Image Warping and Morphing

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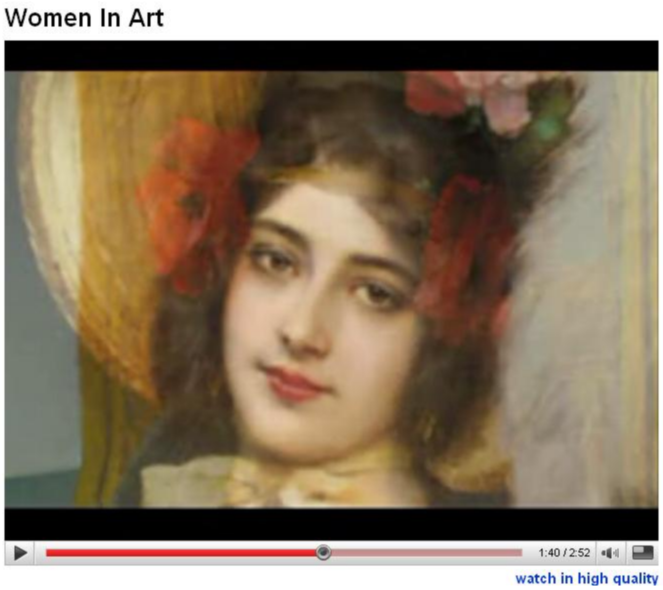
15-463: Computational Photography
Alexei Efros, CMU, Fall 2010

Women in Art video



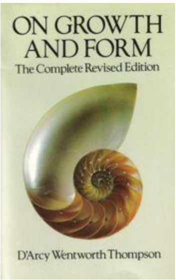
http://youtube.com/watch?v=nUDIoN-_Hxs

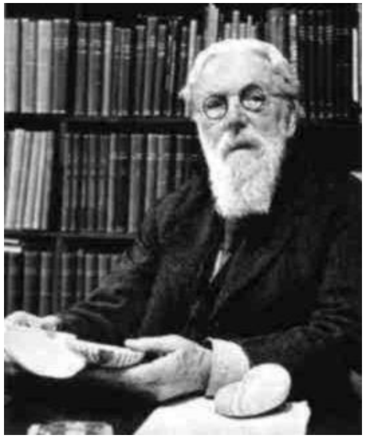
Image Warping in Biology

D'Arcy Thompson

- http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html
- http://en.wikipedia.org/wiki/D'Arcy_Thompson

Importance of shape and structure in evolution





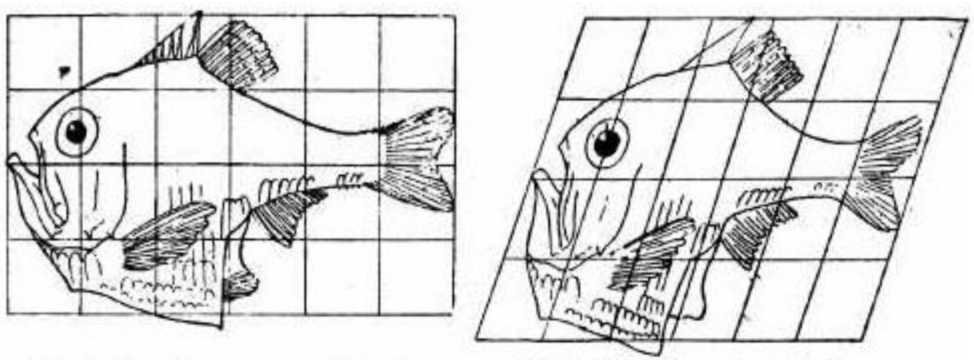
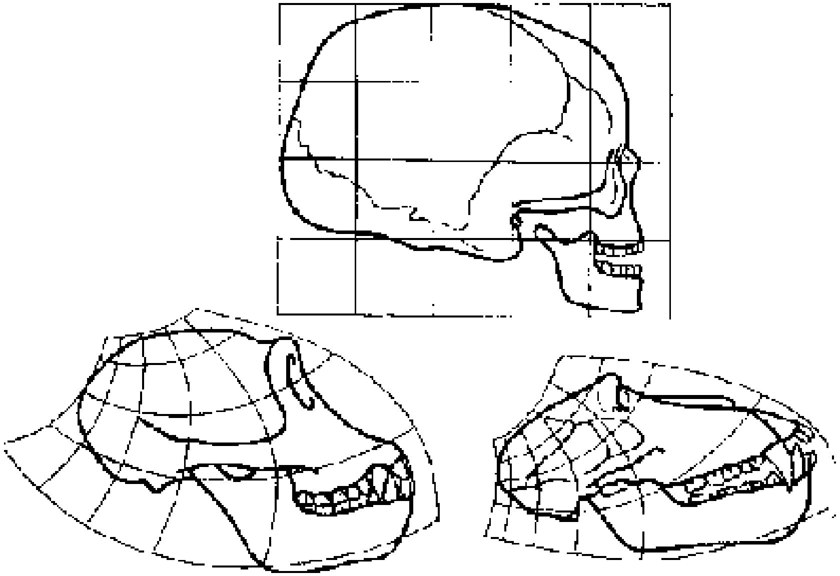


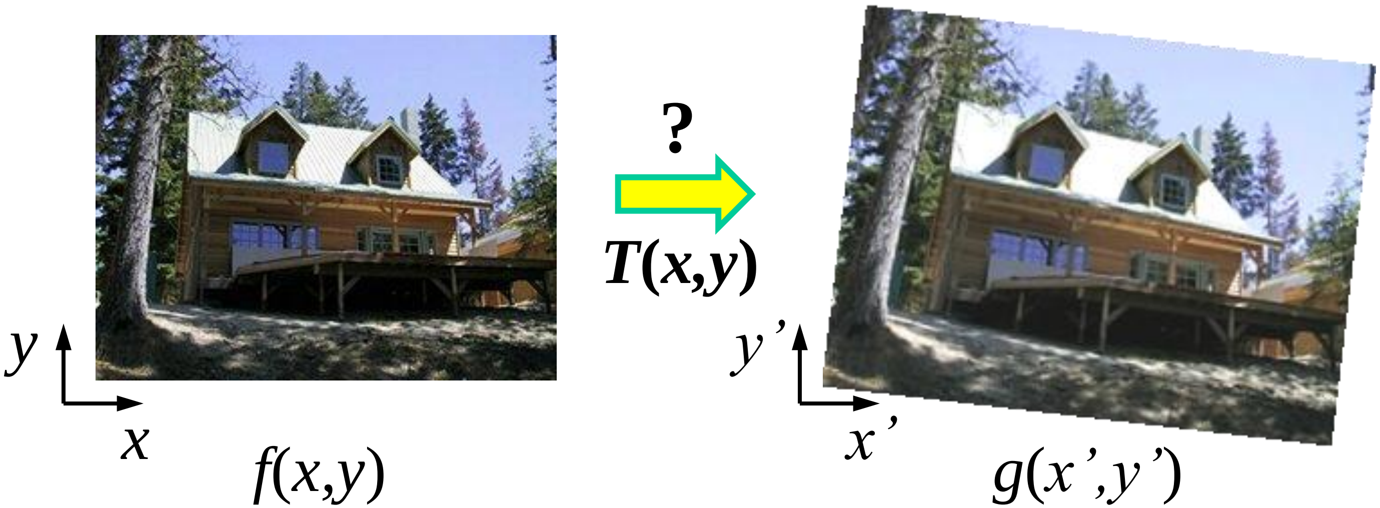
Fig. 517. *Argyropelecus Olfersi.* Fig. 518. *Sternoptyx diaphana.*



Skulls of a human, a chimpanzee and a baboon and transformations between them

Slide by Durand and Freeman

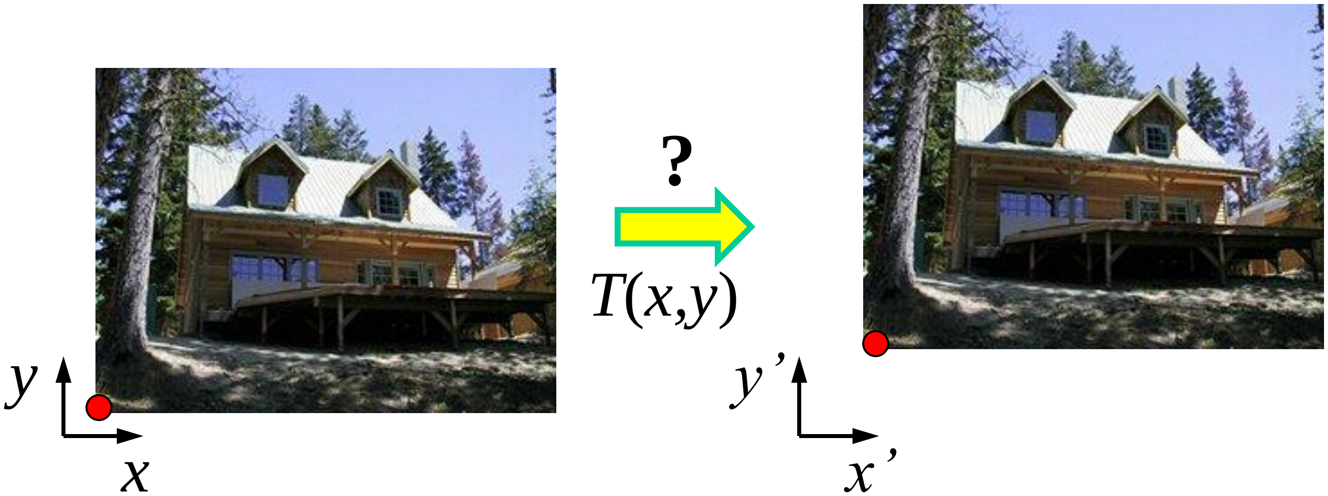
Recovering Transformations



What if we know *f* and *g* and want to recover the transform T?

