More Mosaic Madness

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with a lot of slides stolen from
Steve Seitz and Rick Szeliski

15-463: Computational Photography
Alexei Efros, CMU, Fall 2007

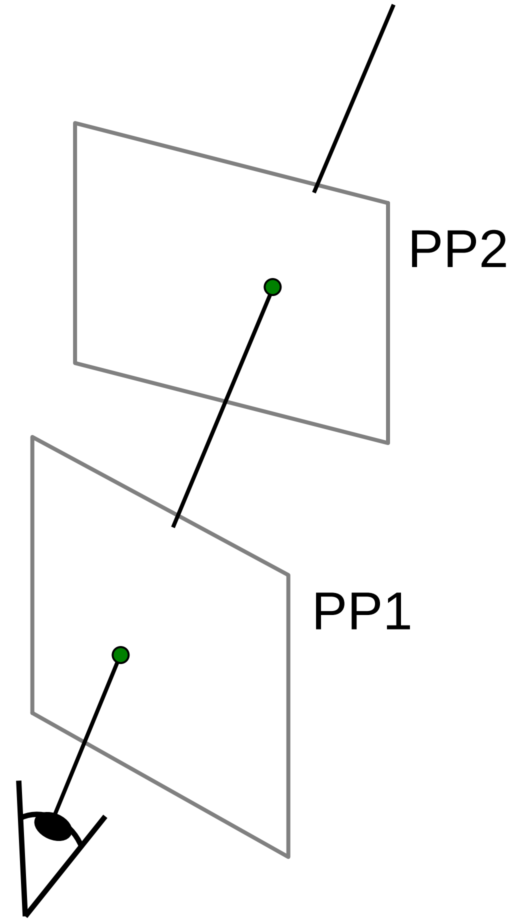
Homography

A: Projective – mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't
- but must preserve straight lines
- same as: project, rotate, reproject

called Homography

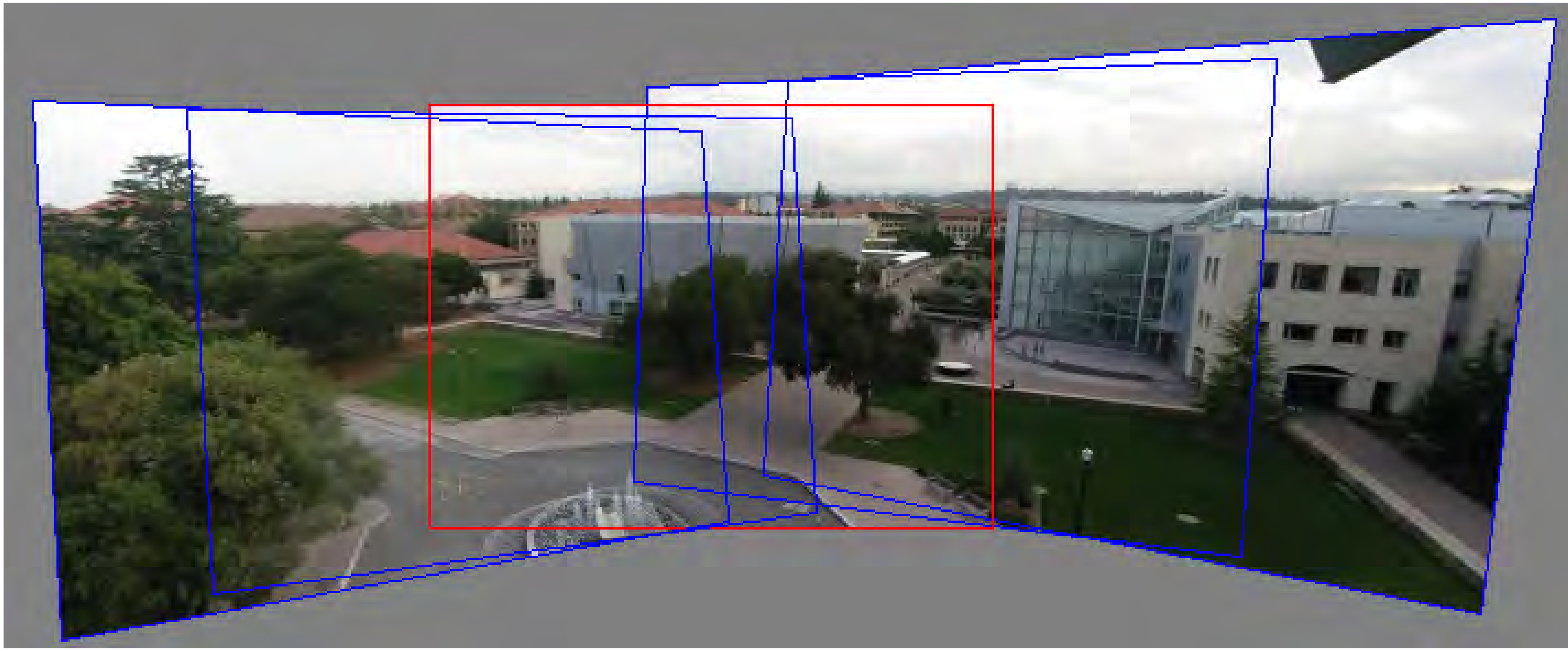
$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$



To apply a homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

Rotational Mosaics

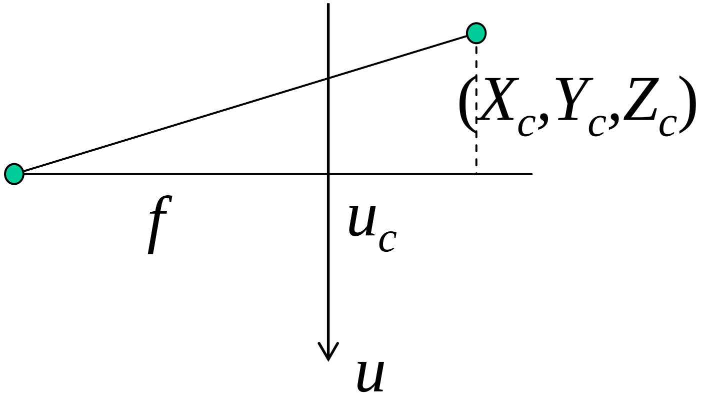


Can we say something more about rotational mosaics?

i.e. can we further constrain our H?

