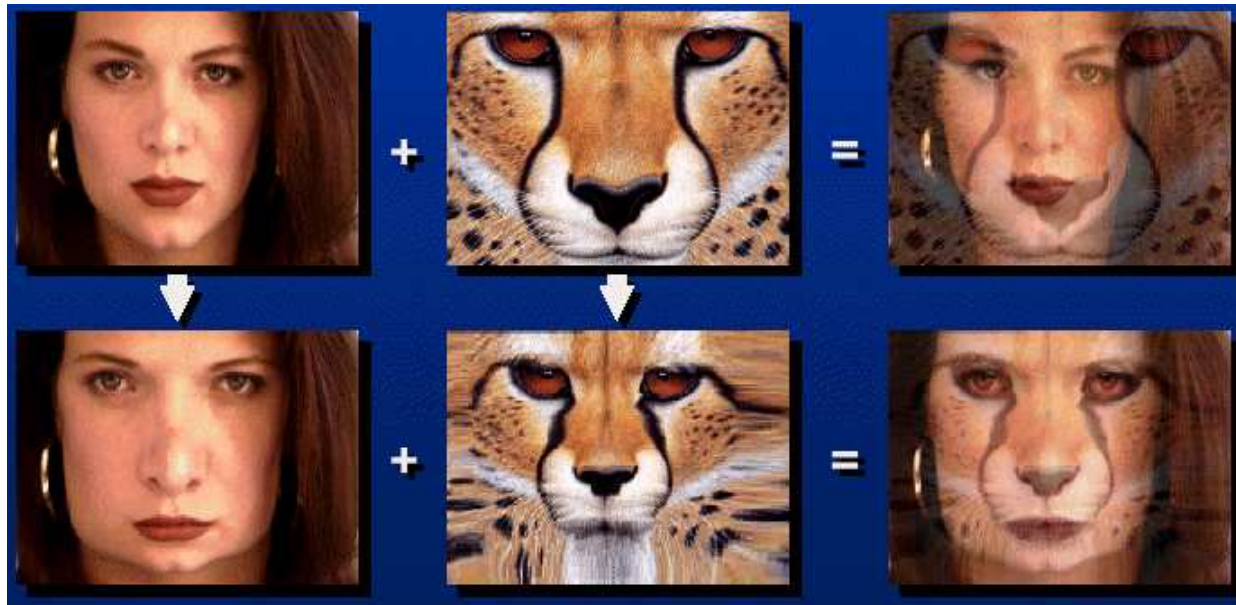


# Image Morphing

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We know how to warp one image into the other, but how do we create a morphing sequence?

1. Create an intermediate shape (by interpolation)
2. Warp both images towards it
3. Cross-dissolve the colors in the newly warped images

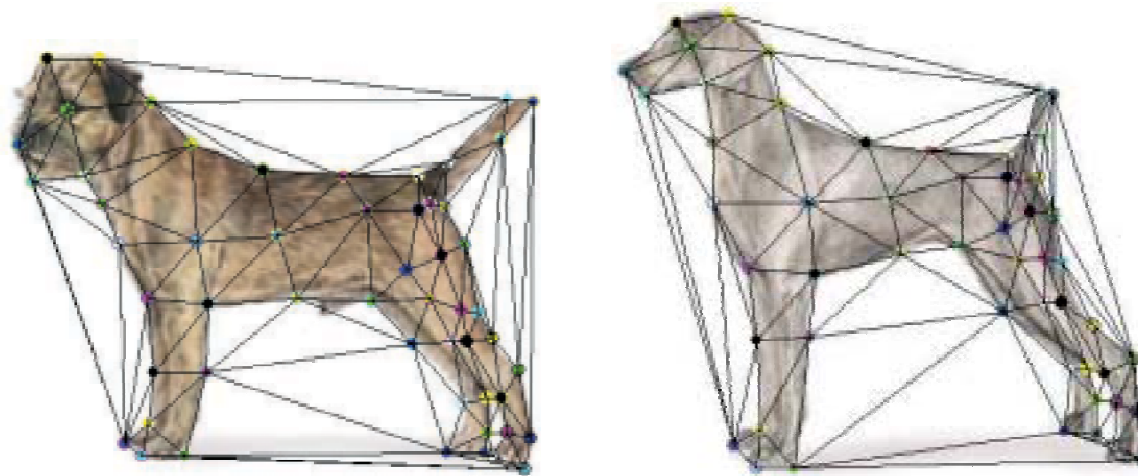


# Warp interpolation

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How do we create an intermediate warp at time  $t$ ?

- Assume  $t = [0,1]$
- Simple linear interpolation of each feature pair
- $(1-t)*p1+t*p0$  for corresponding features  $p0$  and  $p1$



# Morphing & matting

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Extract foreground first to avoid artifacts in the background



(c)  $\alpha = 0.0$

(d)  $\alpha = 0.2$

(e)  $\alpha = 0.4$



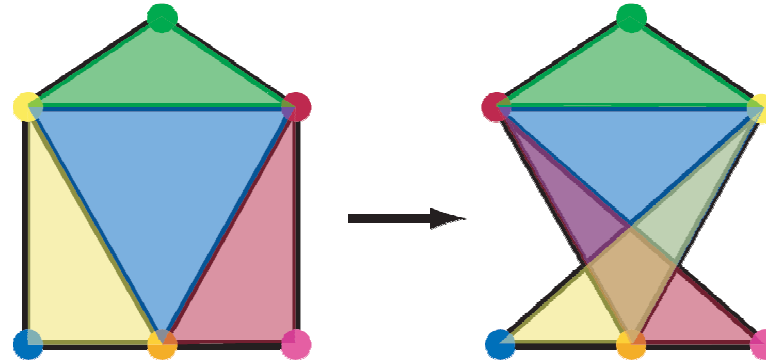
(f)  $\alpha = 0.6$

(g)  $\alpha = 0.8$

(h)  $\alpha = 1.0$

# Other Issues

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Beware of folding

- You are probably trying to do something 3D-ish

Morphing can be generalized into 3D

- If you have 3D data, that is!

Extrapolation can sometimes produce interesting effects

- Caricatures

# Dynamic Scene

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## Project #3

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- Given two photos, produce a 60-frame morph animation
  - Use triangulation-based morphing (lots of helpful Matlab tools)
  - Need to write triangle-to-triangle warp (can't use Matlab tools)
- We put all animations together into a movie!

See last year's movie!