- e.g. better align images from Project 1
- willing to let user provide correspondences
  - How many do we need?

Translation: # correspondences?



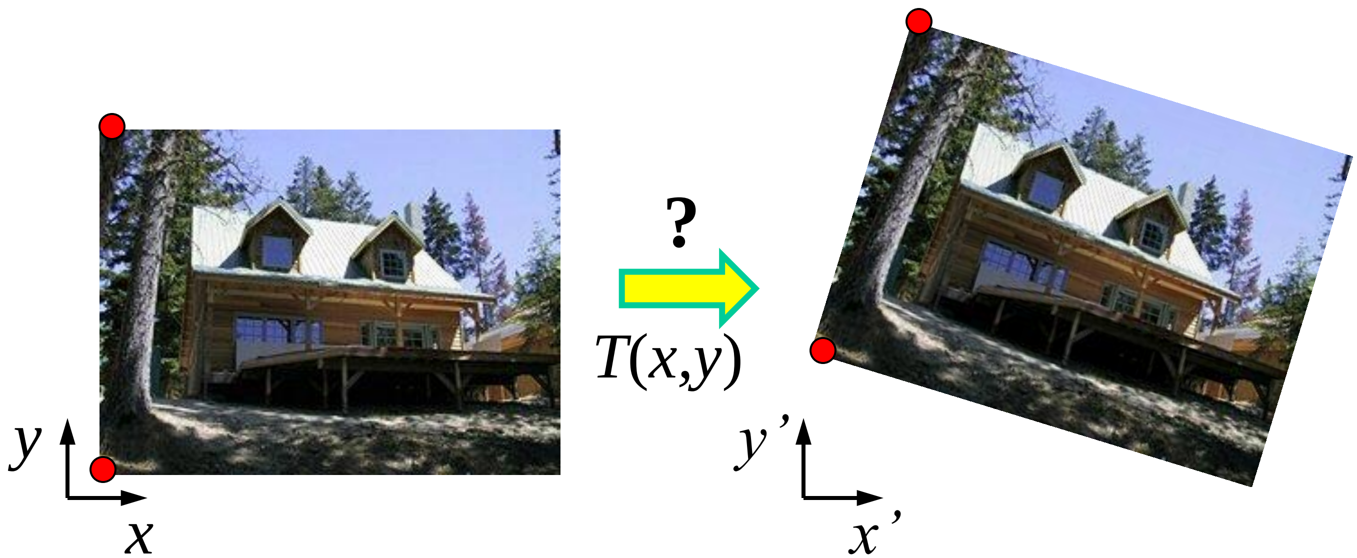
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

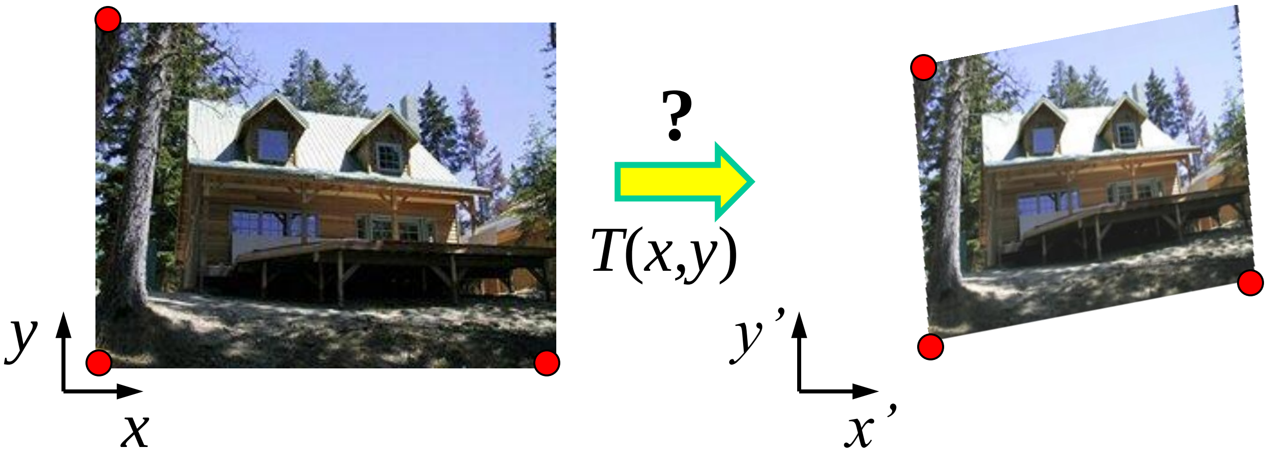
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

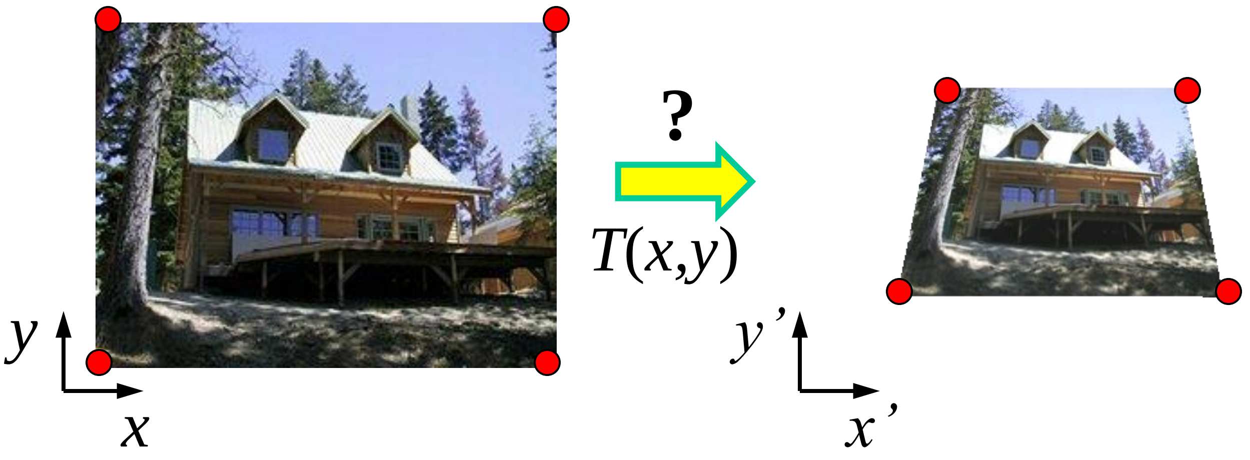
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

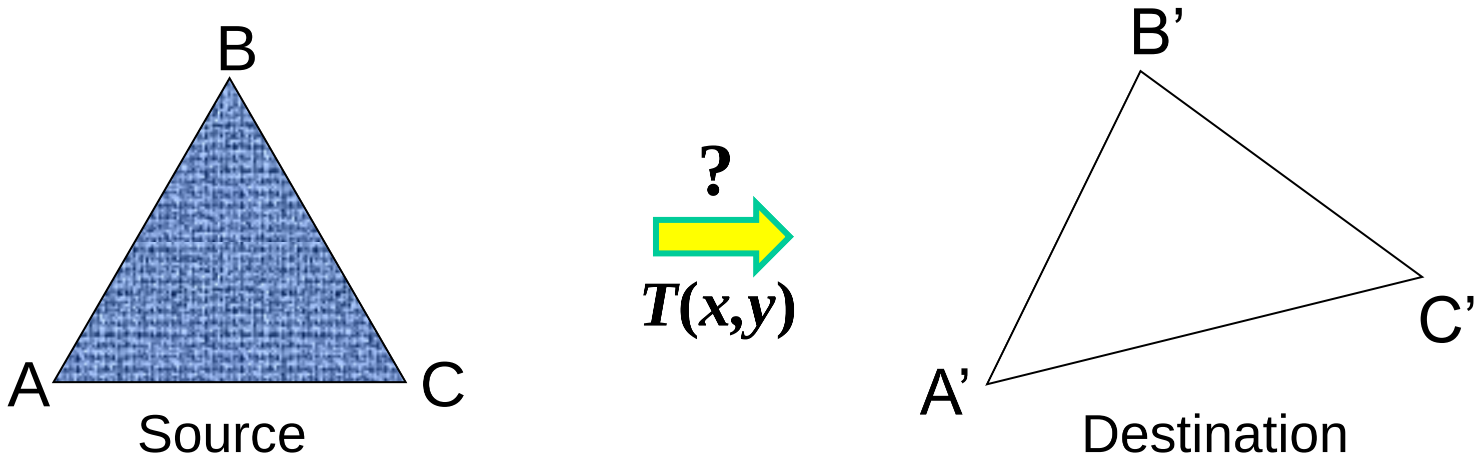
Projective: # correspondences?