3D → 2D Perspective Projection

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & u_c \\ 0 & f & v_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

K

3D Rotation Model

Projection equations

1. Project from image to 3D ray

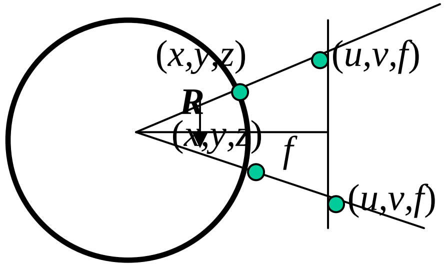
   $(x_0,y_0,z_0) \quad = (u_0-u_c,v_0-v_c,f)$

2. Rotate the ray by camera motion

   $(x_1,y_1,z_1) \quad = \boldsymbol{R}_{01}\,(x_0,y_0,z_0)$

3. Project back into new (source) image

   $(u_1,v_1) \quad = (fx_1/z_1+u_c,fy_1/z_1+v_c)$



Therefore:

$$\mathbf{H} = \mathbf{K_0 R_{01} K_1^{-1}}$$

Our homography has only 3,4 or 5 DOF, depending if focal length is known, same, or different.

- This makes image registration much better behaved

Pairwise alignment

Procrustes Algorithm [Golub & VanLoan]

Given two sets of matching points, compute R

$p_i' = \boldsymbol{R}\, p_i$ with 3D rays

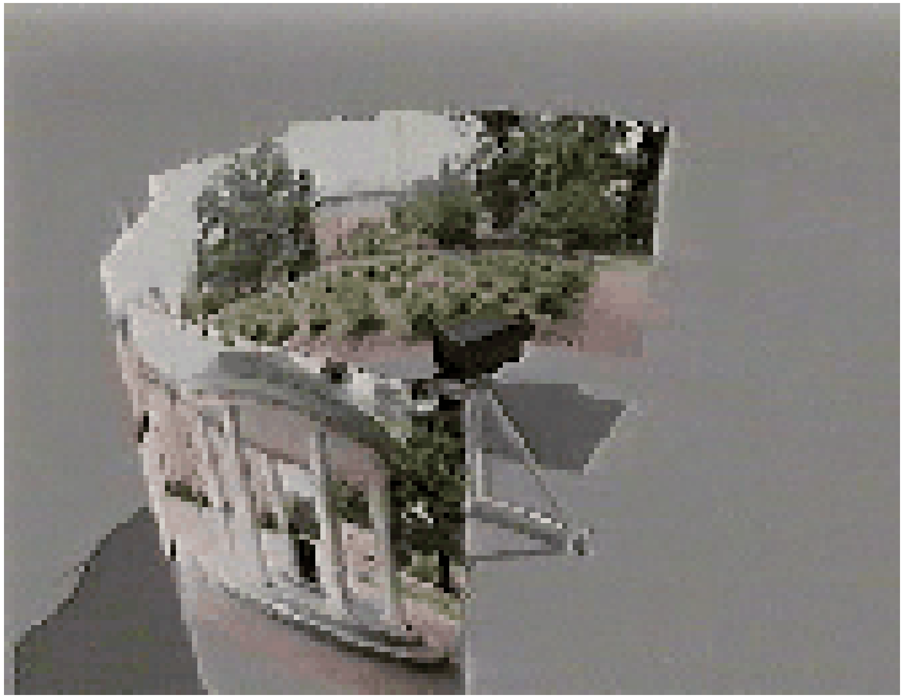
$$p_i = N(x_i, y_i, z_i) = N(u_i - u_c, v_i - v_c, f)$$

$$\boldsymbol{A} = \Sigma_\mathbf{i}\, p_i\, p_i'^T = \Sigma_\mathbf{i}\, p_i\, p_i^T\, \boldsymbol{R}^T = \boldsymbol{U}\,\boldsymbol{S}\,\boldsymbol{V}^T = (\boldsymbol{U}\,\boldsymbol{S}\,\boldsymbol{U}^T)\,\boldsymbol{R}^T$$

$$\boldsymbol{V}^T = \boldsymbol{U}^T \boldsymbol{R}^T$$

$$\boldsymbol{R} = \boldsymbol{V}\,\boldsymbol{U}^T$$

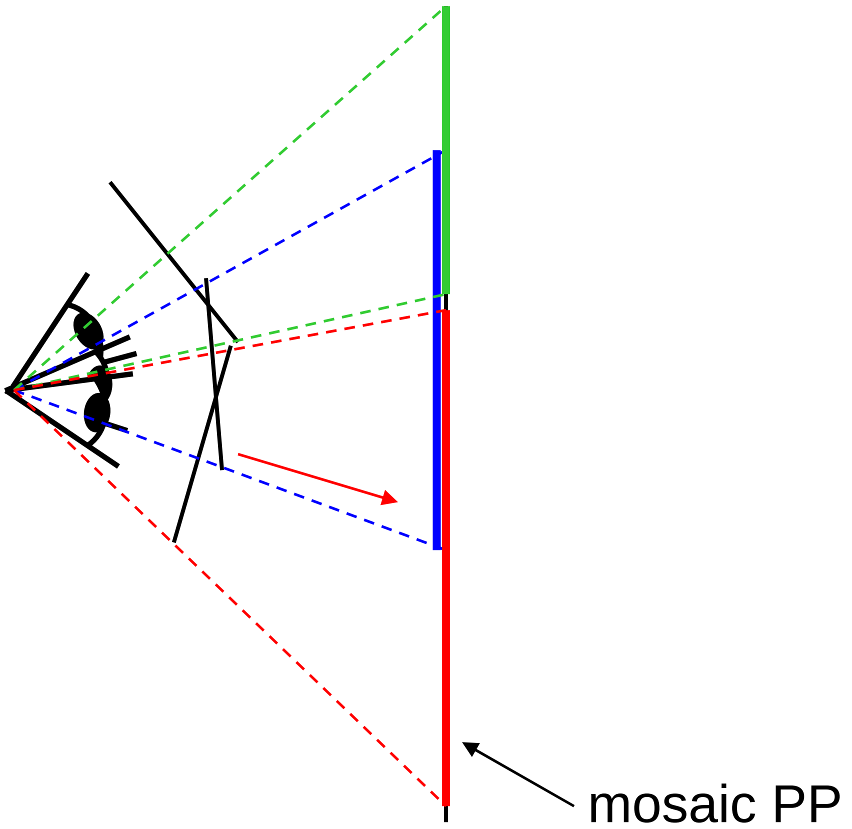
Rotation about vertical axis



What if our camera rotates on a tripod?

