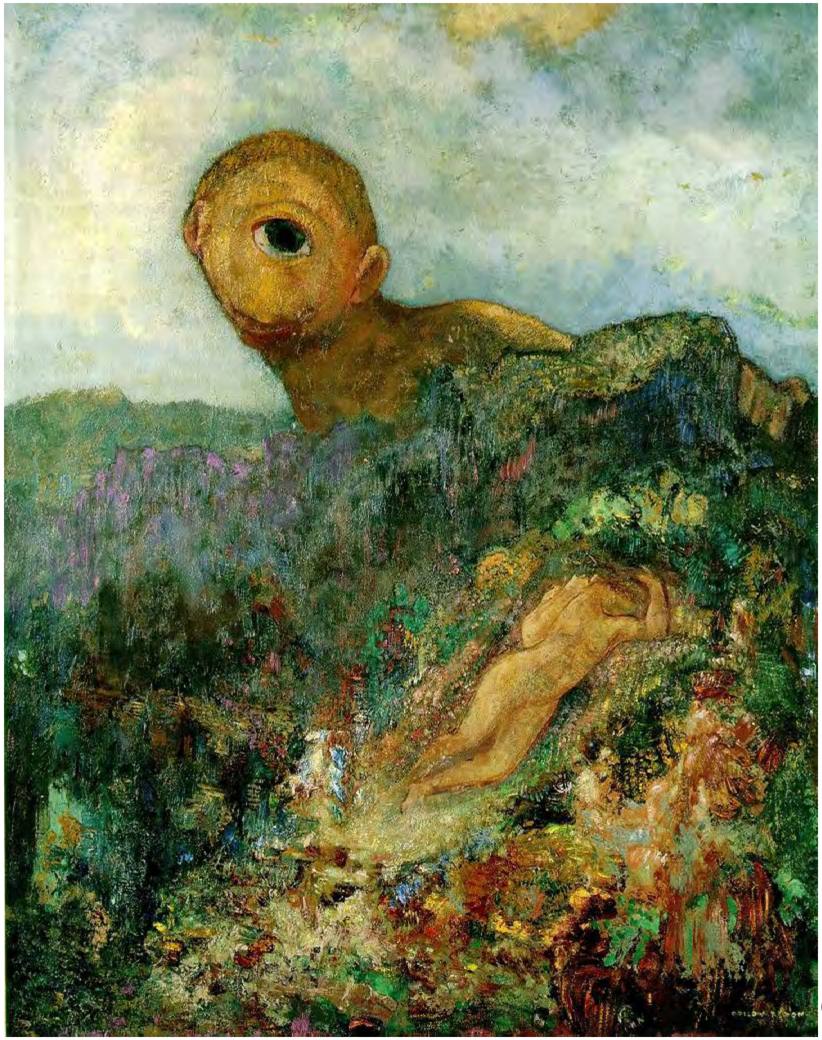
More Single View Geometry



Cyclops **Odilon Redon** 1904

15-463: Computational Photography
Alexei Efros, CMU, Fall 2007

…with a lot of slides stolen from Steve Seitz

Final Projects

Are coming up fast!

Undergrads can work in pairs, but project must be bigger.

Sample Topics:

- Full 360 panorama construction (spherical or cylindrical)
- Render in synthetic object into real scene
- Automatic Tour into the Picture (can use Pop-up labeling code)
- Build a virtual CMU campus environment
- Implement a paper discussed in class (e.g. Video Textures)
- Come up with art project that uses Comp. Photography
- Etc.

Project proposals due next Tuesday!