How many correspondences needed for projective?

How many DOF?

Example: warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

Need to find transform T to transfer all pixels from one to the other.

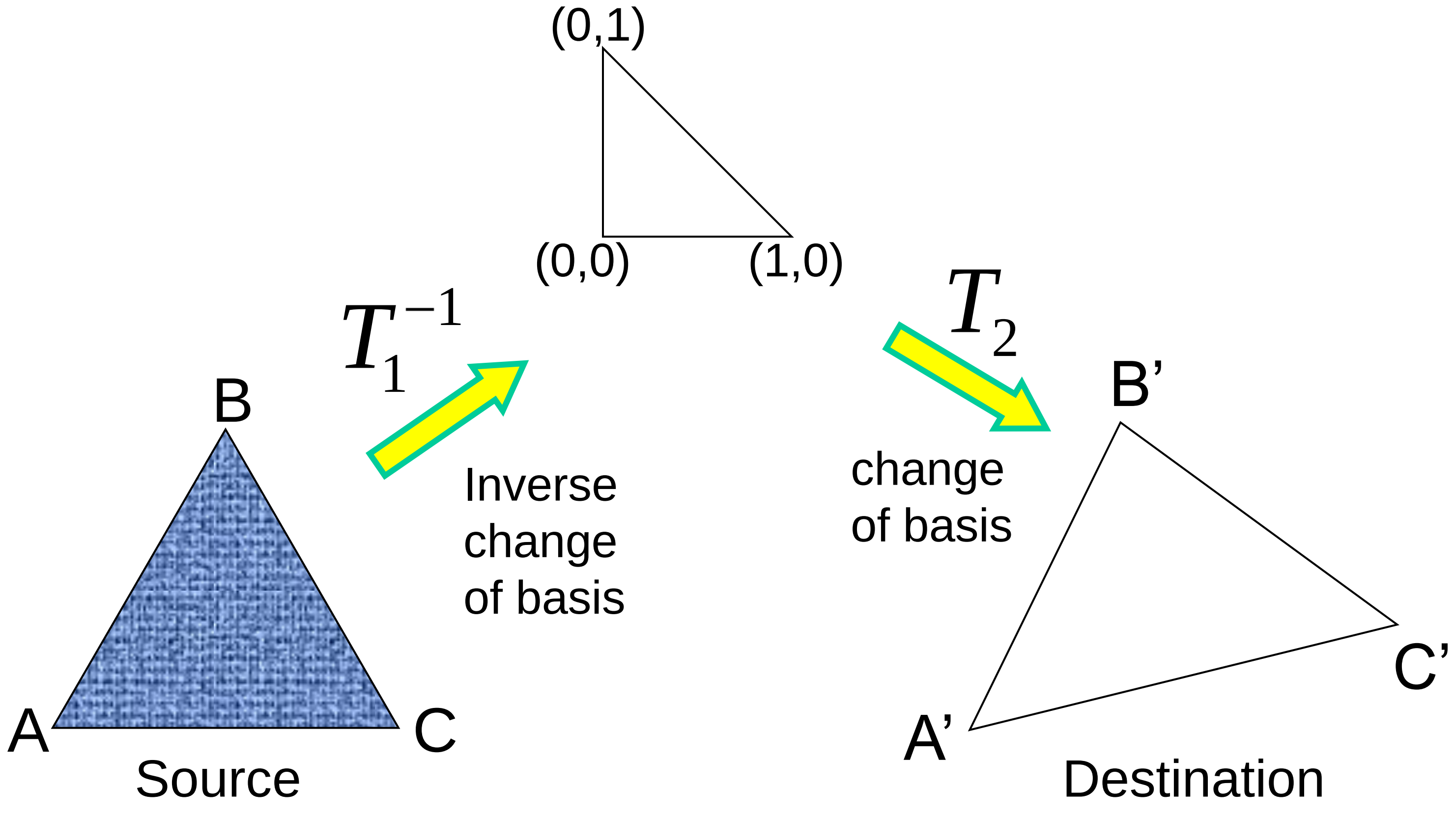
What kind of transformation is T?

How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways:
Algebraic and geometric

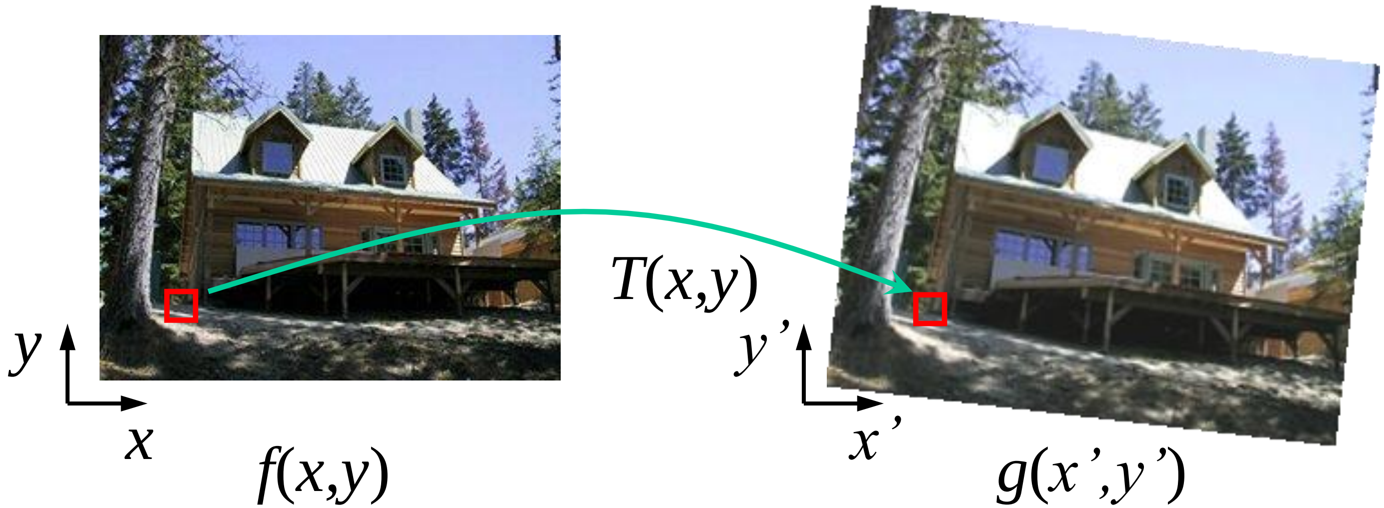
warping triangles (Barycentric Coordinaes)



Don't forget to move the origin too!