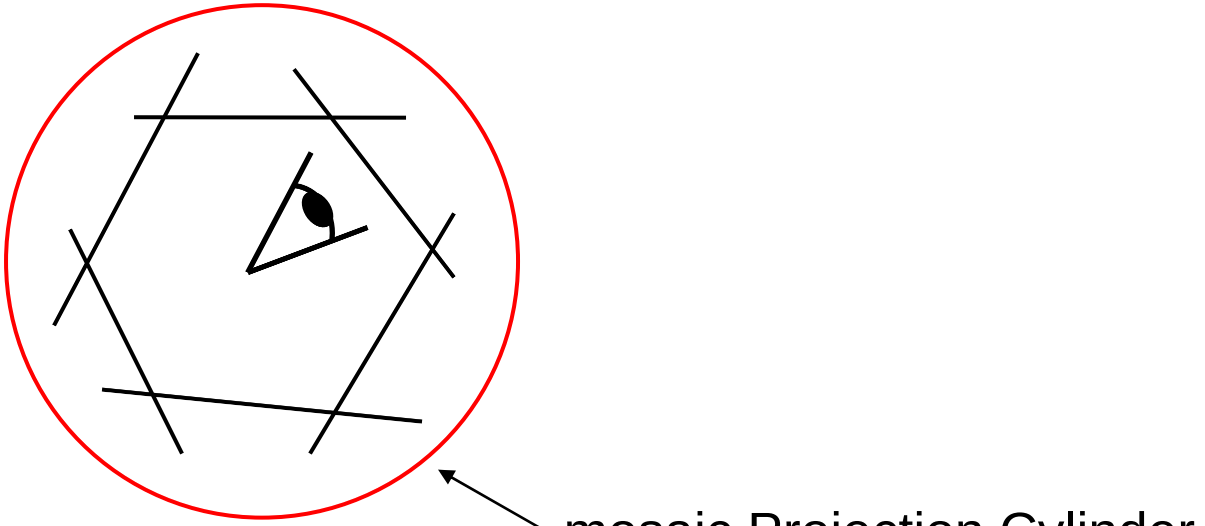
What's the structure of H?

Do we have to project onto a plane?

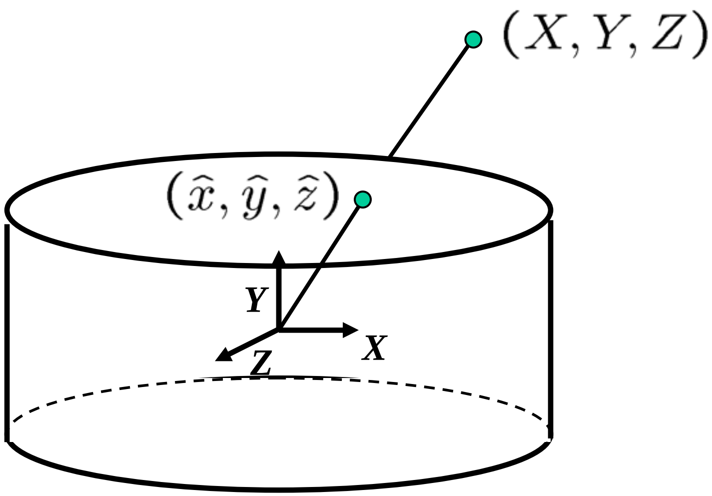


Full Panoramas

What if you want a 360° field of view?



Cylindrical projection



unit cylinder

- Map 3D point (X,Y,Z) onto cylinder

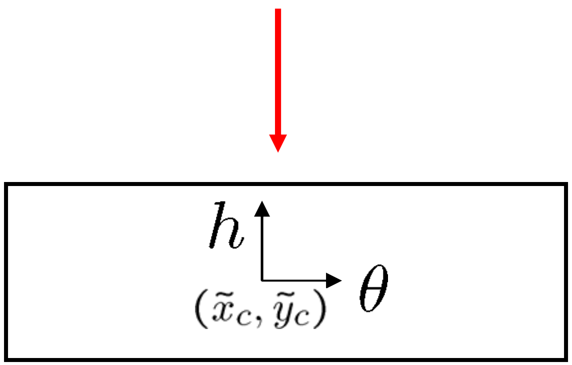
$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2+Z^2}}(X, Y, Z)$$

- Convert to cylindrical coordinates

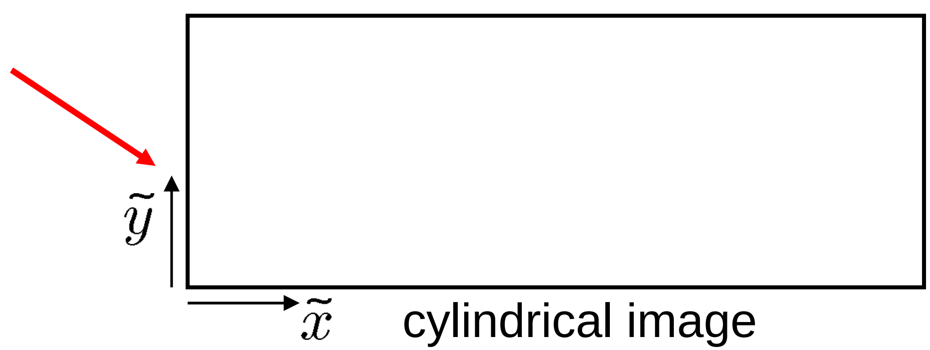
$$(sin\theta, h, cos\theta) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to cylindrical image coordinates