Pop Quiz: which is 1,2,3-point perspective

Image A

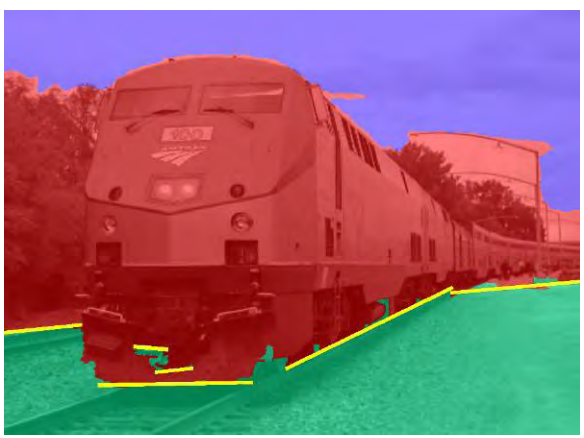
Image B

Image C

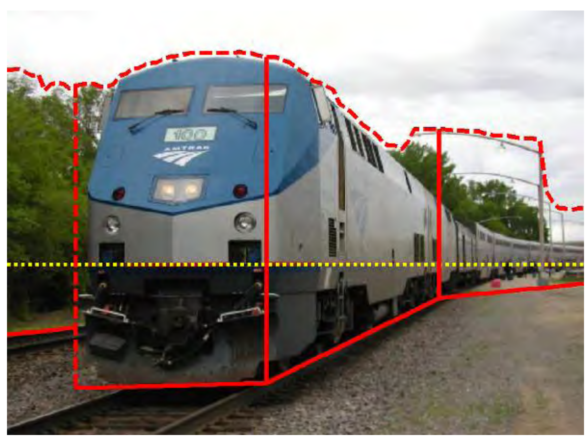
Automatic Photo Pop-up

Original Image

Geometric Labels



Fit Segments

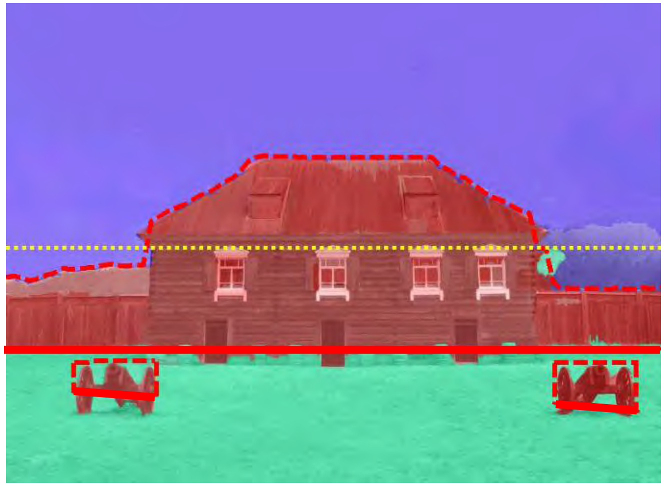


Cut and Fold

Novel View

Results

Input Image

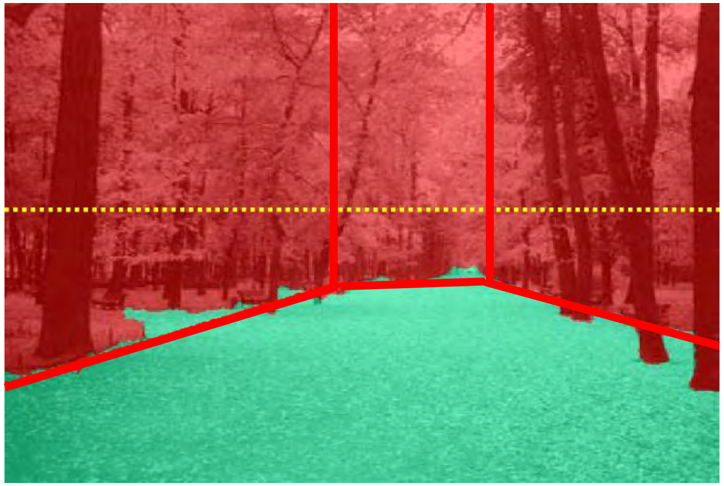


Cut and Fold

Automatic Photo Pop-up

Results

Input Image

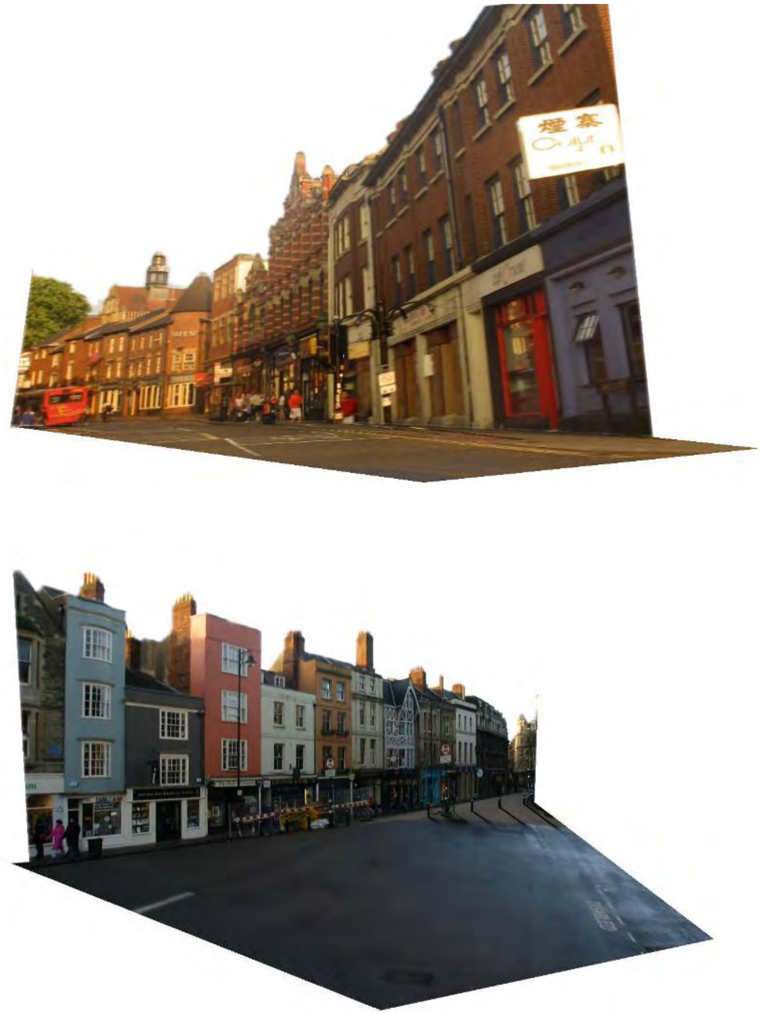


Cut and Fold

Automatic Photo Pop-up

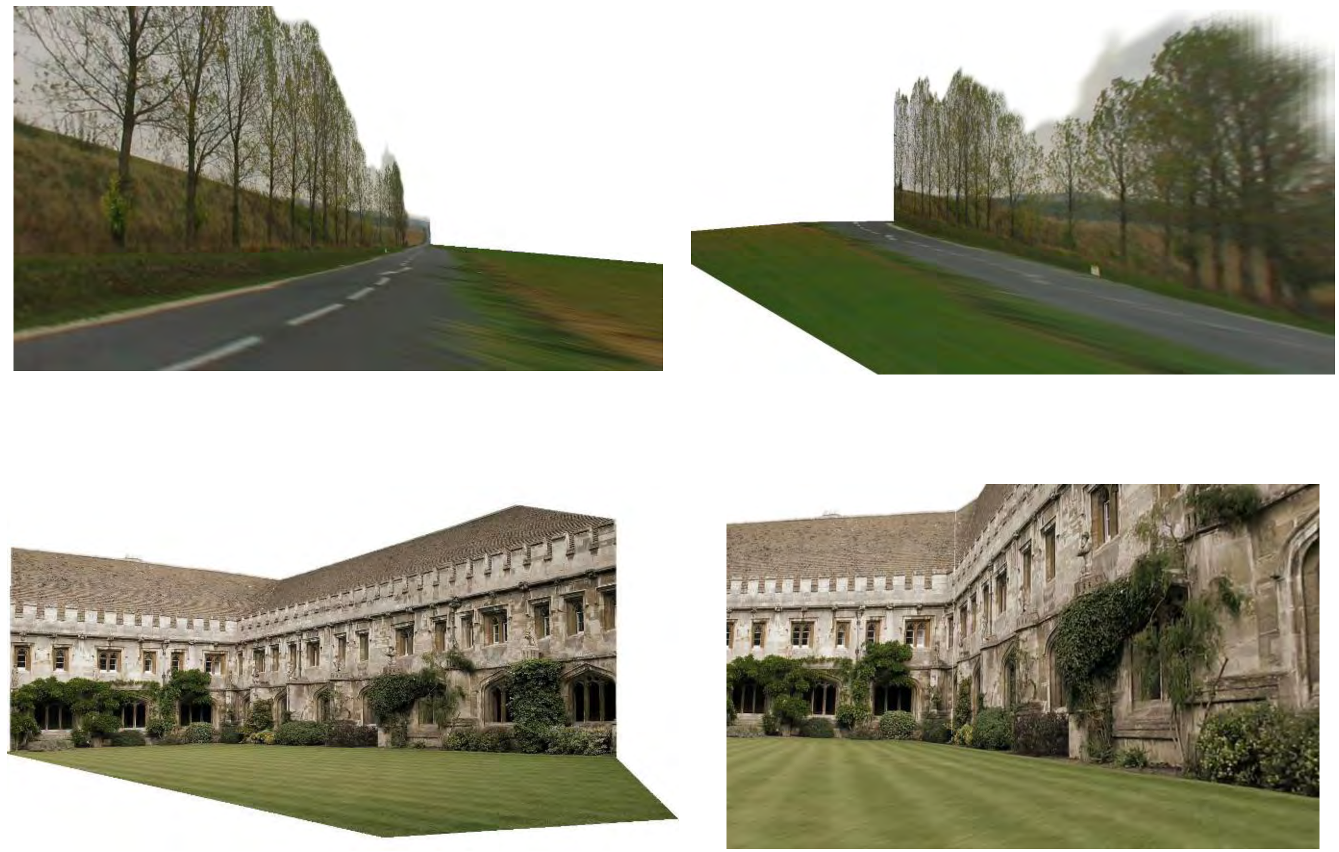
Results

Input Image



Automatic Photo Pop-up

Results



Input Images

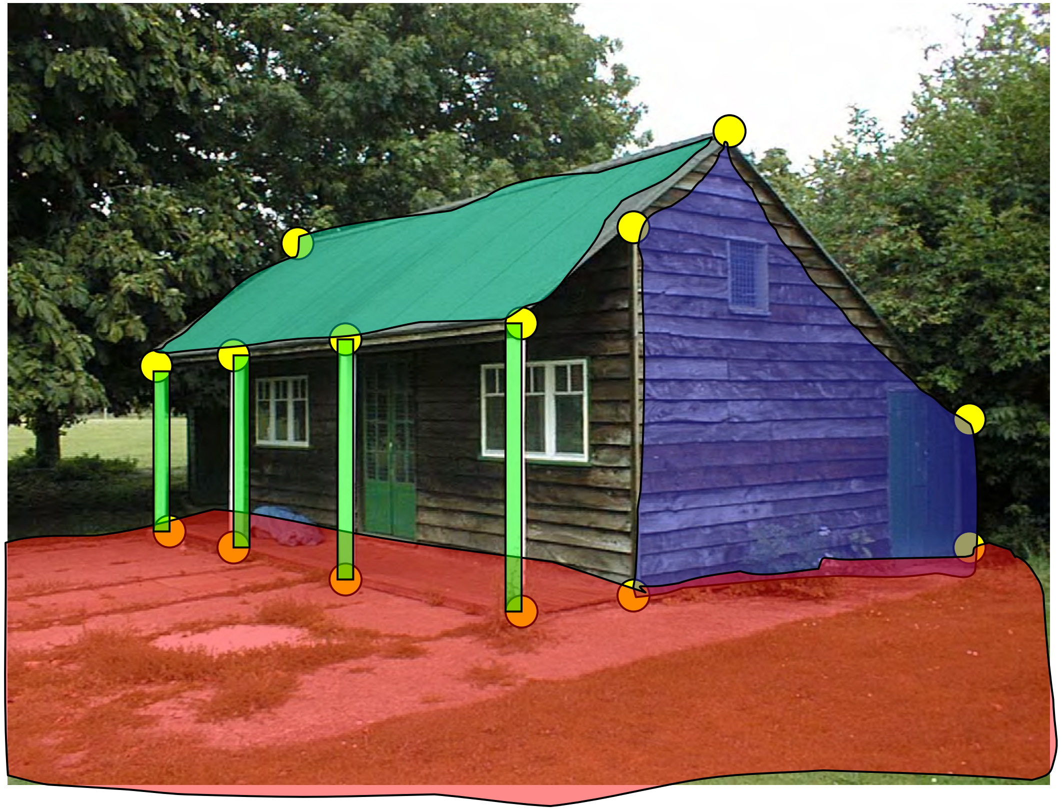
Automatic Photo Pop-up

Results

Input Image

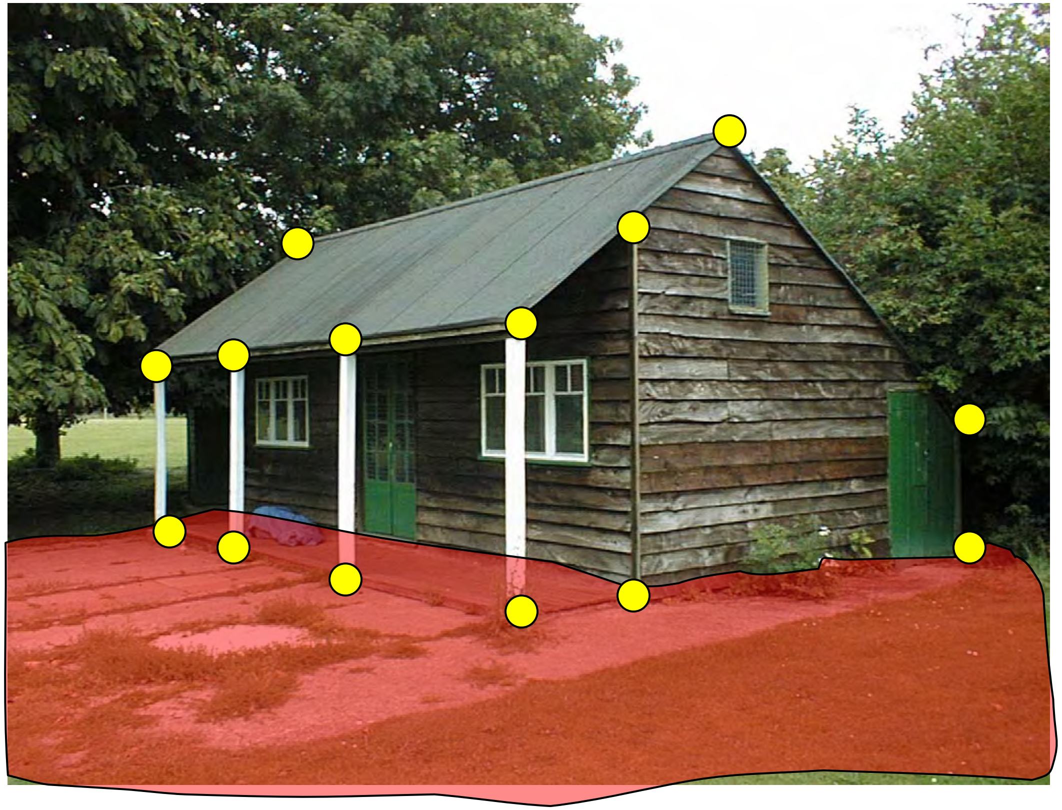
Automatic Photo Pop-up

How can we model this scene?



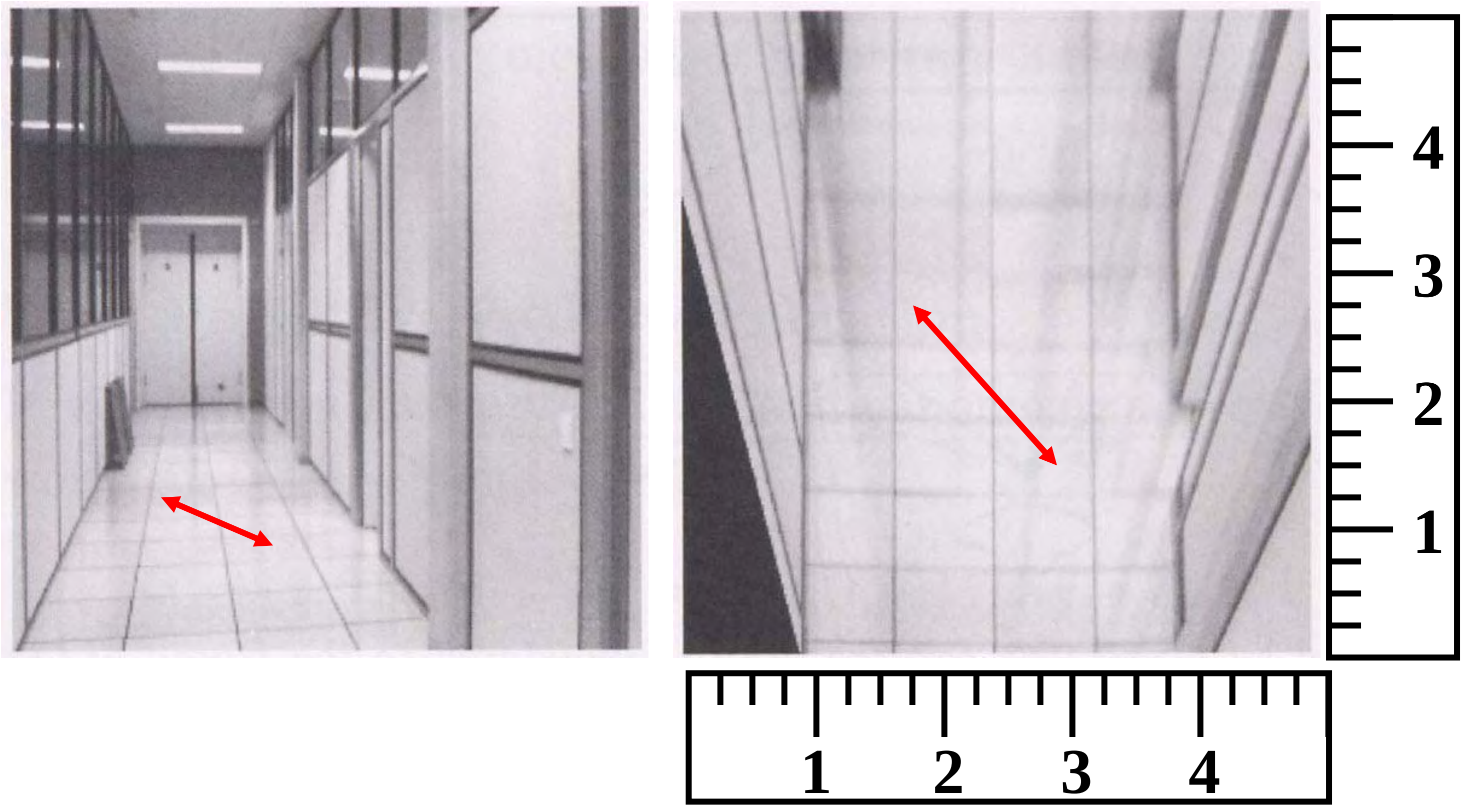
1. Find world coordinates (X,Y,Z) for a few points
2. Connect the points with planes to model geometry
   - Texture map the planes