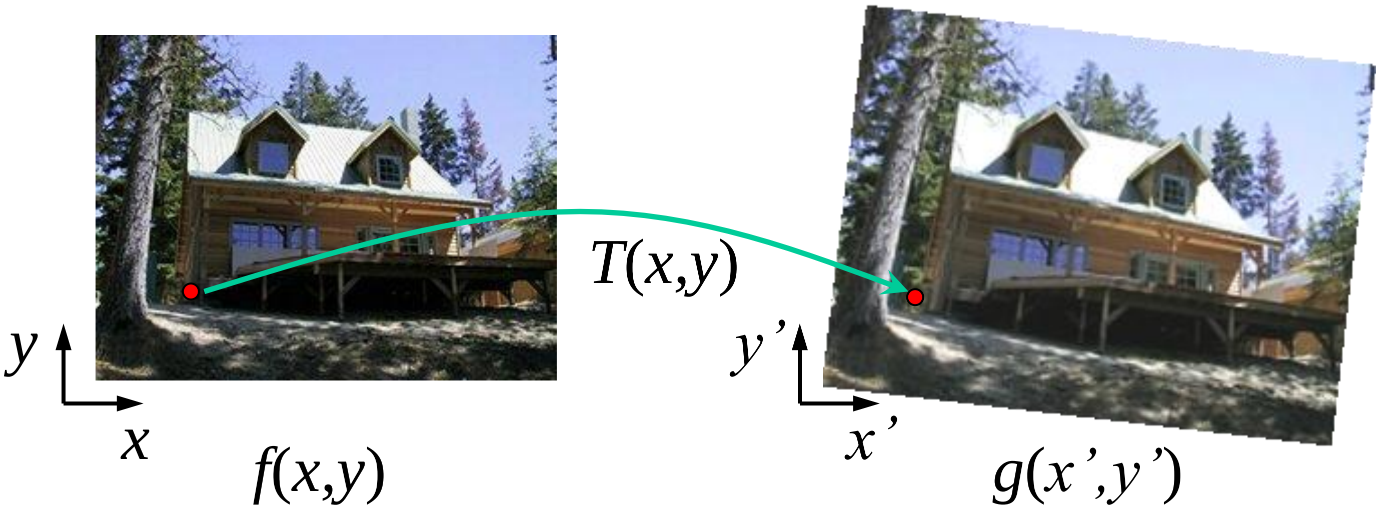
Very useful for Project 3… (hint,hint,nudge,nudge)

Image warping



Given a coordinate transform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

Forward warping

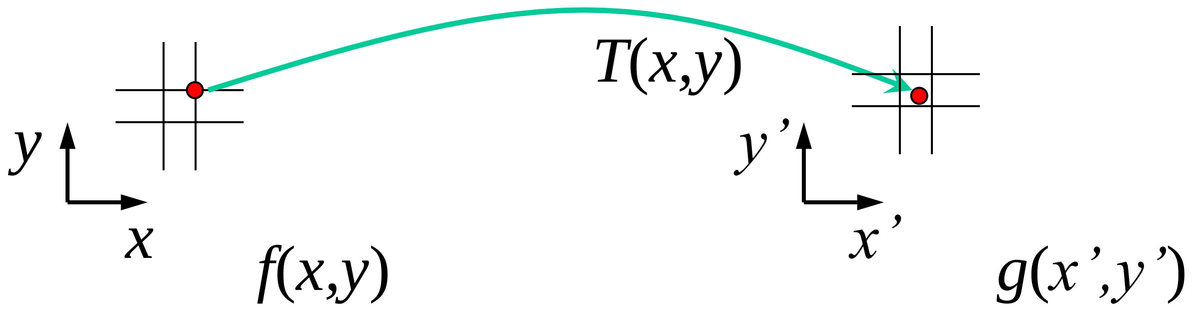


Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

Forward warping



Send each pixel $f(x,y)$ to its corresponding location

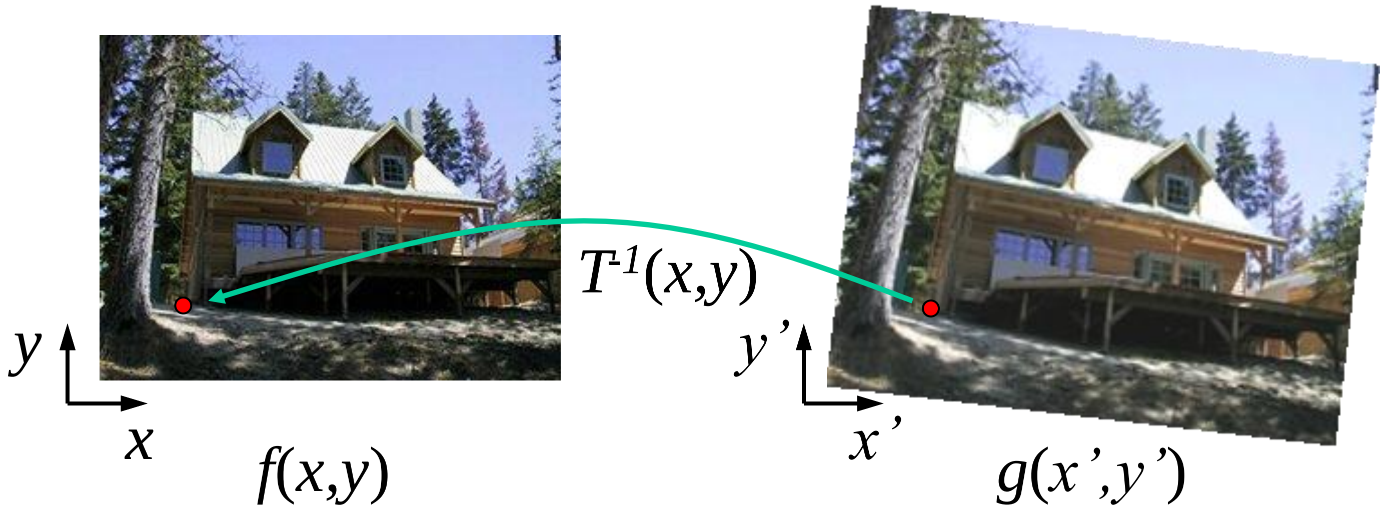
$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"
- Check out `griddata` in Matlab

Inverse warping

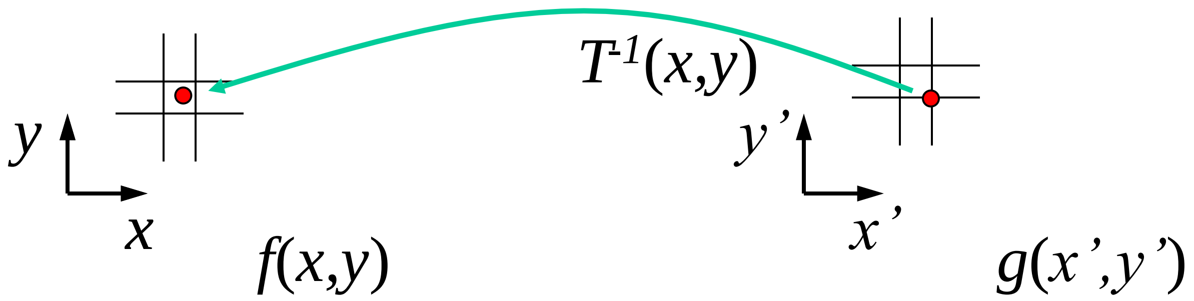


Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab

Forward vs. inverse warping

Q: which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Morphing = Object Averaging



The aim is to find "an average" between two objects

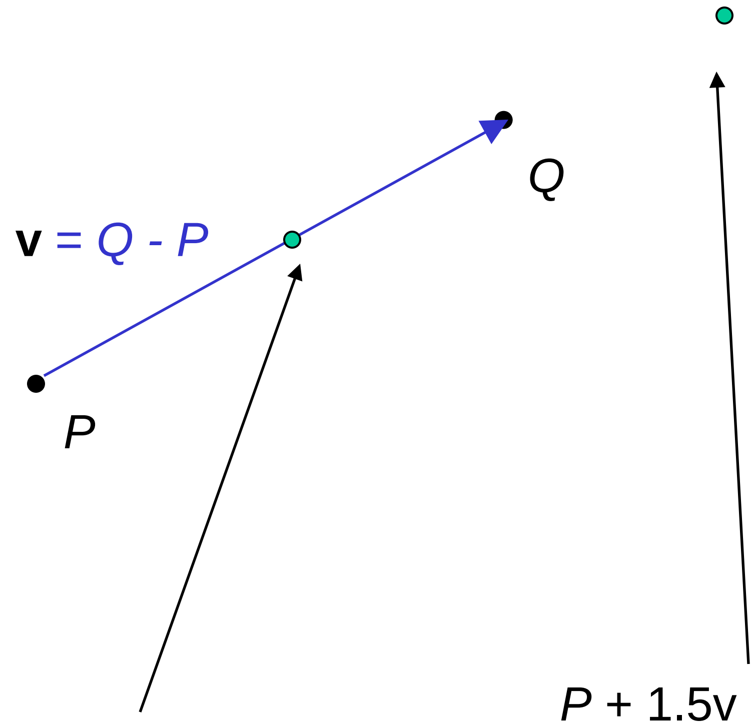
- Not an average of two <u>images of objects</u>…
- …but an image of the <u>average object</u>!
- How can we make a smooth transition in time?
  - Do a "weighted average" over time t

How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
  - Usually required user/artist input

Averaging Points

What's the average of P and Q?



$P + 0.5\mathrm{v}$
$= P + 0.5(Q - P)$
$= 0.5P + 0.5\,Q$

$P + 1.5\mathrm{v}$
$= P + 1.5(Q - P)$
$= -0.5P + 1.5\,Q$
(extrapolation)

Linear Interpolation
(Affine Combination):
New point $aP + bQ$,
defined only when $a+b = 1$
So $aP+bQ = aP+(1-a)Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

Interpolate whole images:

$$Image_{halfway} = (1-t)*Image_1 + t*image_2$$

This is called **cross-dissolve** in film industry

But what is the images are not aligned?