$$(\tilde{x}, \tilde{y}) = (f\theta, fh) + (\tilde{x}_c, \tilde{y}_c)$$

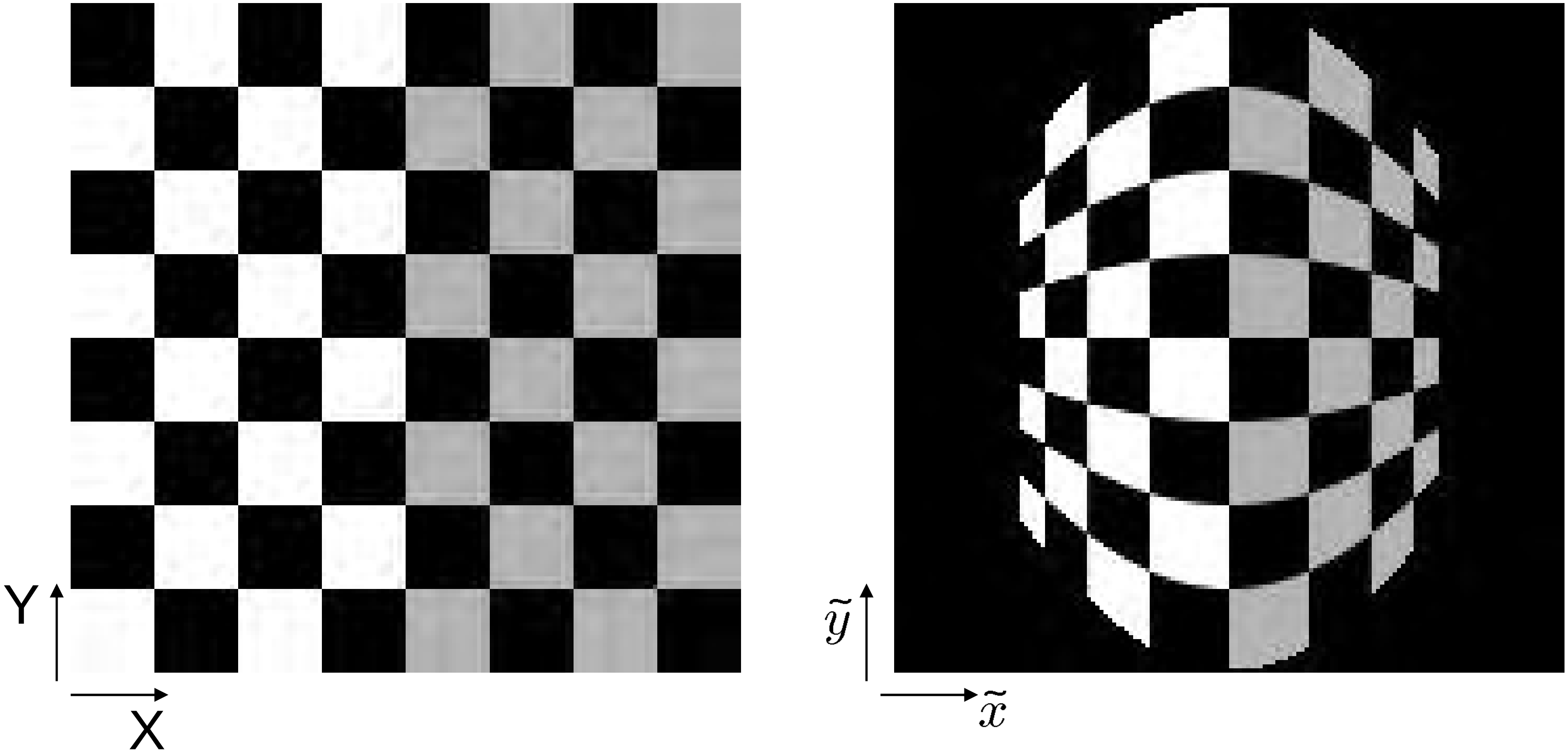


unwrapped cylinder

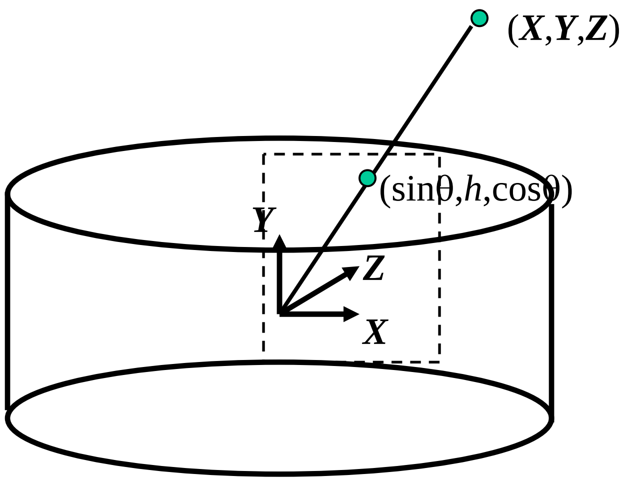


cylindrical image

Cylindrical Projection

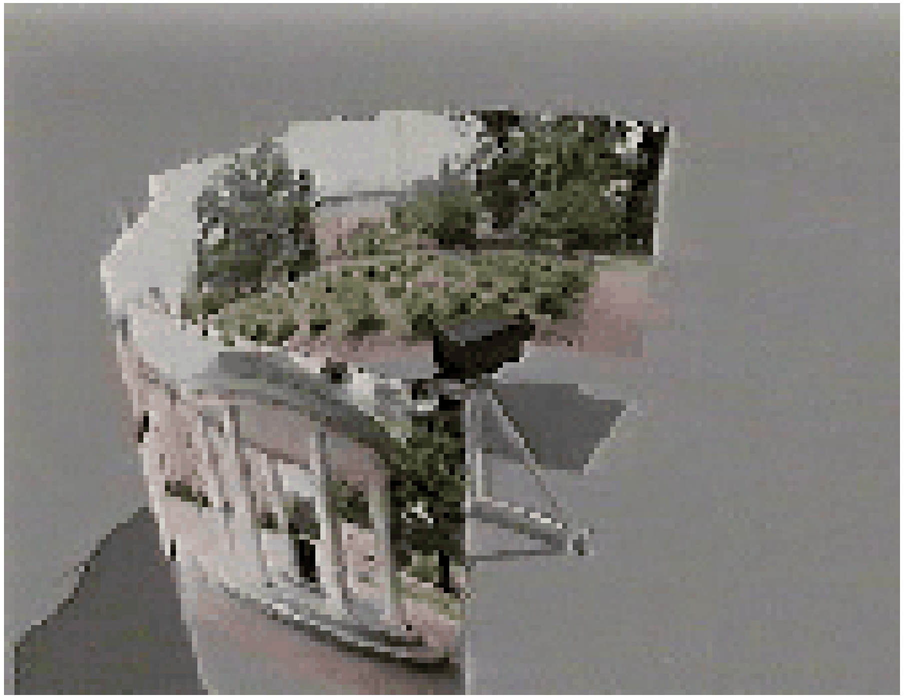


Inverse Cylindrical projection



$$\begin{aligned}
\theta &= (x_{cyl} - x_c)/f \\
h &= (y_{cyl} - y_c)/f \\
\hat{x} &= \sin\theta \\
\hat{y} &= h \\
\hat{z} &= \cos\theta \\
x &= f\hat{x}/\hat{z} + x_c \\
y &= f\hat{y}/\hat{z} + y_c
\end{aligned}$$

Cylindrical panoramas



Steps

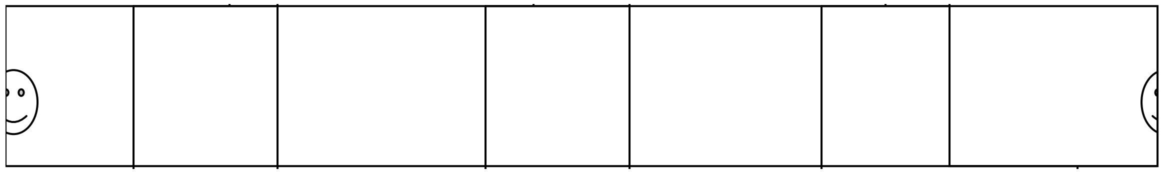