Finding world coordinates (X,Y,Z)



1. Define the ground plane (Z=0)
2. Compute points (X,Y,0) on that plane
3. Compute the *heights* Z of all other points

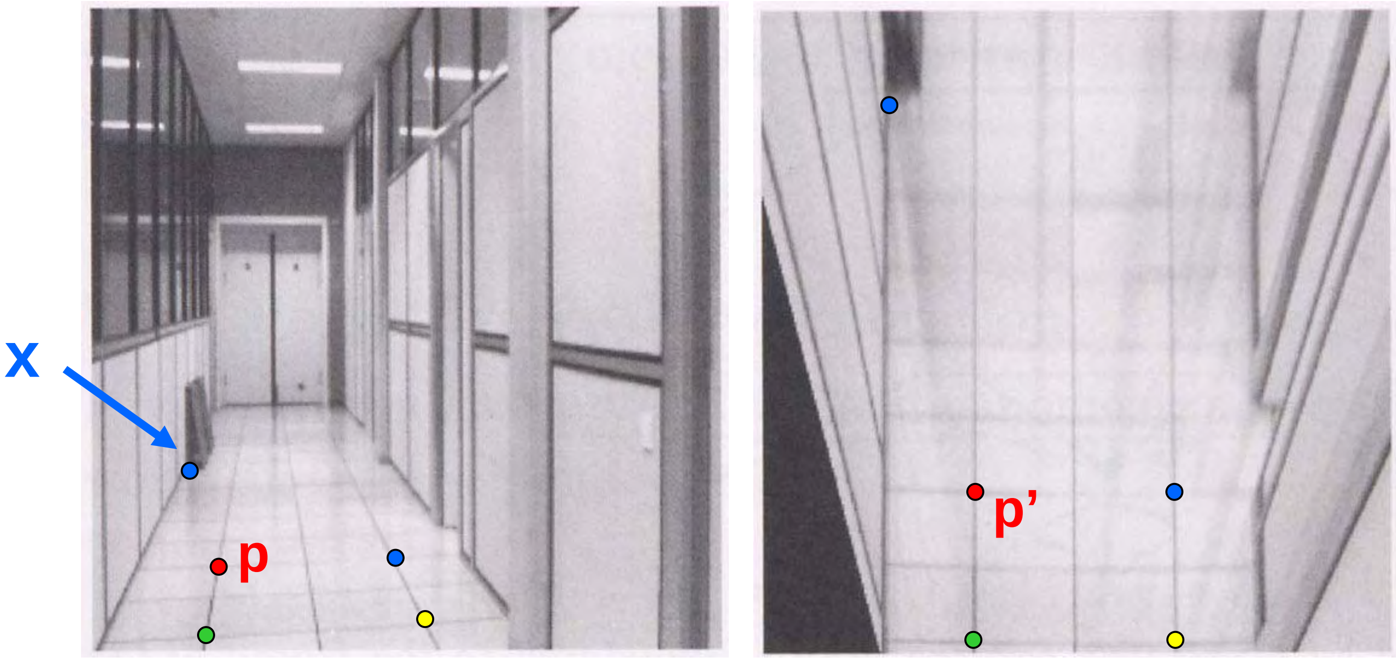
Measurements on planes



Approach: unwarp, then measure

What kind of warp is this?

Unwarp ground plane



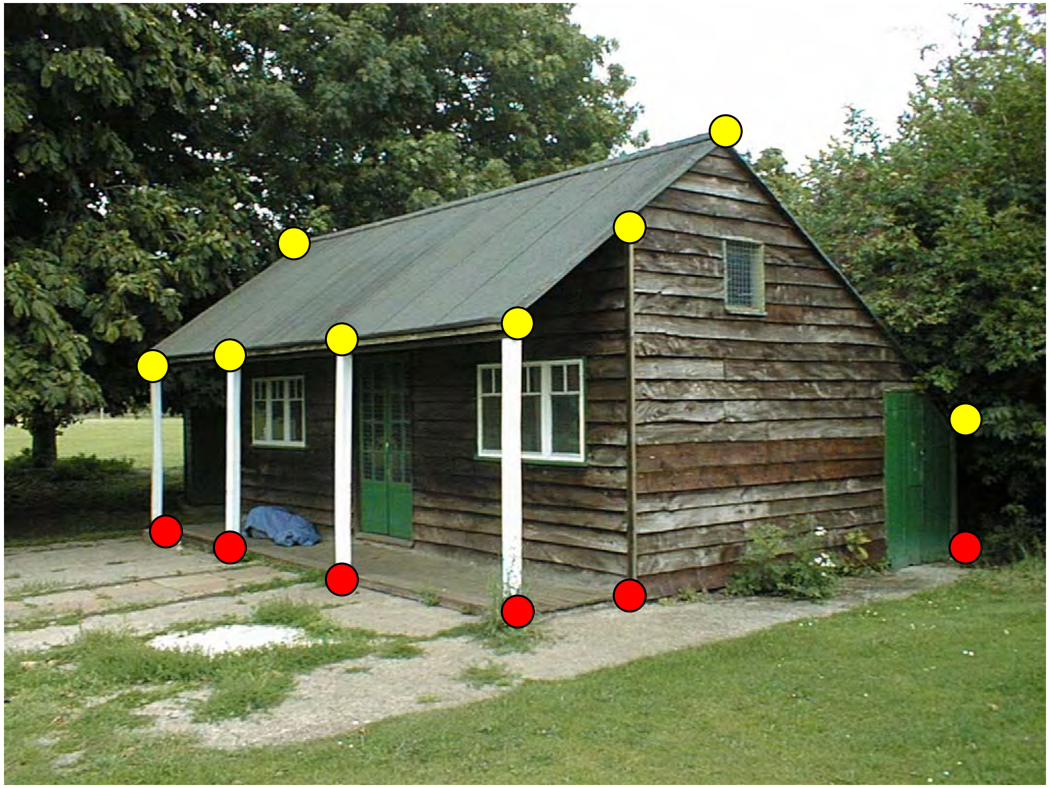
Our old friend – the homography

Need 4 reference points with world coordinates

p = (x,y)

p’ = (X,Y,0)

Finding world coordinates (X,Y,Z)