Idea #2: Align, then cross-disolve

Align first, then cross-dissolve

- Alignment using global warp – picture still valid

Dog Averaging

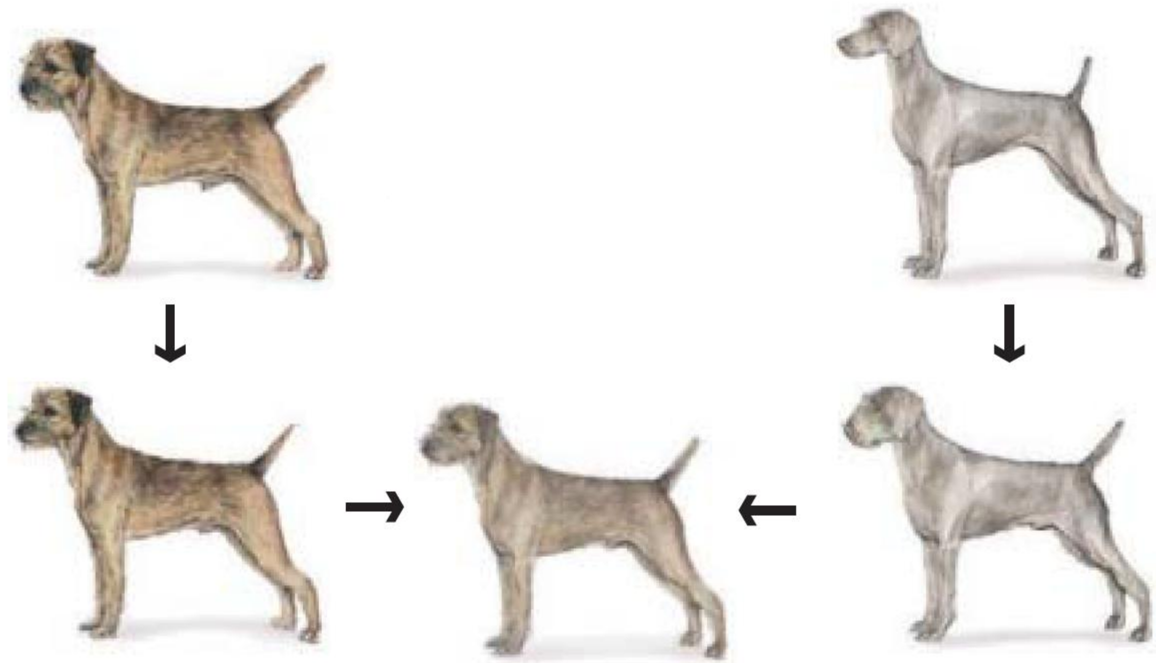
What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

- Nose to nose, tail to tail, etc.
- This is a local (non-parametric) warp

Idea #3: Local warp, then cross-dissolve



Morphing procedure:

for every t,

1. Find the average shape (the "mean dog"☺)
   - local warping
2. Find the average color
   - Cross-dissolve the warped images

Local (non-parametric) Image Warping

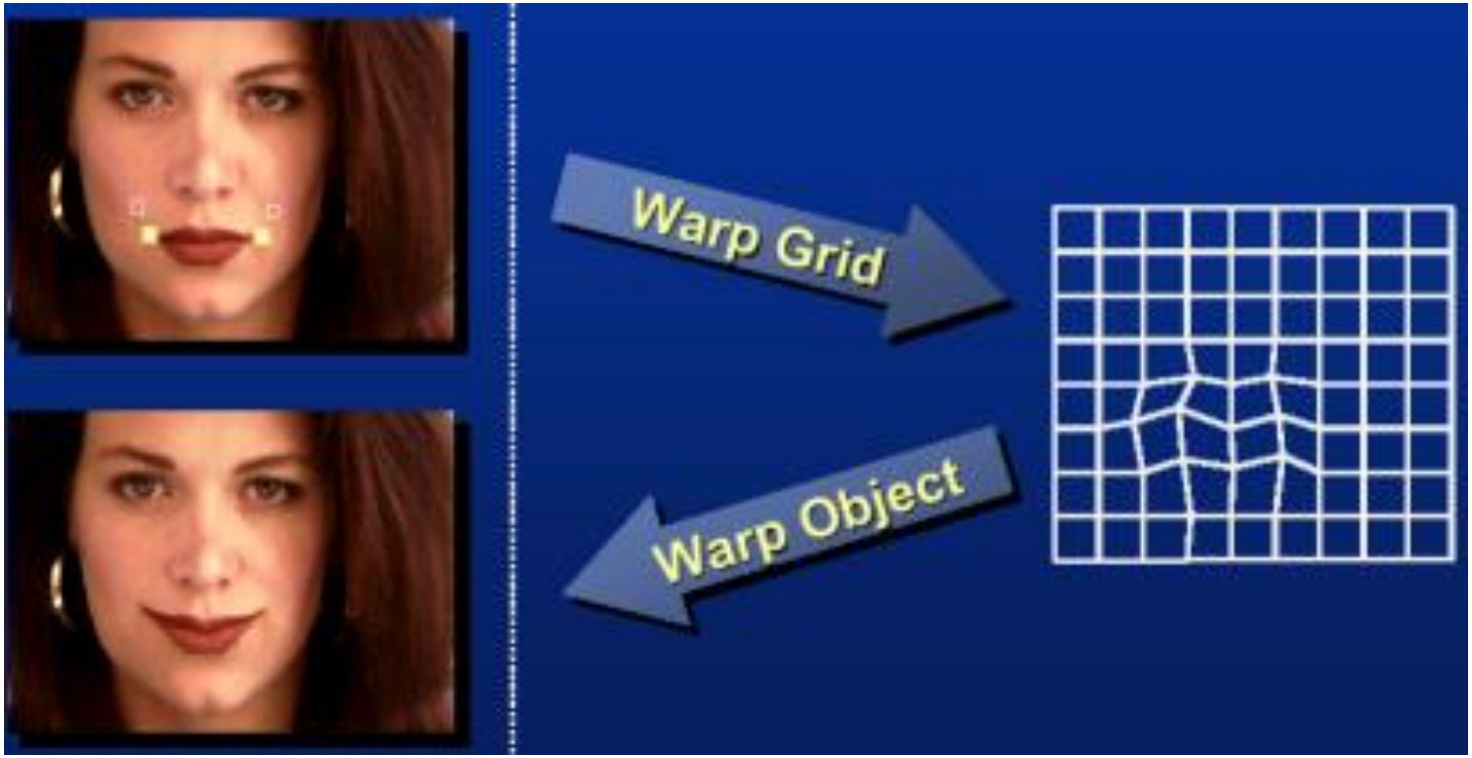
Need to specify a more detailed warp function

- Global warps were functions of a few (2,4,8) parameters
- Non-parametric warps u(x,y) and v(x,y) can be defined independently for every single location x,y!
- Once we know vector field u,v we can easily warp each pixel (use backward warping with interpolation)

Image Warping – non-parametric

Move control points to specify a spline warp

Spline produces a smooth vector field

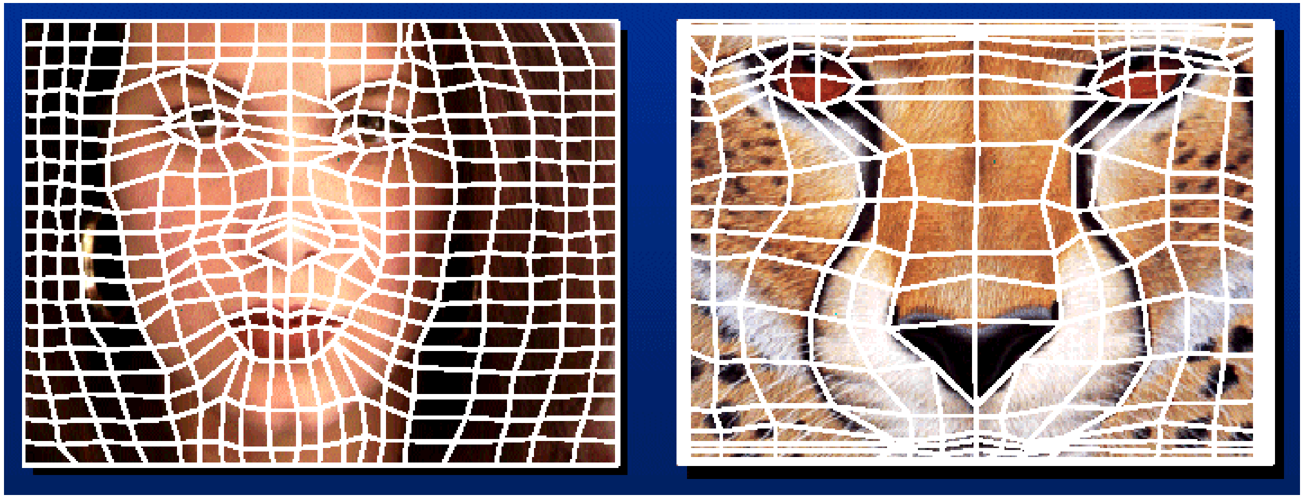


Warp specification - dense

How can we specify the warp?