- Reproject each image onto a cylinder
- Blend
- Output the resulting mosaic

Cylindrical image stitching

What if you don't know the camera rotation?

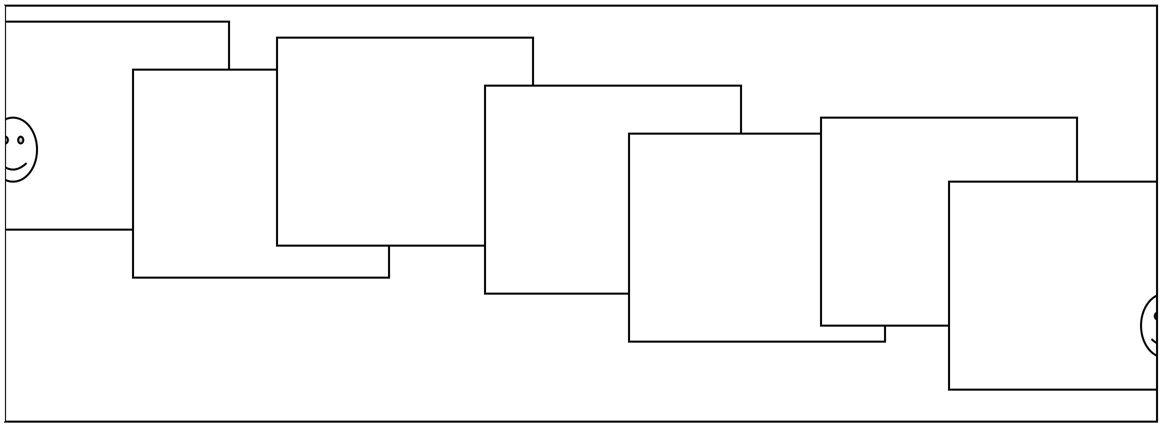
- Solve for the camera rotations
  - Note that a rotation of the camera is a **translation** of the cylinder!

Assembling the panorama



Stitch pairs together, blend, then crop

Problem: Drift



Vertical Error accumulation

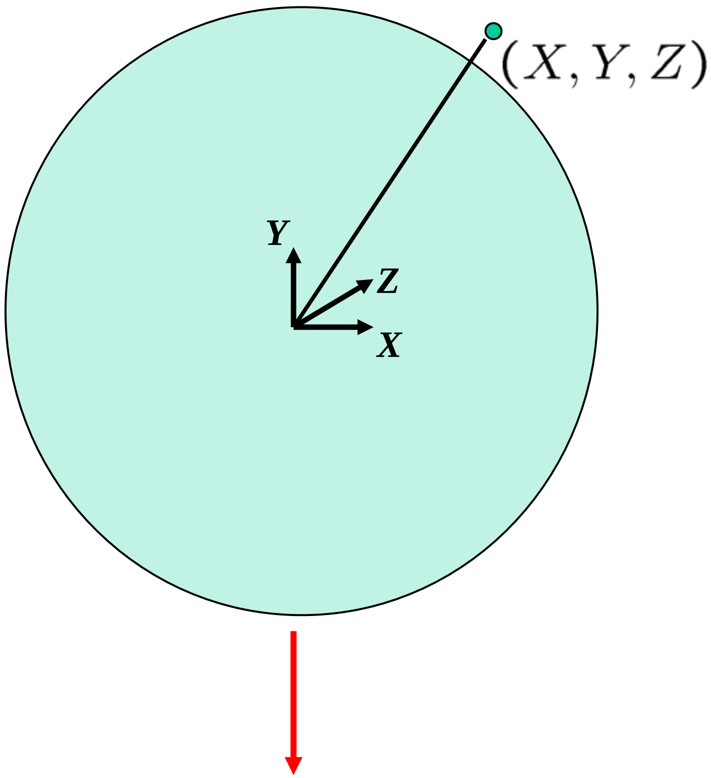
- small (vertical) errors accumulate over time
- apply correction so that sum = 0 (for 360° pan.)