1. Define the ground plane (Z=0)
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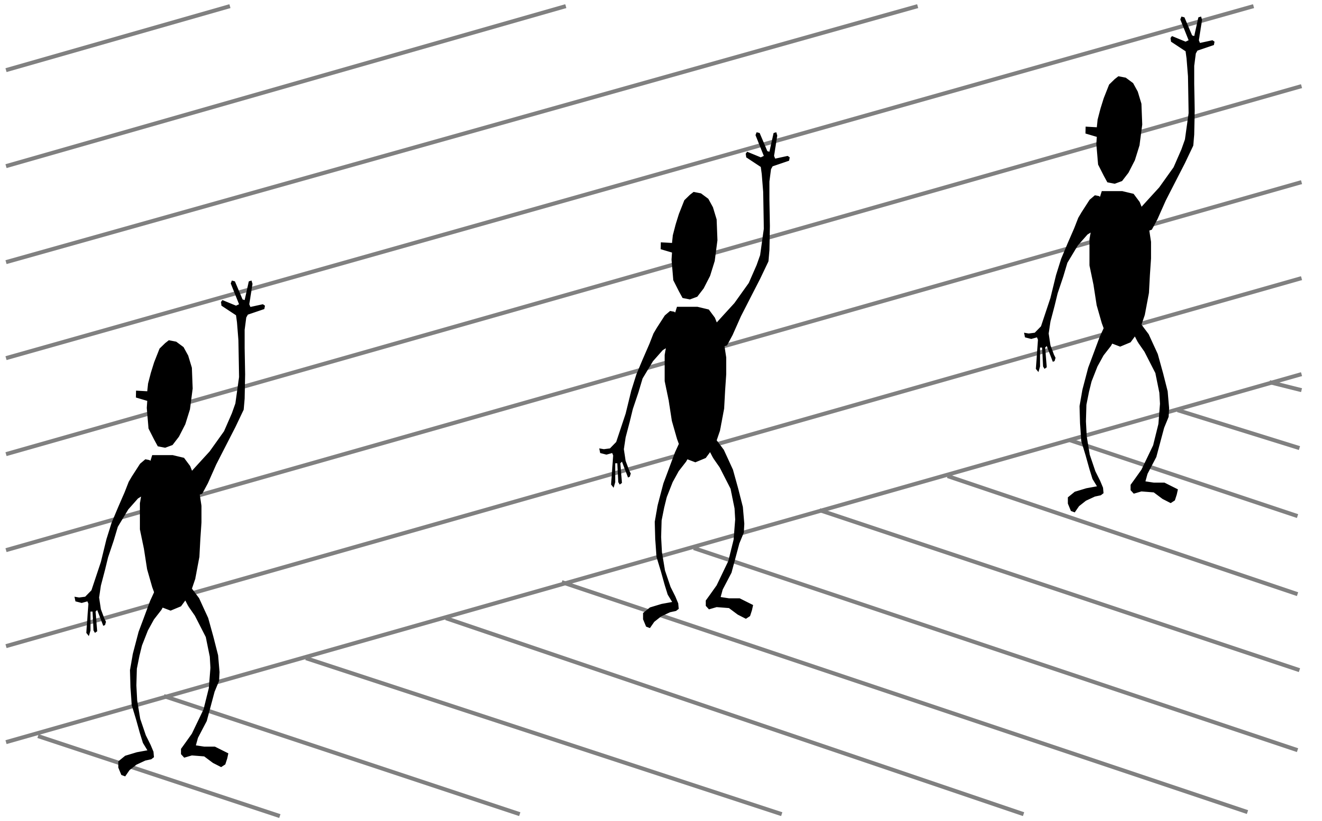
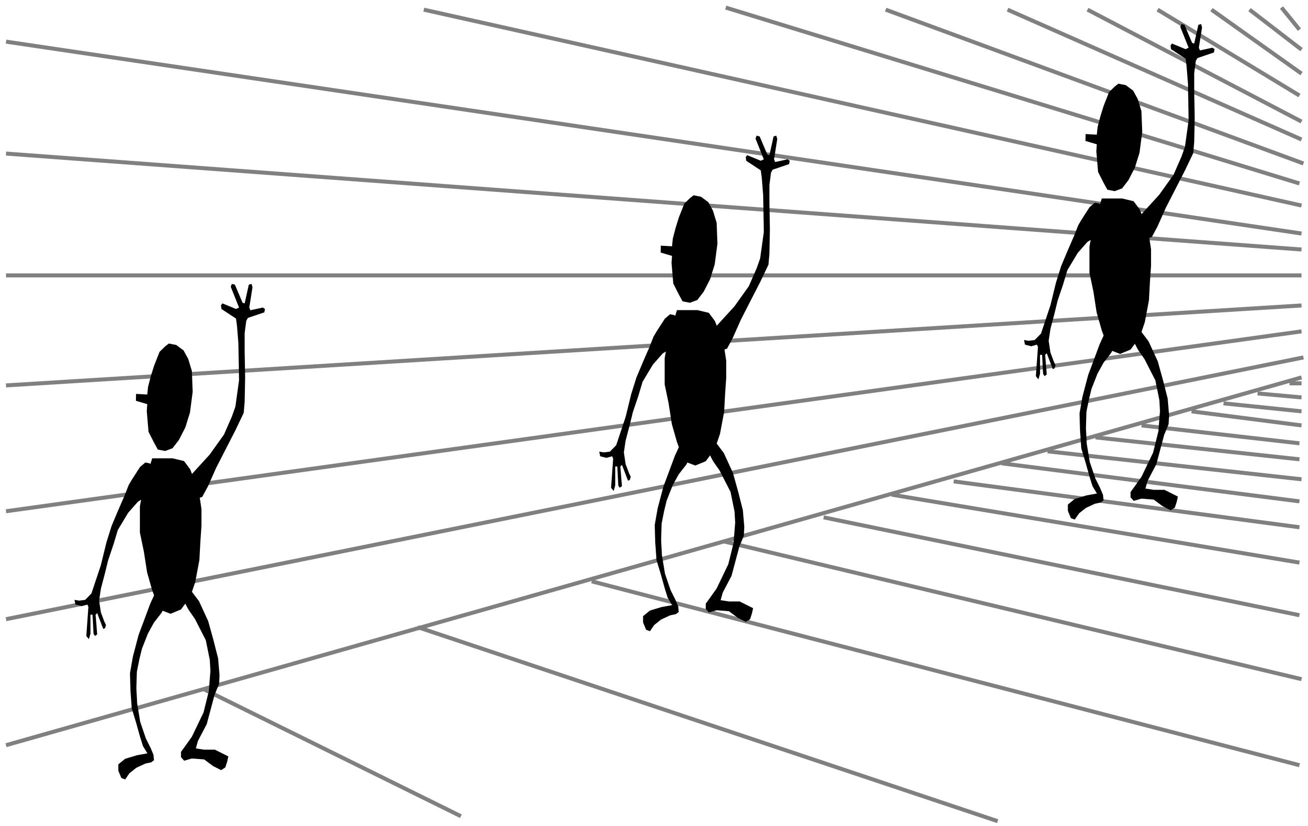
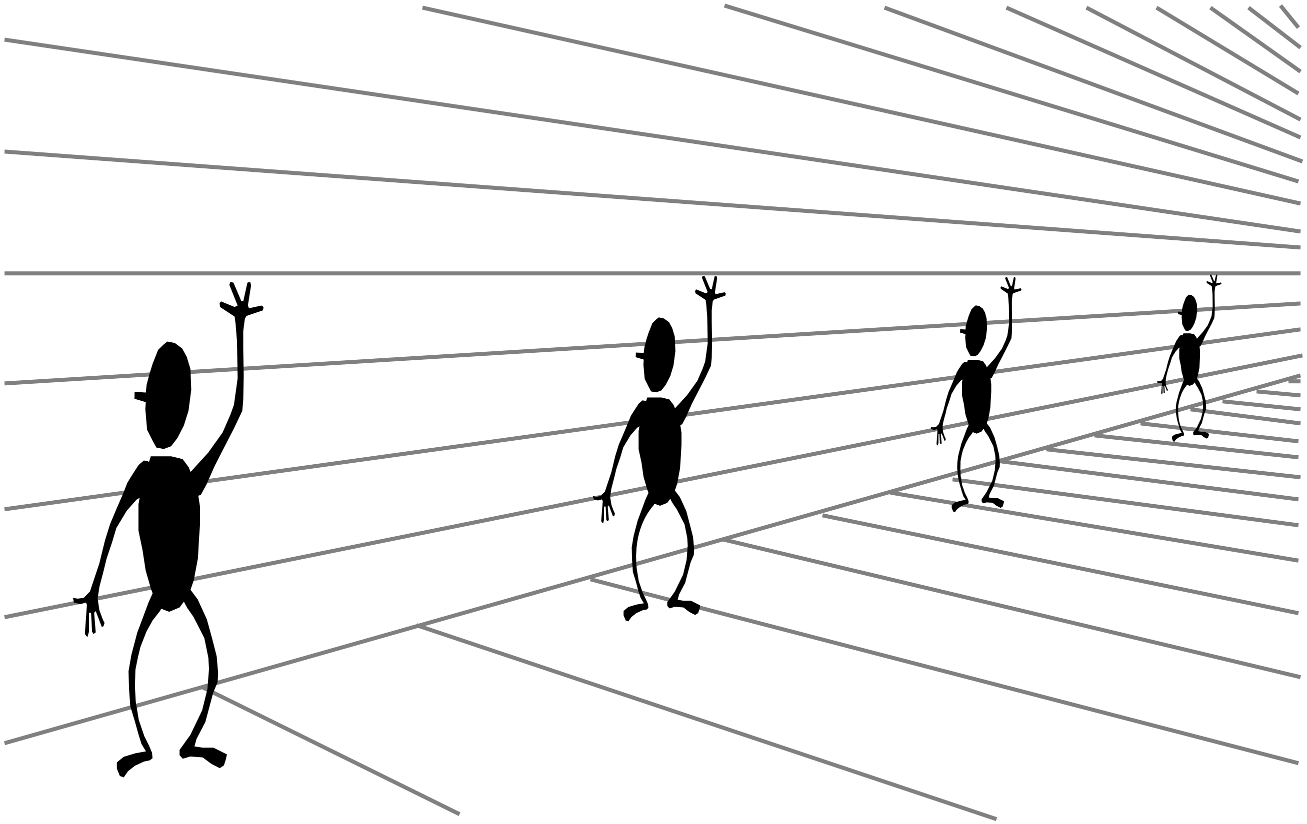
Comparing heights