Specify corresponding *spline control points*

- *interpolate* to a complete warping function



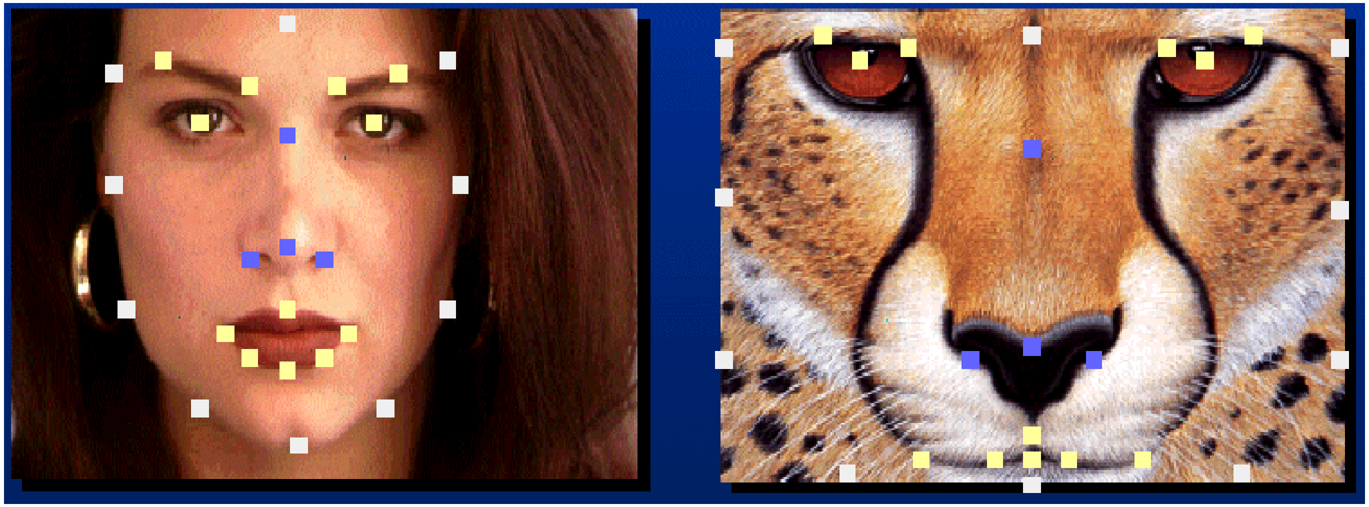
But we want to specify only a few points, not a grid

Warp specification - sparse

How can we specify the warp?

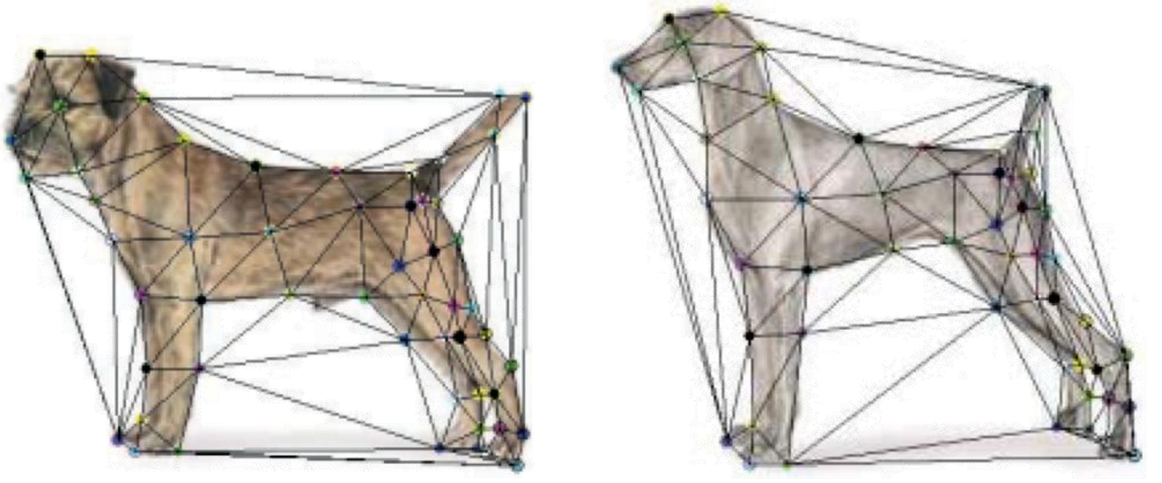
Specify corresponding *points*

- *interpolate* to a complete warping function
- How do we do it?



How do we go from feature points to pixels?

Triangular Mesh

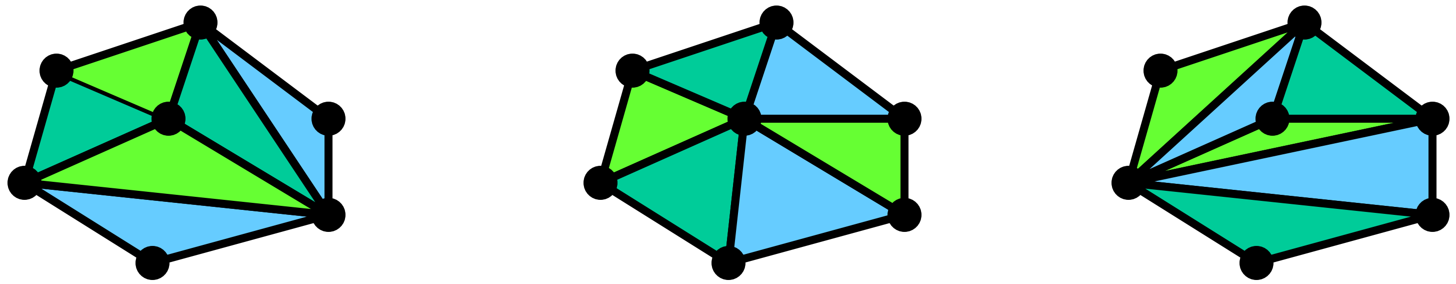


1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
   - How do we warp a triangle?
   - 3 points = affine warp!
   - Just like texture mapping

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

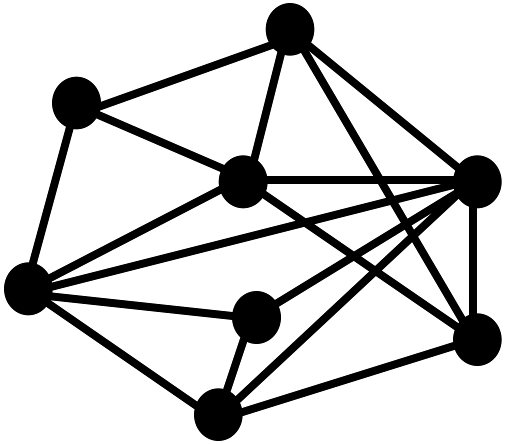
There are an exponential number of triangulations of a point set.



An O(n^3) Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



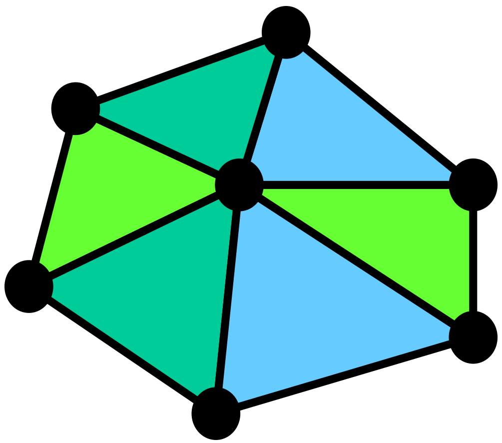
“Quality” Triangulations

Let $\alpha(T) = (\alpha_1, \alpha_2, \ldots, \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.