Horizontal Error accumulation

- can reuse first/last image to find the right panorama radius

Full-view (360°) panoramas

Spherical projection



- Map 3D point (X,Y,Z) onto sphere

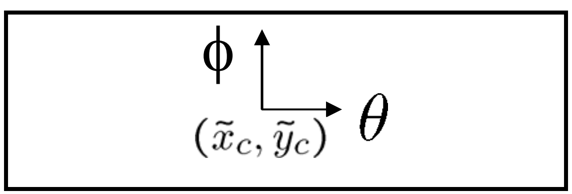
$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2 + Y^2 + Z^2}}(X, Y, Z)$$

- Convert to spherical coordinates

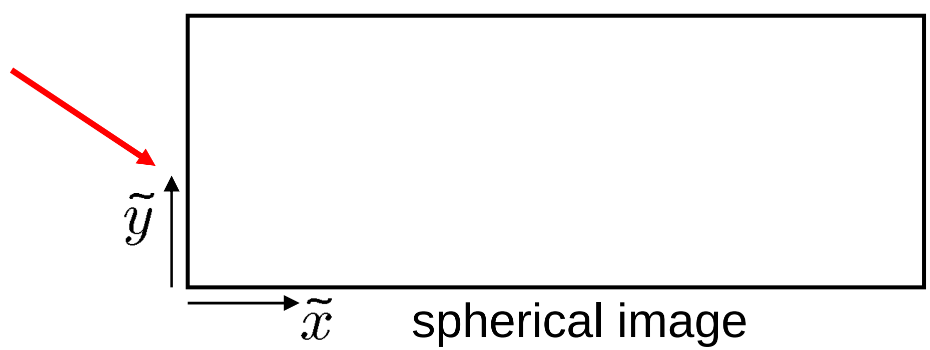
$$(\sin\theta\cos\phi, \sin\phi, \cos\theta\cos\phi) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to spherical image coordinates