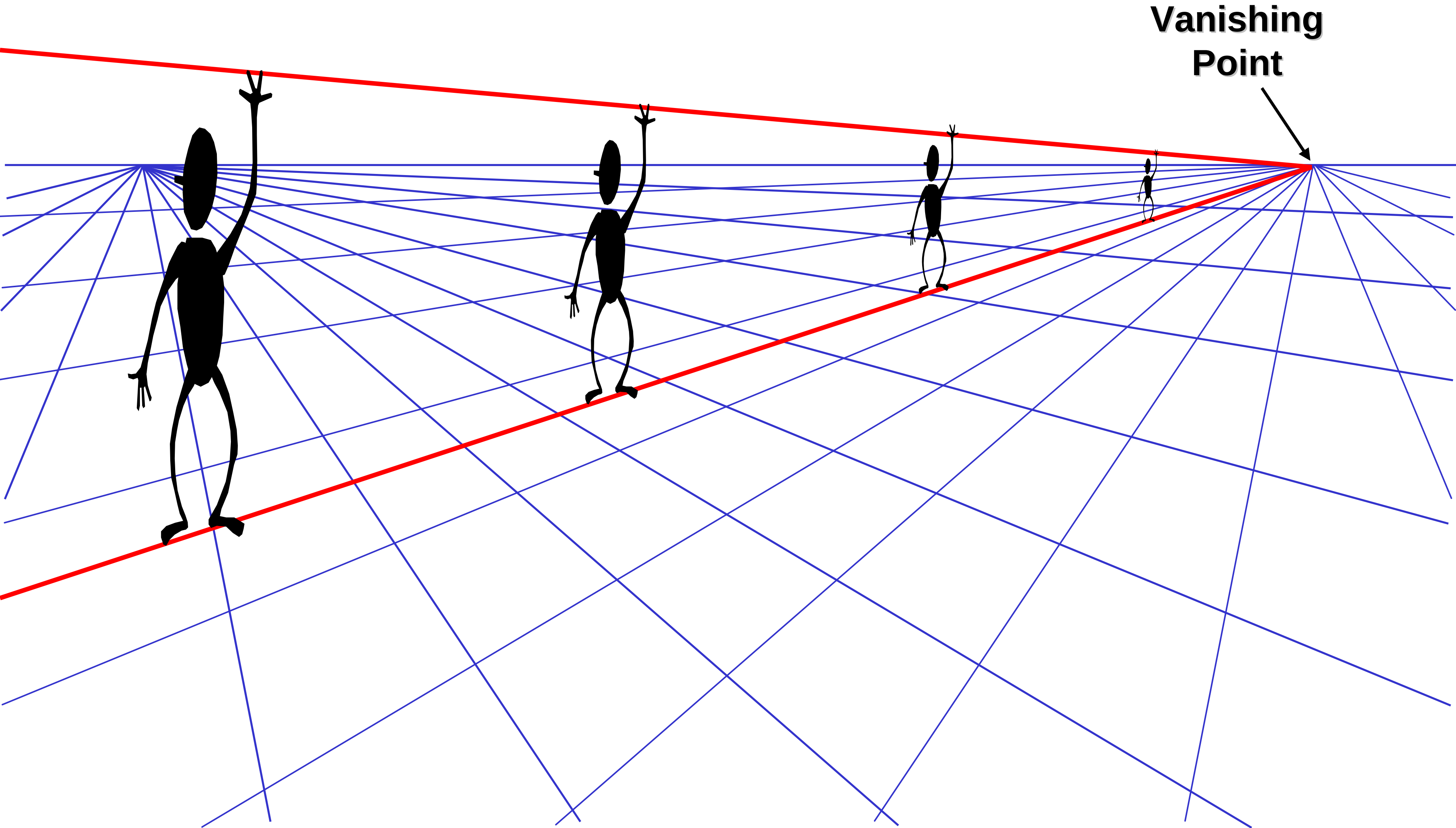
Perspective cues



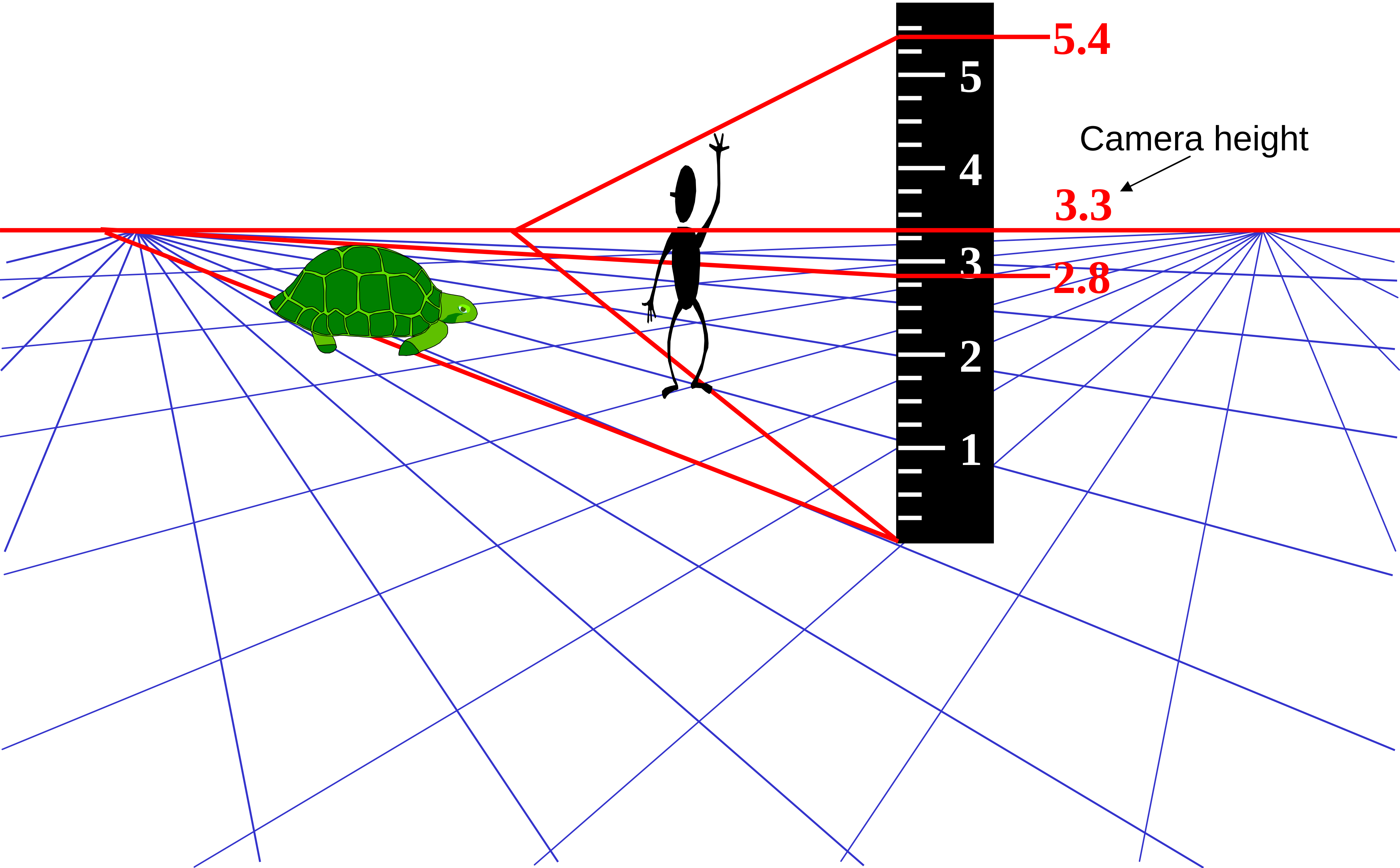
Perspective cues



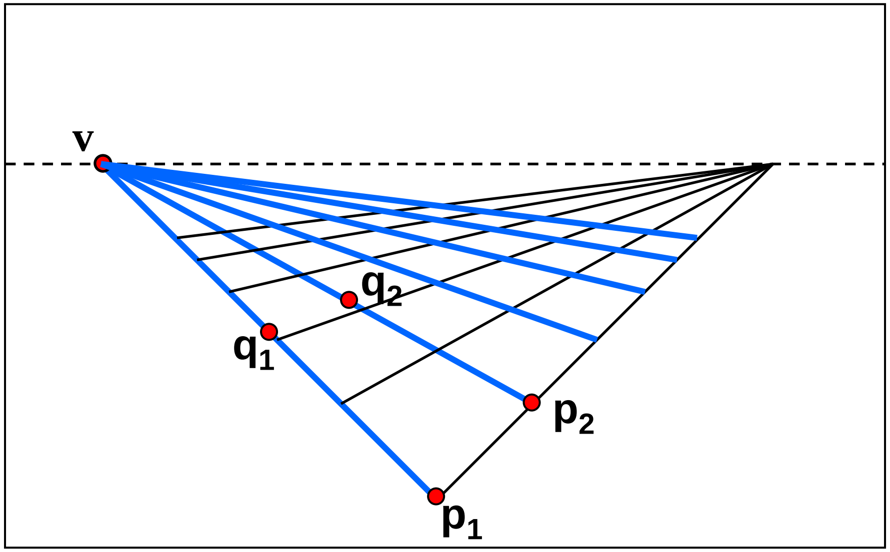
Comparing heights



Measuring height



Computing vanishing points (from lines)



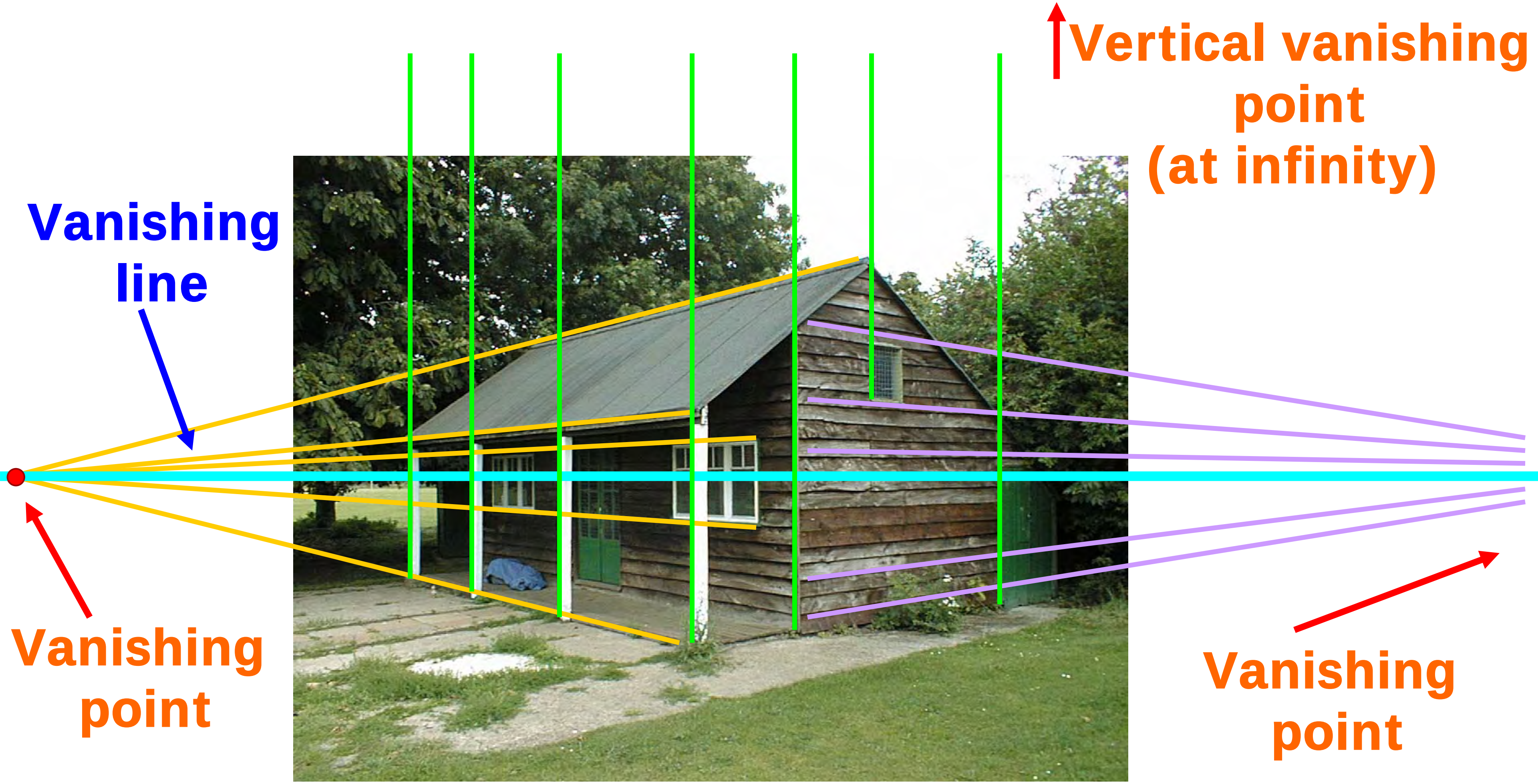
Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

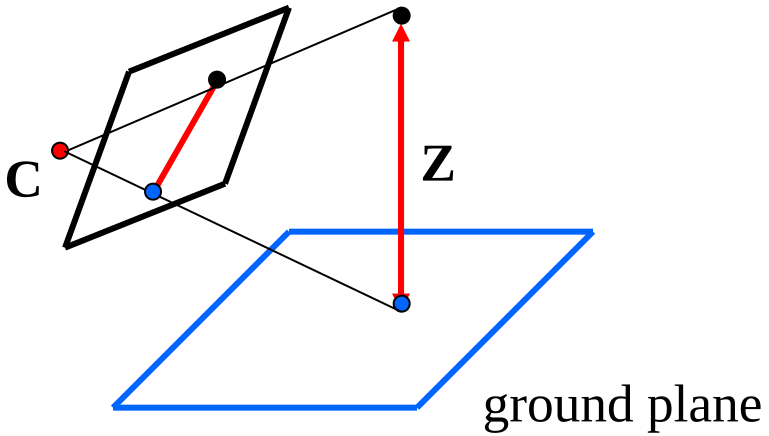
Least squares version

- Better to use more than two lines and compute the "closest" point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Criminisi ’99