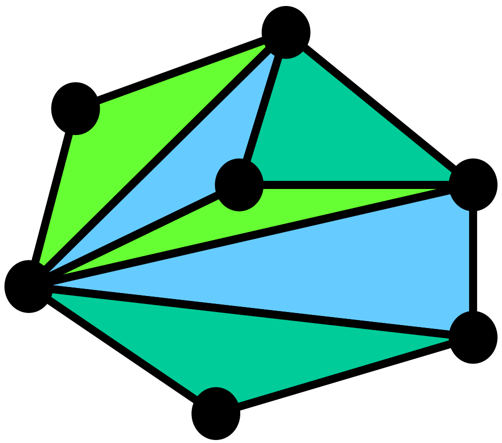
A triangulation T_1 will be “better” than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles



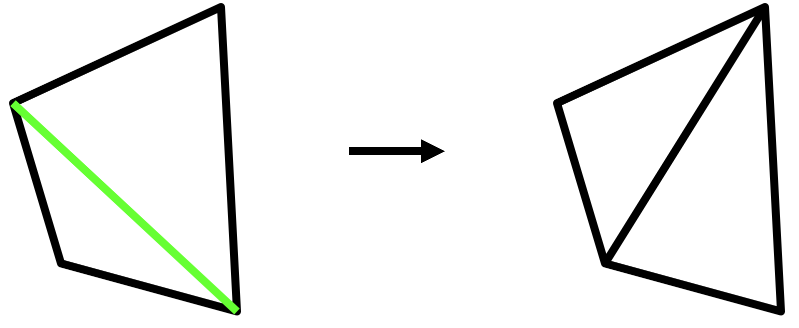
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

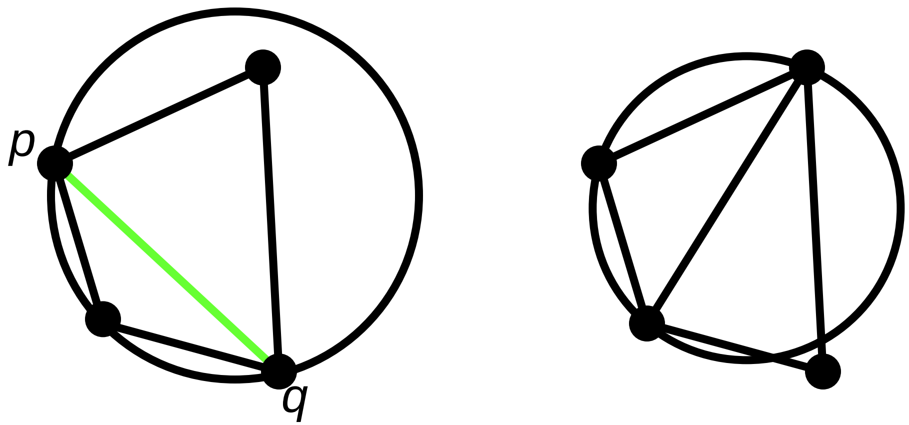


If an edge flip improves the triangulation, the first edge is called *illegal*.

Illegal Edges

Lemma: An edge *pq* is illegal iff one of its opposite vertices is inside the circle defined by the other three vertices.

Proof: By Thales' theorem.



Theorem: A Delaunay triangulation does not contain illegal edges.

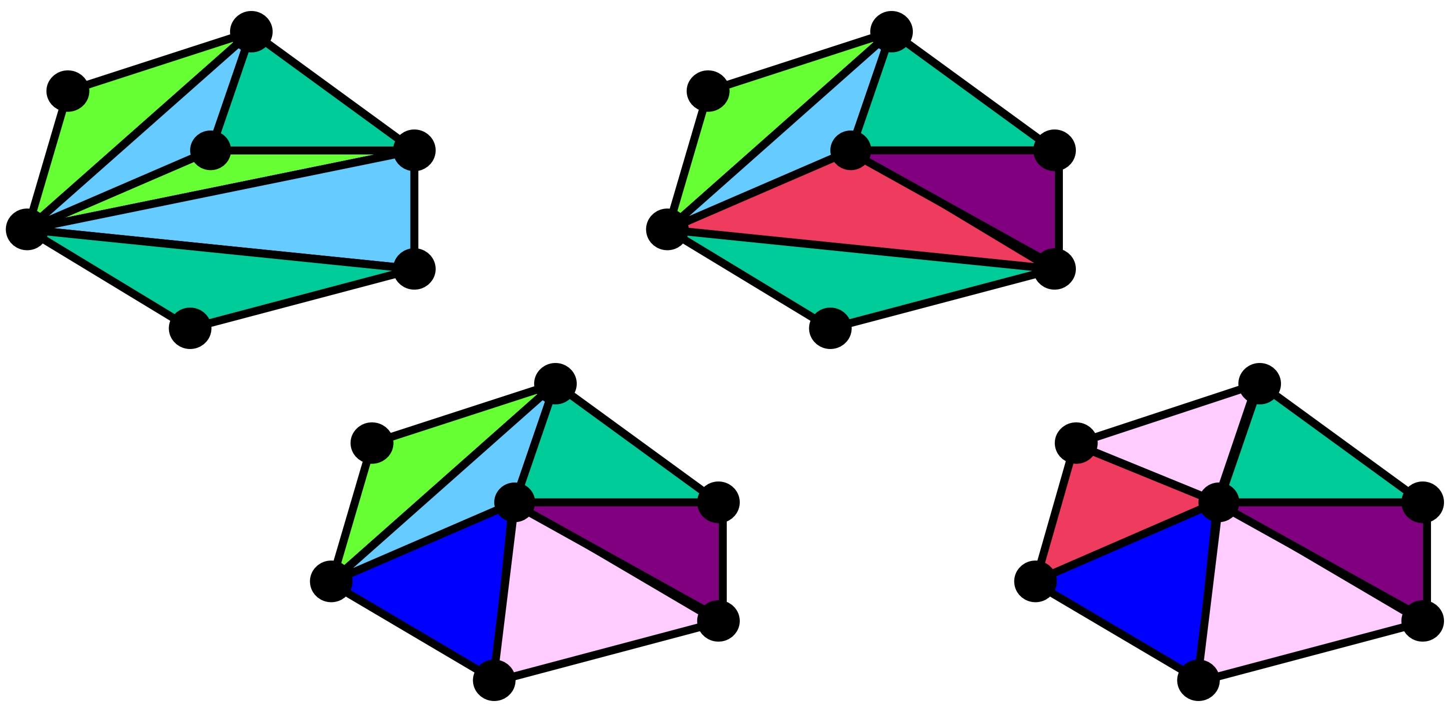
Corollary: A triangle is Delaunay iff the circle through its vertices is empty of other sites.

Corollary: The Delaunay triangulation is not unique if more than three sites are co-circular.

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

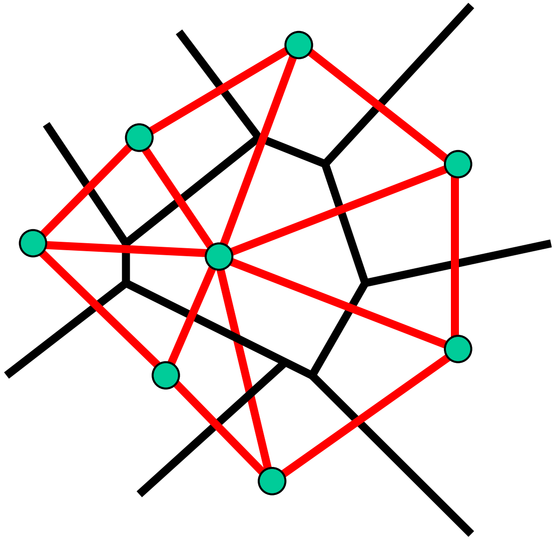
Could take a long time to terminate.



Delaunay Triangulation by Duality

General position assumption: There are no four co-circular points.

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.