$$(\tilde{x}, \tilde{y}) = (f\theta, fh) + (\tilde{x}_c, \tilde{y}_c)$$

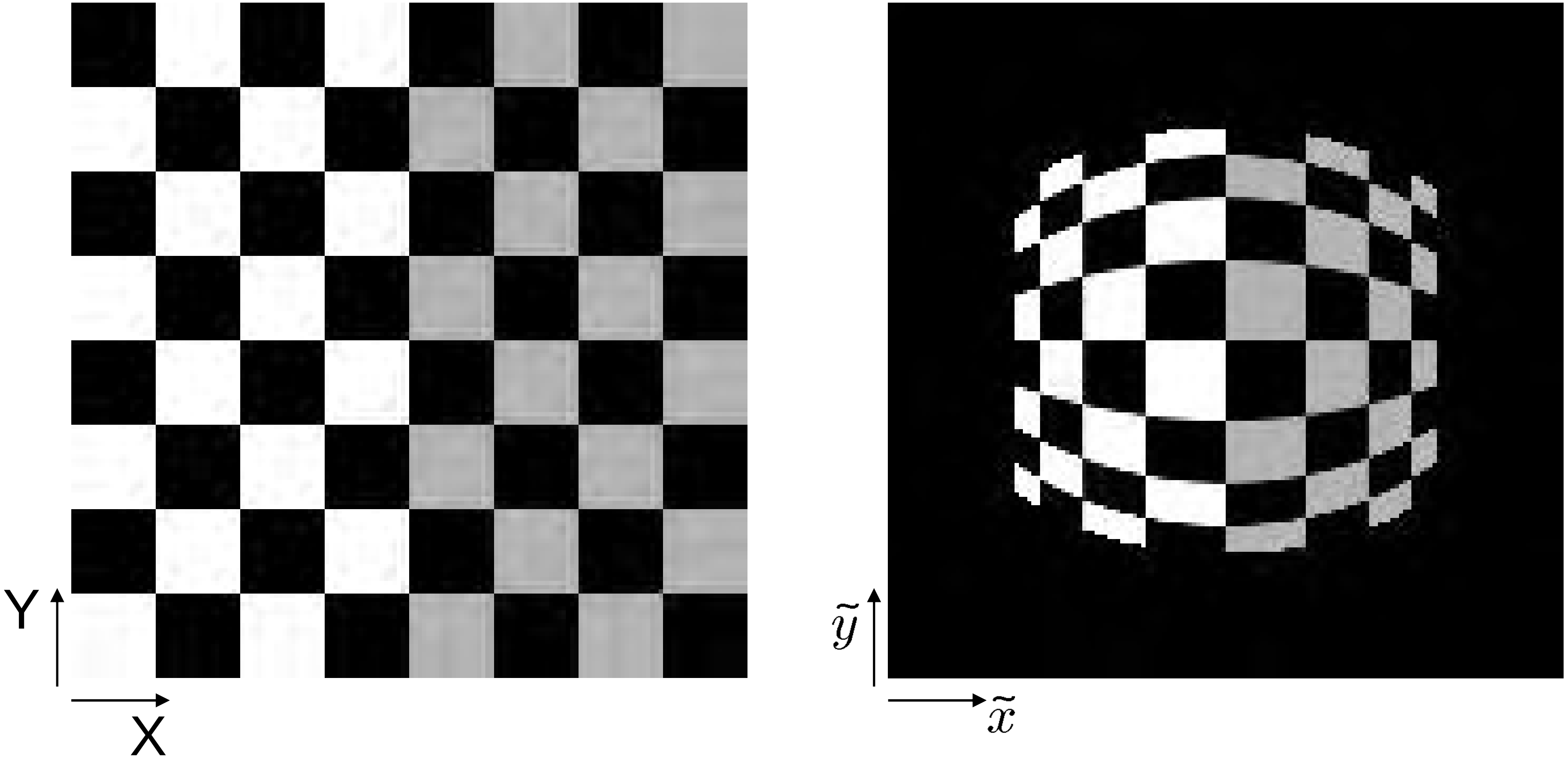


unwrapped sphere

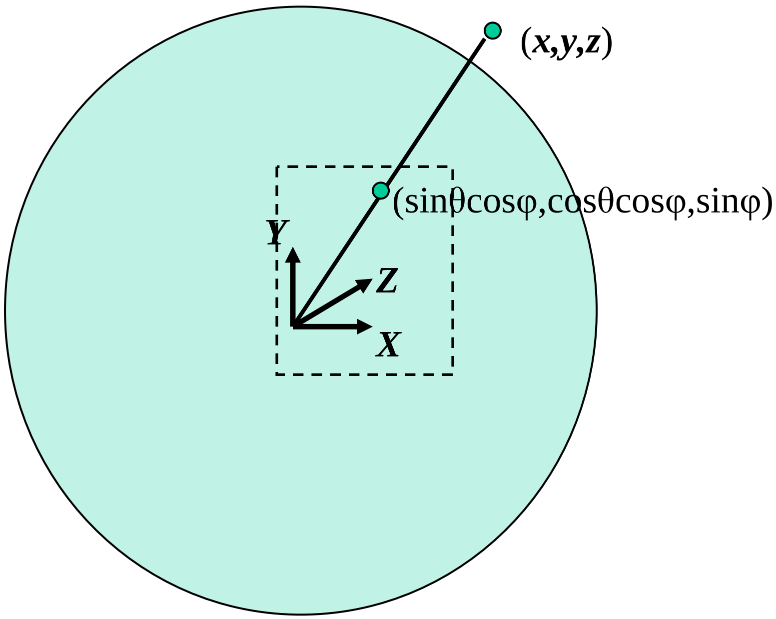


spherical image

Spherical Projection



Inverse Spherical projection



$$\theta = (x_{sph} - x_c)/f$$

$$\varphi = (y_{sph} - y_c)/f$$

$$\hat{x} = \sin\theta \cos\varphi$$

$$\hat{y} = \sin\varphi$$

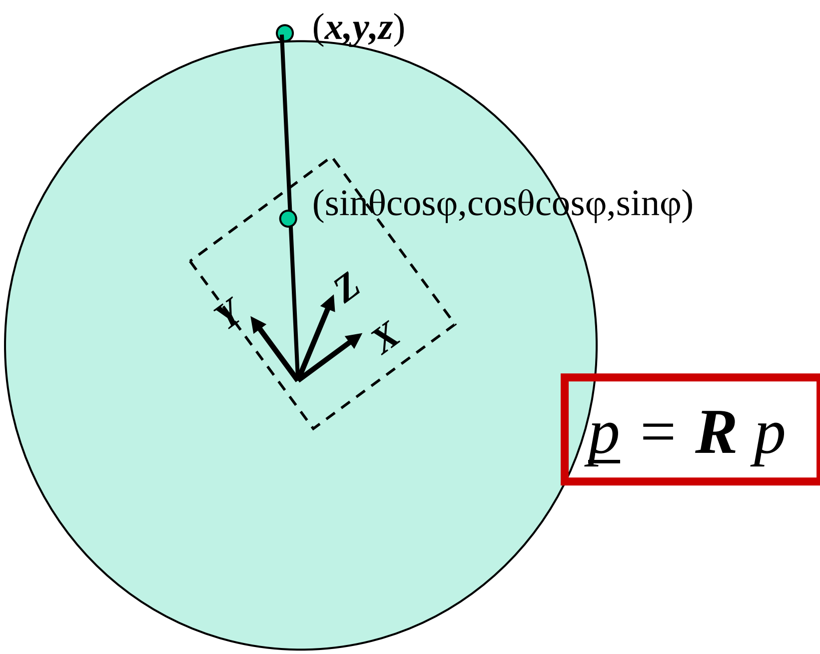
$$\hat{z} = \cos\theta \cos\varphi$$

$$x = f\hat{x}/\hat{z} + x_c$$

$$y = f\hat{y}/\hat{z} + y_c$$

3D rotation

Rotate image before placing on unrolled sphere



$$\theta = (x_{sph} - x_c)/f$$

$$\varphi = (y_{sph} - y_c)/f$$

$$\hat{x} = \sin\theta \cos\varphi$$

$$\hat{y} = \sin\varphi$$

$$\hat{z} = \cos\theta \cos\varphi$$

$$x = f\hat{x}/\hat{z} + x_c$$

$$y = f\hat{y}/\hat{z} + y_c$$

Full-view Panorama

Other projections are possible

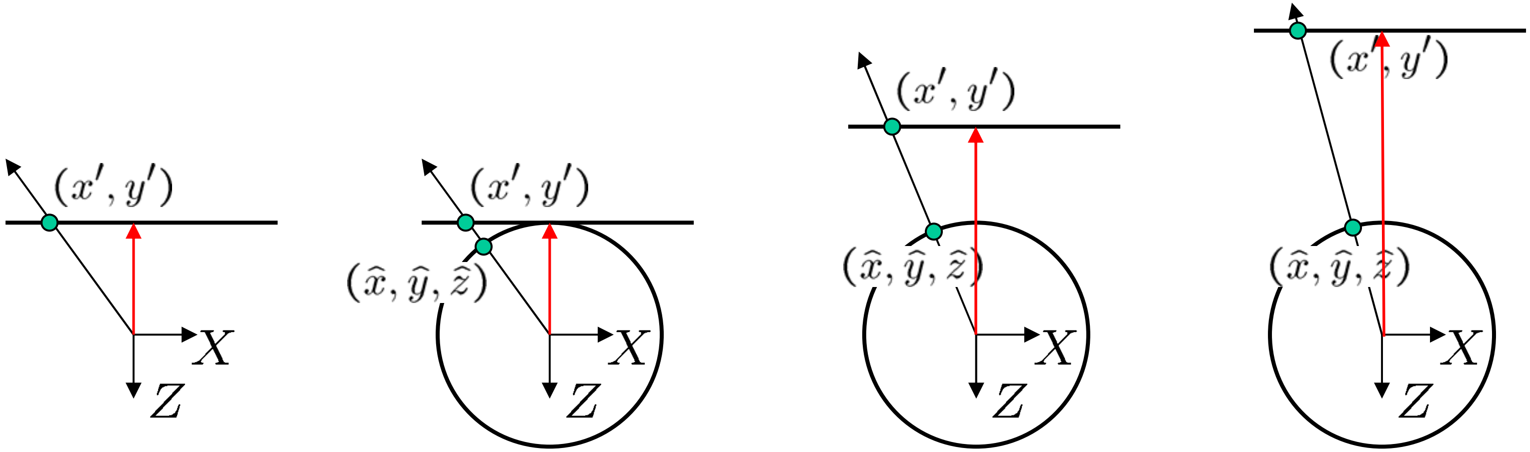
You can stitch on the plane and then warp the resulting panorama

- What's the limitation here?