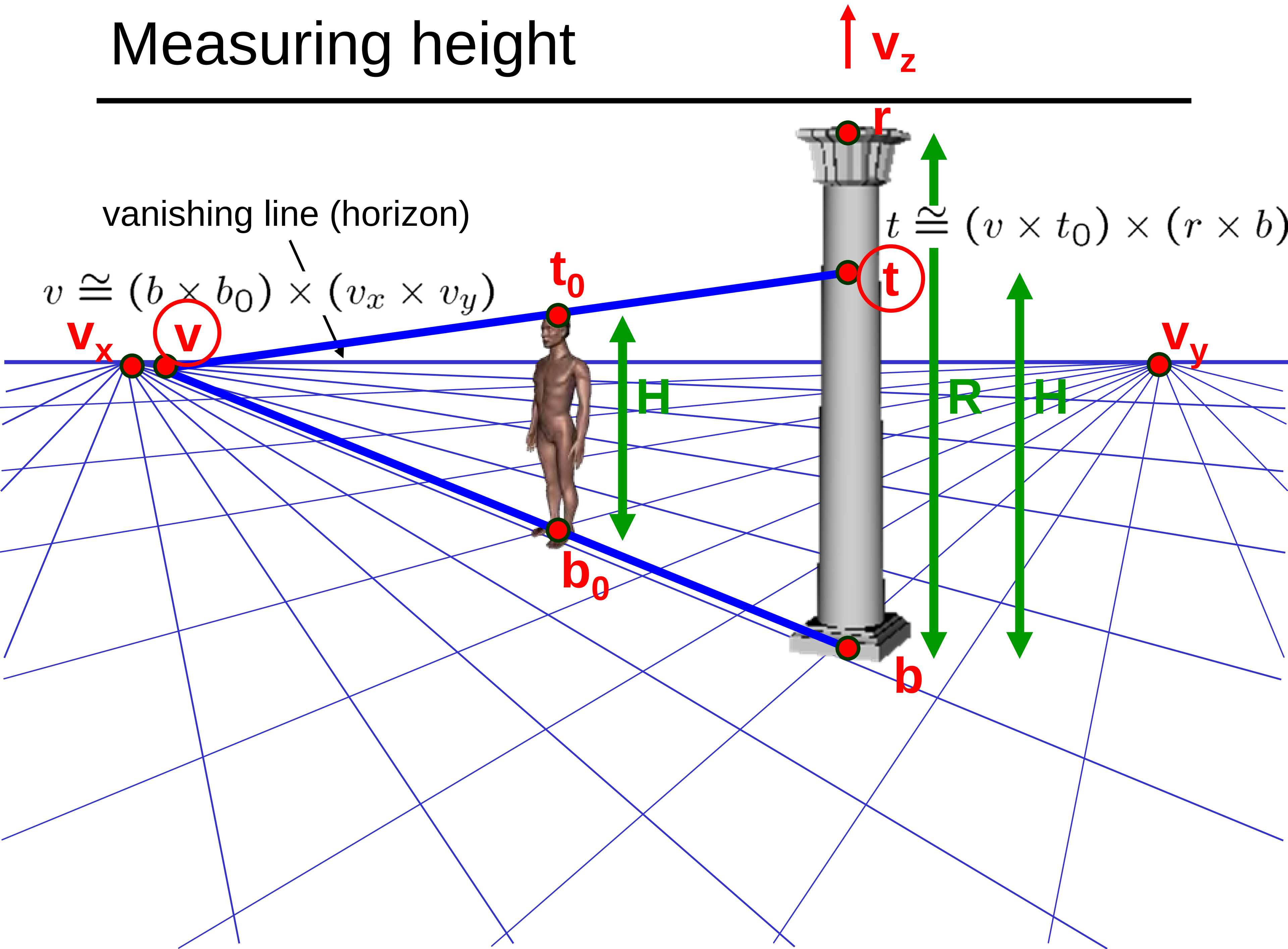
Measuring height without a ruler



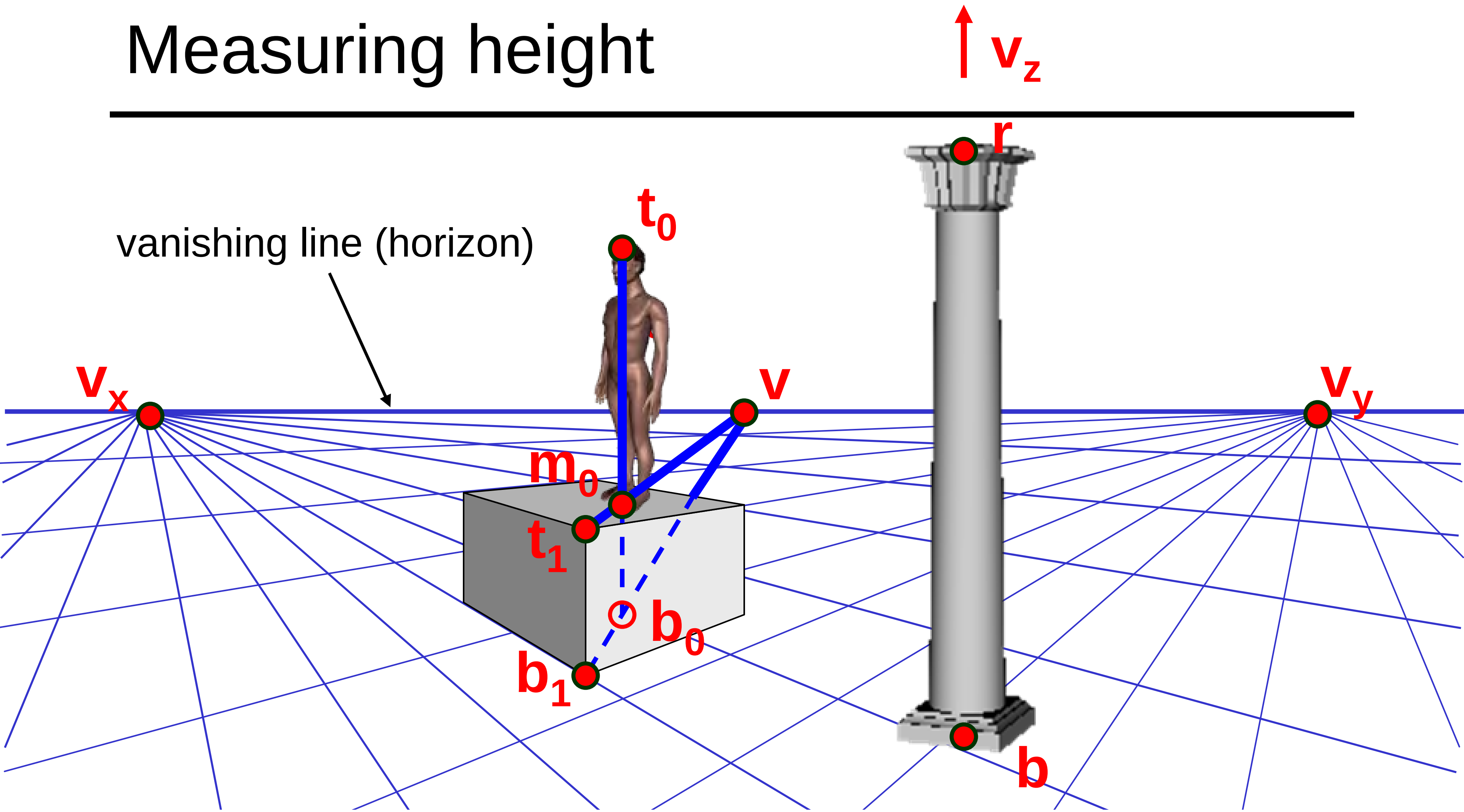
Compute Z from image measurements

- Need more than vanishing points to do this

Measuring height



Measuring height



What if the point on the ground plane $\mathbf{b_0}$ is not known?

- Here the guy is standing on the box
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

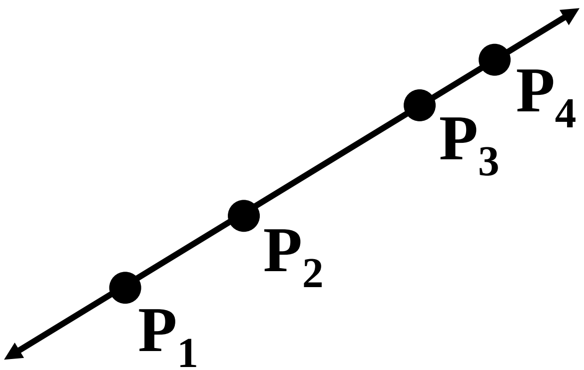
What if v_z is not infinity?

