The Camera
How do we see the world?

Let’s design a camera

• Idea 1: put a piece of film in front of an object
• Do we get a reasonable image?
Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**
- How does this transform the image?
Pinhole camera model

Pinhole model:

• Captures **pencil of rays** – all rays through a single point
• The point is called **Center of Projection (COP)**
• The image is formed on the **Image Plane**
• **Effective focal length** $f$ is distance from COP to Image Plane
Dimensionality Reduction Machine (3D to 2D)

What have we lost?

- Angles
- Distances (lengths)

Figures © Stephen E. Palmer, 2002
Funny things happen…
Parallel lines aren’t...
Lengths can’t be trusted...
...but humans adopt!

Müller-Lyer Illusion

We don’t make measurements in the image plane

http://www.michaelbach.de/ot/sze_muelue/index.html
Modeling projection

The coordinate system

• We will use the pin-hole model as an approximation
• Put the optical center (Center Of Projection) at the origin
• Put the image plane (Projection Plane) in front of the COP
  = Why?
• The camera looks down the negative z axis
  – we need this if we want right-handed-coordinates

Slide by Steve Seitz
Modeling projection

Projection equations

- Compute intersection with PP of ray from \((x,y,z)\) to COP
- Derived using similar triangles (on board)

\[
(x, y, z) \rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}, -d\right)
\]

- We get the projection by throwing out the last coordinate:

\[
(x, y, z) \rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)
\]
Homogeneous coordinates

Is this a linear transformation?
  • no—division by z is nonlinear

Trick: add one more coordinate:

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\]

homogeneous image coordinates
homogeneous scene coordinates

Converting \textit{from} homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \left(\frac{x}{w}, \frac{y}{w}\right)\quad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)
\]
Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1/d & 0 \\
0 & 0 & -1/d & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
z/d \\
1
\end{bmatrix} \Rightarrow \left(-\frac{dx}{z}, -\frac{dy}{z}\right)
\]

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1/d & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
z \\
-z/d
\end{bmatrix} \Rightarrow \left(-\frac{dx}{z}, -\frac{dy}{z}\right)
\]

divide by fourth coordinate

Slide by Steve Seitz
Orthographic Projection

Special case of perspective projection

- Distance from the COP to the PP is infinite

- Also called “parallel projection”
- What’s the projection matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
1
\end{bmatrix} \Rightarrow (x, y)
\]
What if PP is spherical with center at COP? In spherical coordinates, projection is trivial:

$$(\theta, \phi) = (\theta, \phi, d)$$

Note: doesn’t depend on focal length $d$!
Building a real camera
Camera Obscura

The first camera
- Known to Aristotle
- Depth of the room is the effective focal length

Camera Obscura, Gemma Frisius, 1558
Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/
Shrinking the aperture

Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…
Shrinking the aperture
The reason for lenses
Focus
A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance
Thin Lenses

Thin lens equation: \[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: [http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html](http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html) (by Fu-Kwun Hwang)
Varying Focus

Ren Ng
Depth Of Field
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm
Aperture controls Depth of Field

Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure
Varying the aperture

Large aperture = small DOF

Small aperture = large DOF
Nice Depth of Field effect
Field of View (Zoom)
Field of View (Zoom)

From London and Upton
Field of View (Zoom)

From London and Upton
FOV depends on Focal Length

Size of field of view governed by size of the camera retina:

\[ \varphi = \tan^{-1}\left(\frac{d}{2f}\right) \]

Smaller FOV = larger Focal Length
From Zisserman & Hartley
Field of View / Focal Length

Large FOV, small f
Camera close to car

Small FOV, large f
Camera far from the car
Fun with Focal Length (Jim Sherwood)

http://www.hash.com/users/jsherwood/tutes/focal/Zoomin.mov
Lens Flaws
Lens Flaws: Chromatic Aberration

Dispersion: wavelength-dependent refractive index
  • (enables prism to spread white light beam into rainbow)

Modifies ray-bending and lens focal length: \( f(\lambda) \)

color fringes near edges of image

Corrections: add ‘doublet’ lens of flint glass, etc.
Chromatic Aberration

Near Lens Center

Near Lens Outer Edge
Radial Distortion (e.g. ‘Barrel’ and ‘pin-cushion’)

straight lines curve around the image center
Radial Distortion

Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens
Radial Distortion

(a) Orthoscopic

(b) Barrel

(c) Pin-cushion