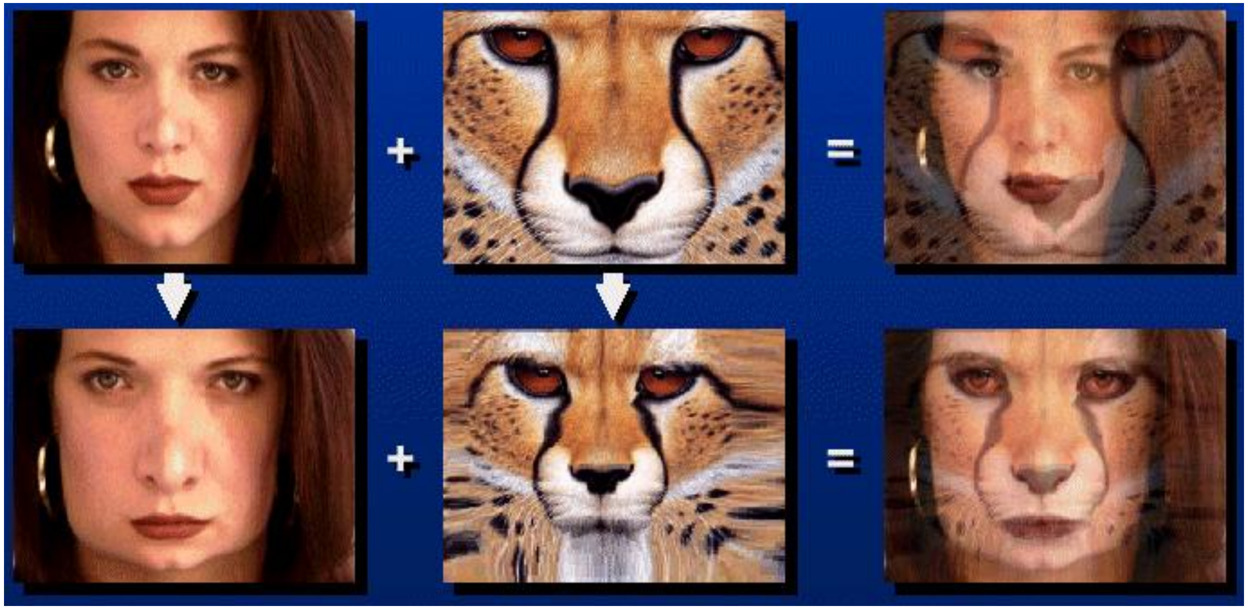
Corollary: The DT may be constructed in O(*n*log*n*) time.

This is what Matlab's `delaunay` function uses.

Image Morphing

We know how to warp one image into the other, but how do we create a morphing sequence?

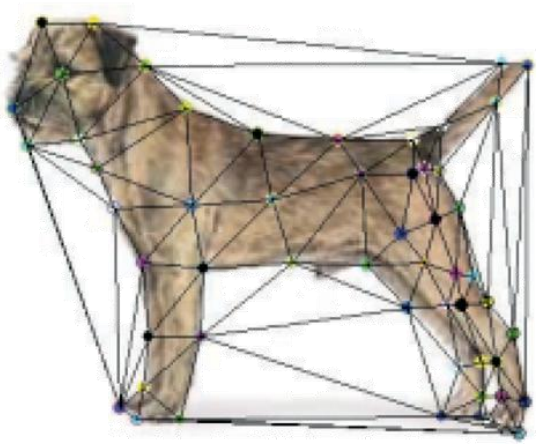
1. Create an intermediate shape (by interpolation)
2. Warp both images towards it
3. Cross-dissolve the colors in the newly warped images

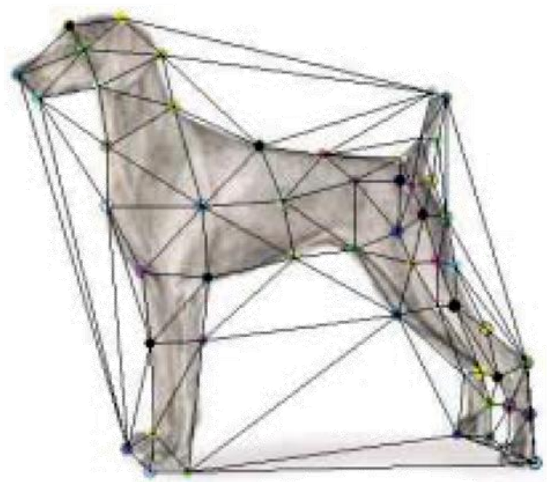


Warp interpolation

How do we create an intermediate warp at time t?

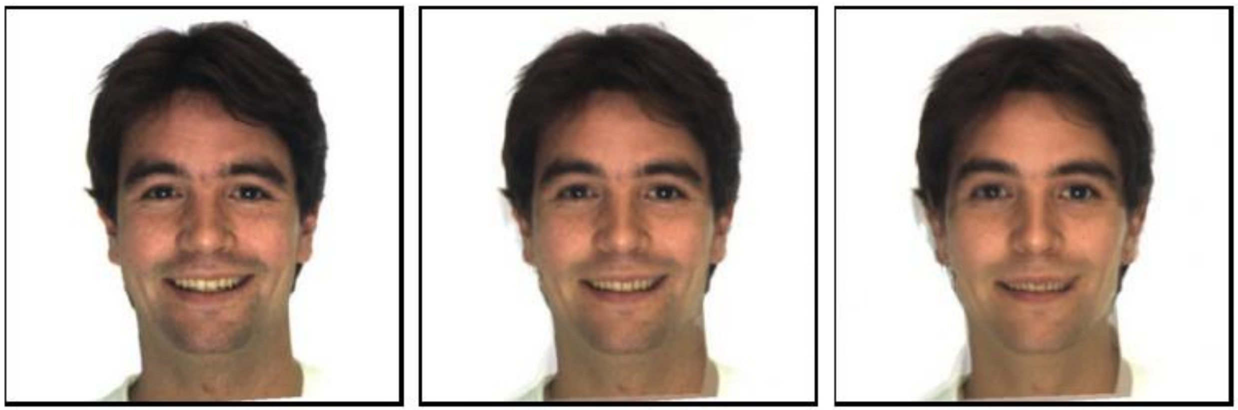
- Assume t = [0,1]
- Simple linear interpolation of each feature pair
- (1-t)*p1+t*p0 for corresponding features p0 and p1





Morphing & matting

Extract foreground first to avoid artifacts in the background

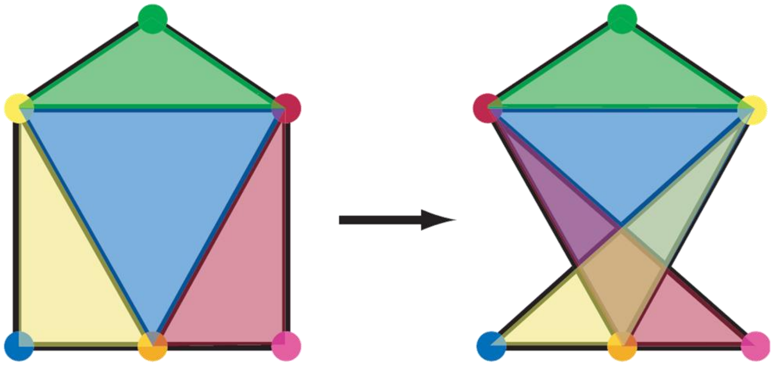


(c) $\alpha = 0.0$ (d) $\alpha = 0.2$ (e) $\alpha = 0.4$

(f) $\alpha = 0.6$ (g) $\alpha = 0.8$ (h) $\alpha = 1.0$

Slide by Durand and Freeman

Other Issues



Beware of folding

- You are probably trying to do something 3D-ish

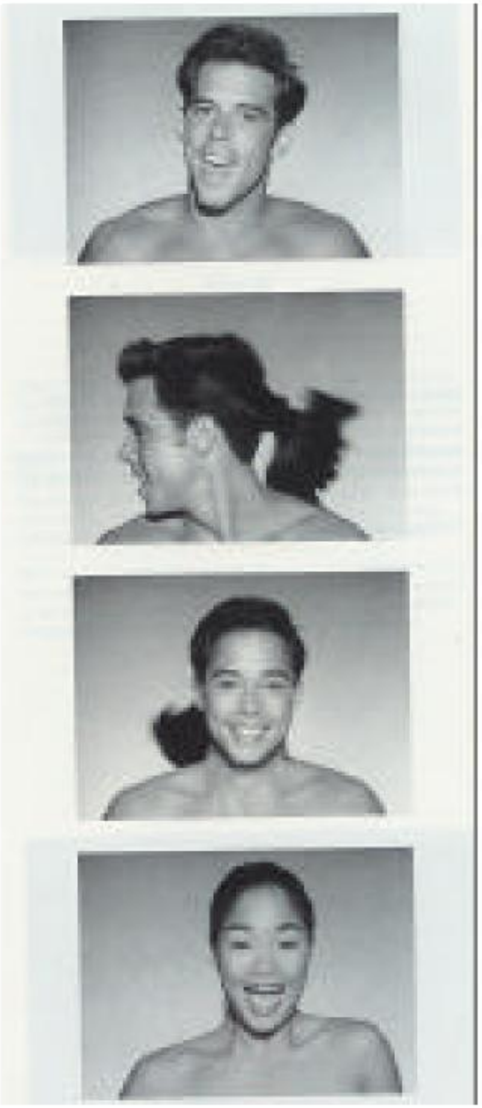
Morphing can be generalized into 3D

- If you have 3D data, that is!

Extrapolation can sometimes produce interesting effects

- Caricatures

Dynamic Scene



Project #3

- Given two photos, produce a 60-frame morph animation
  - Use triangulation-based morphing (lots of helpful Matlab tools)
  - Need to write triangle-to-triangle warp (can’t use Matlab tools)
- We put all animations together into a movie!

See last year’s movie!