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

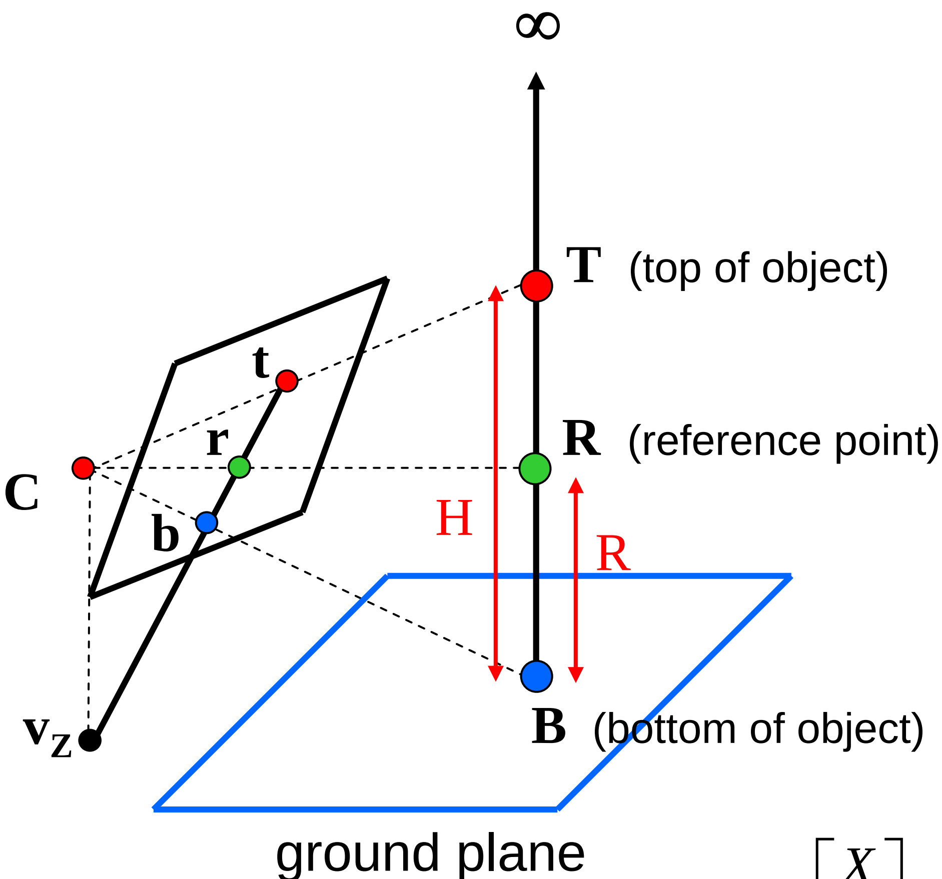
Can permute the point ordering

$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|} = \frac{H}{R}$$

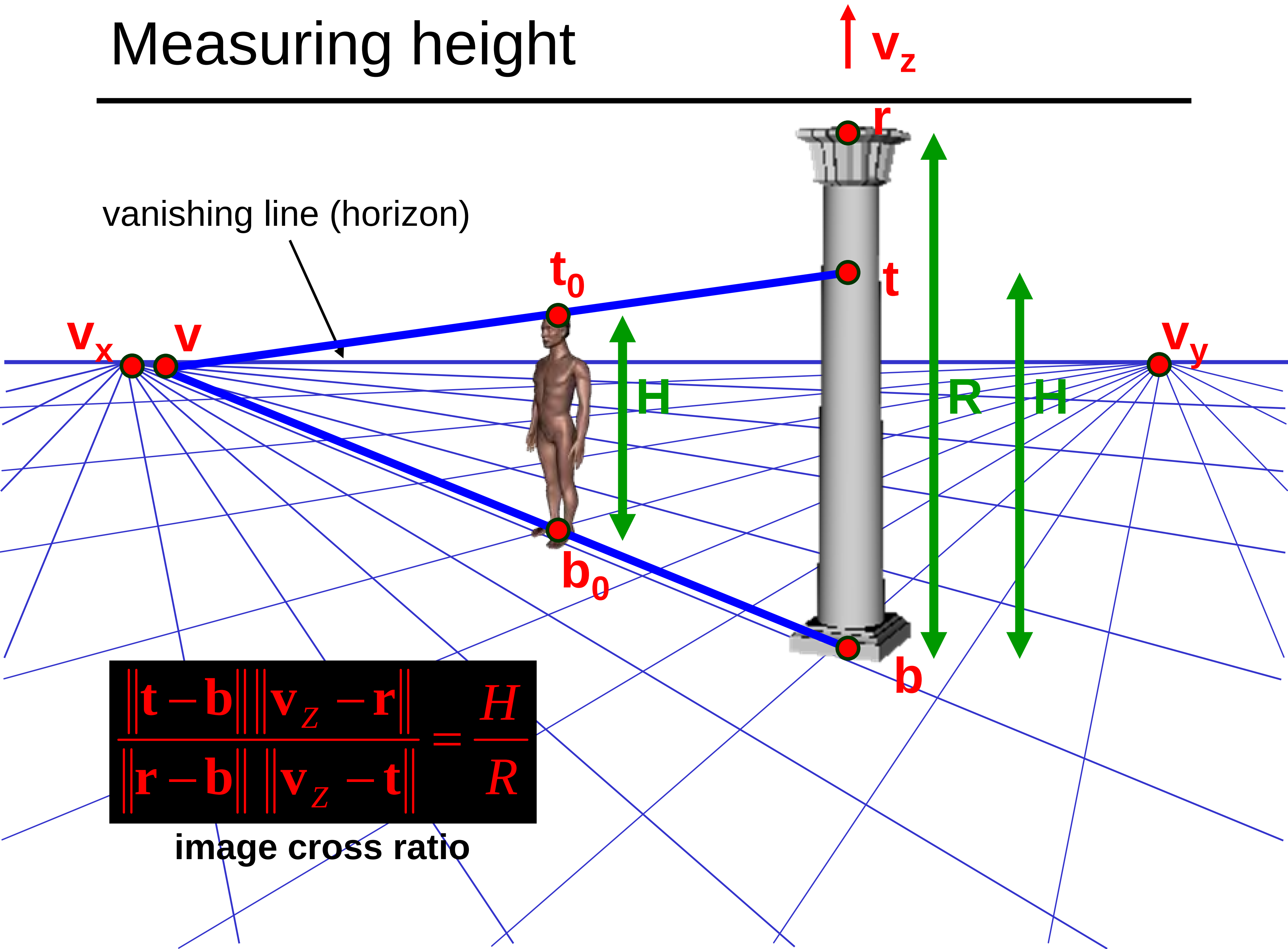
scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

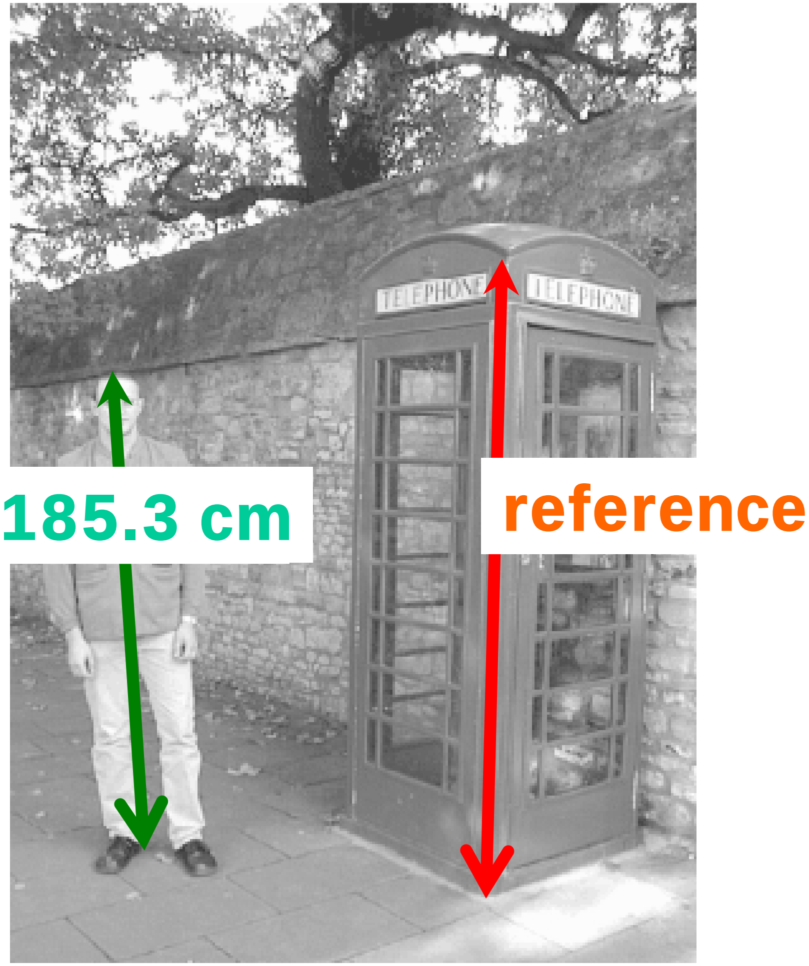
Measuring height



$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

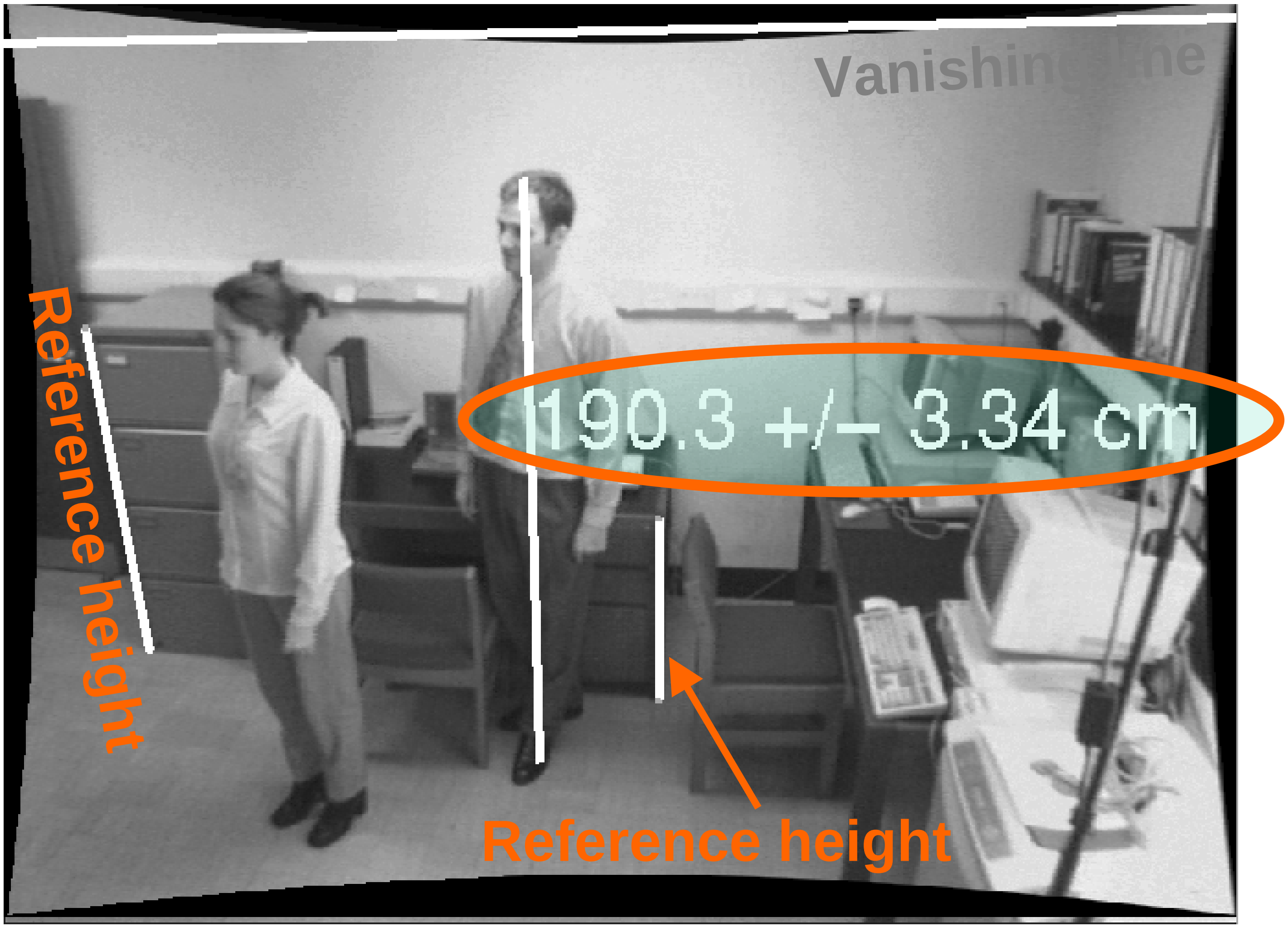
image cross ratio

Measuring heights of people



Here we go !

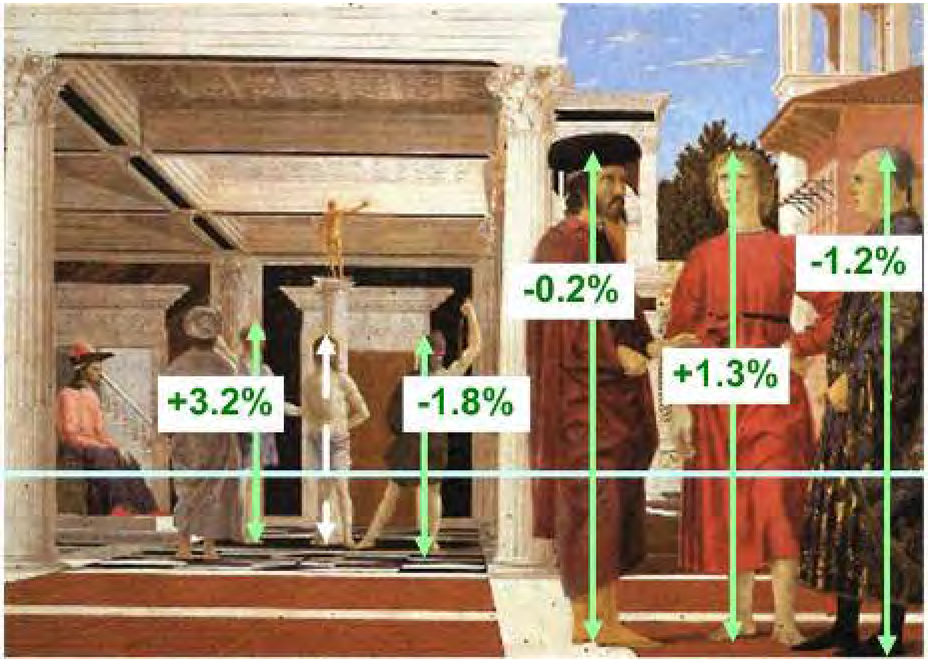
Forensic Science: measuring heights of suspects



Assessing geometric accuracy

Are the heights of the 2 groups of people consistent with each other?

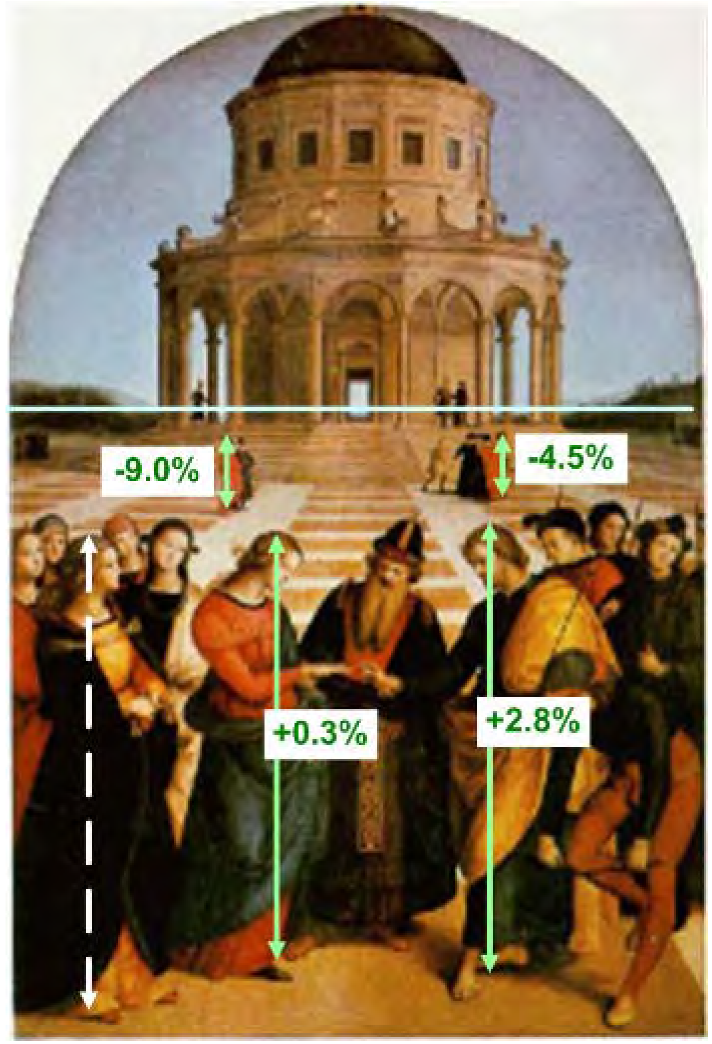
Flagellation, **Piero della Francesca**



Estimated relative heights

Assessing geometric accuracy

***The Marriage of the Virgin*, Raphael**



Estimated relative heights

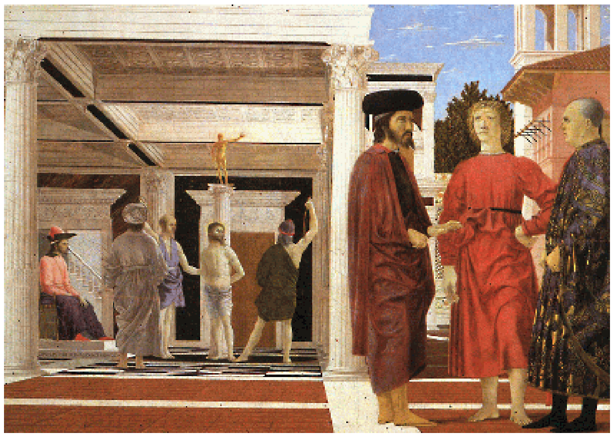
Criminisi et al., ICCV 99

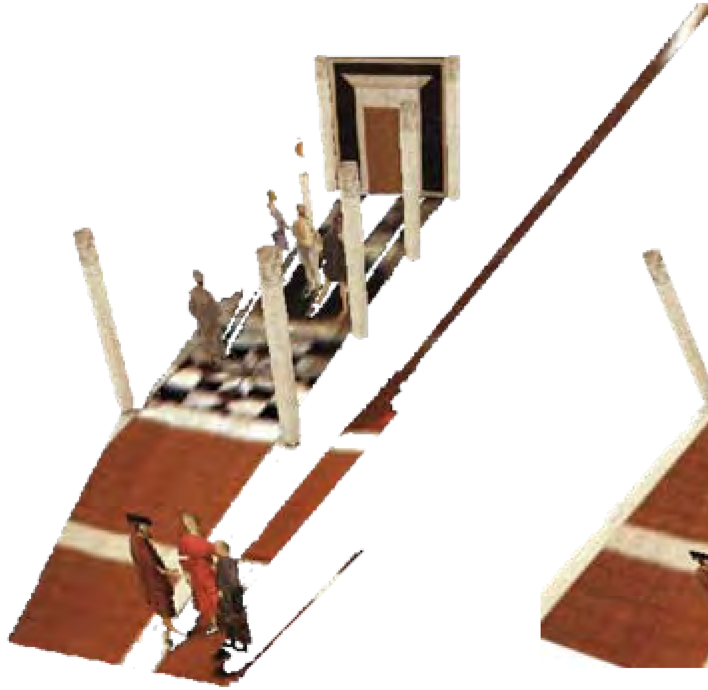
Complete approach

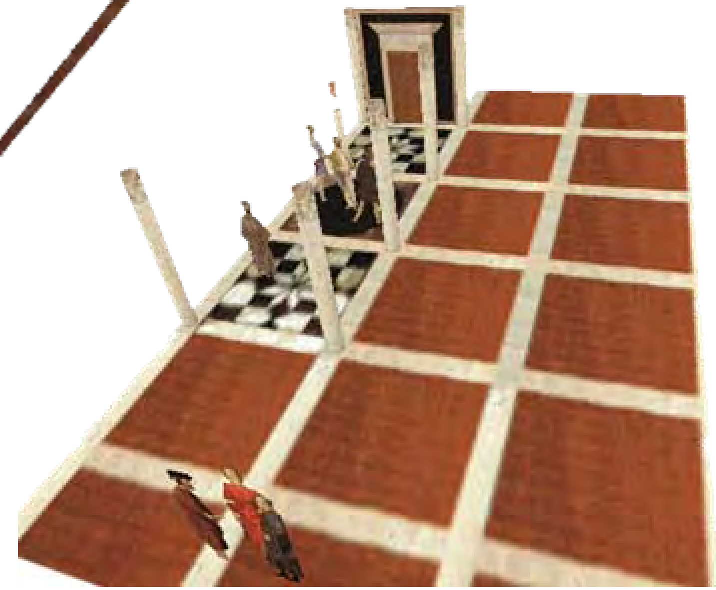
- Load in an image
- Click on lines parallel to X axis
  - repeat for Y, Z axes
- Compute vanishing points
- Specify 3D and 2D positions of 4 points on reference plane
- Compute homography H
- Specify a reference height
- Compute 3D positions of several points
- Create a 3D model from these points
- Extract texture maps
  - Cut out objects
  - Fill in holes
- Output a VRML model

Interactive silhouette cut-out

Occlusion filling







Geometric filling by exploiting:

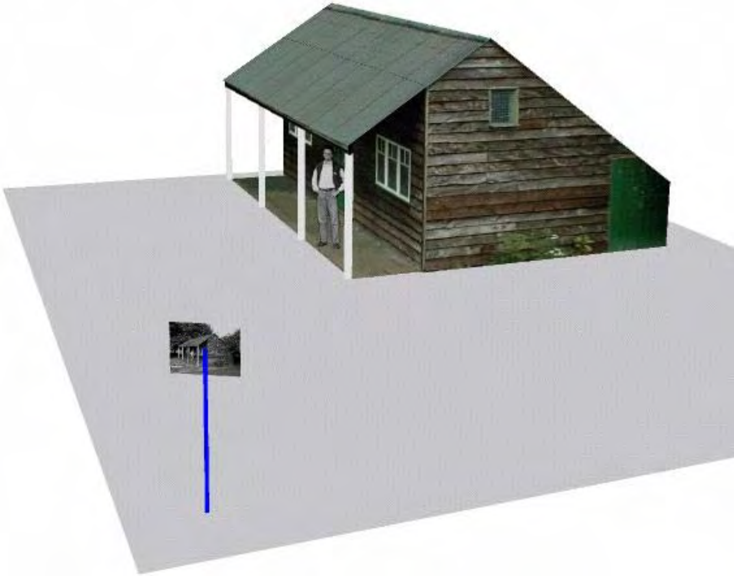
- symmetries
- repeated regular patterns

Texture synthesis

- repeated stochastic patterns

Complete 3D reconstruction

Single View algorithms

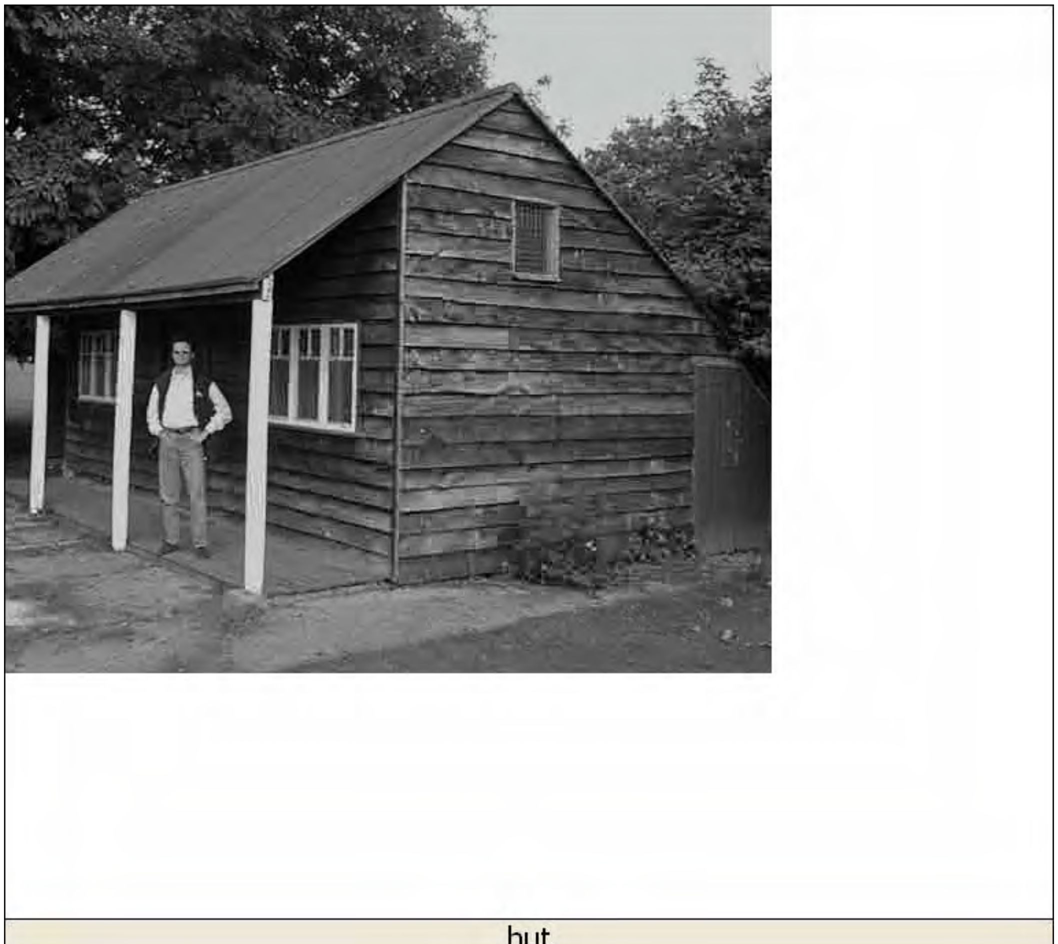


Single image

- Planar measurements
- Height measurements
- Automatic vanishing point / line computation
- Interactive segmentation
- Occlusion filling
- Object placement in 3D model

3D model

Reconstruction from single photographs



Reconstruction of the garden Hut from a single image

A virtual museum @ Microsoft