Or, you can use these as stitching surfaces

- But there is a catch…

Cylindrical reprojection



top-down view

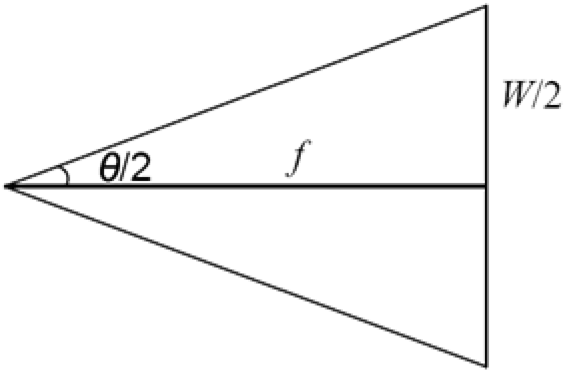
Focal length – the dirty secret…

Image 384x300 **f = 180 (pixels)** **f = 280** **f = 380**

What's your focal length, buddy?

Focal length is (highly!) camera dependant

- Can get a rough estimate by measuring FOV:

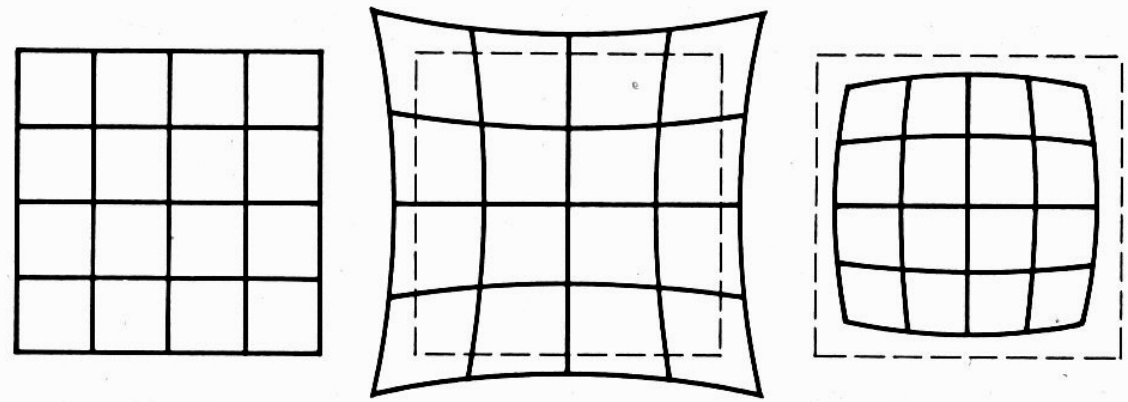


- Can use the EXIF data tag (might not give the right thing)
- Can use several images together and try to find f that would make them match
- Can use a known 3D object and its projection to solve for f
- Etc.

There are other camera parameters too:

- Optical center, non-square pixels, lens distortion, etc.

Distortion



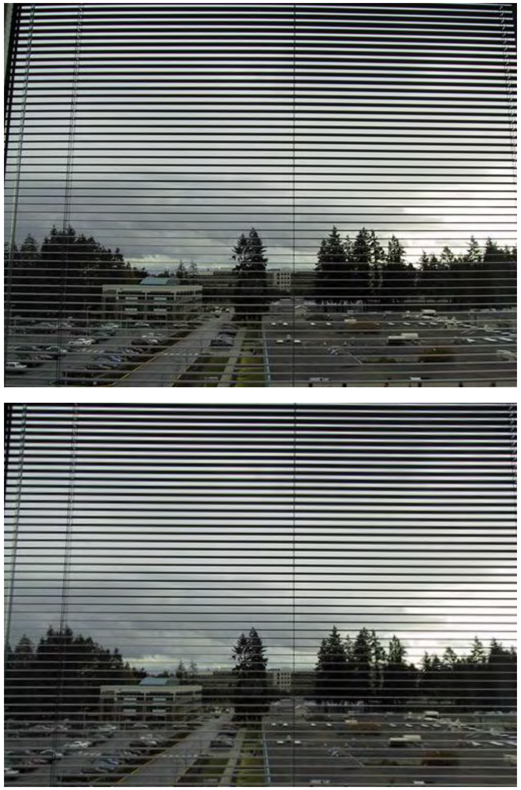
No distortion Pin cushion Barrel

Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

Radial distortion

Correct for “bending” in wide field of view lenses

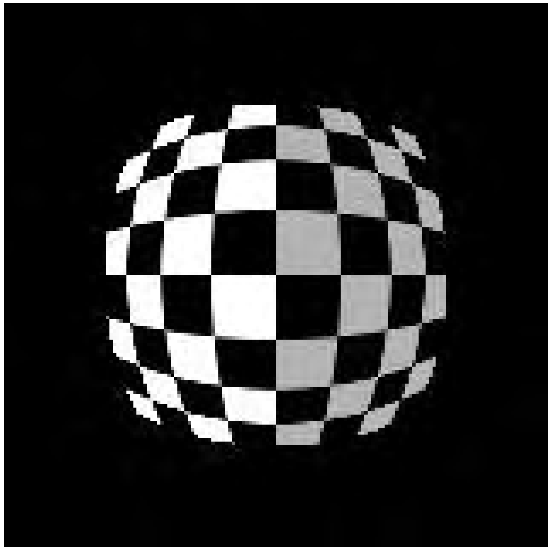


$$\begin{aligned}
\widehat{r}^2 &= \widehat{x}^2 + \widehat{y}^2 \\
\widehat{x}' &= \widehat{x}/(1 + \kappa_1 \widehat{r}^2 + \kappa_2 \widehat{r}^4) \\
\widehat{y}' &= \widehat{y}/(1 + \kappa_1 \widehat{r}^2 + \kappa_2 \widehat{r}^4) \\
x &= f\widehat{x}'/\widehat{z} + x_c \\
y &= f\widehat{y}'/\widehat{z} + y_c
\end{aligned}$$

Use this instead of normal projection

Polar Projection

Extreme "bending" in ultra-wide fields of view



$$\hat{r}^2 = \hat{x}^2 + \hat{y}^2$$

$$(\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi) = s\,(x, y, z)$$

uations become

$$x' = s\phi\cos\theta = s\frac{x}{r}\tan^{-1}\frac{r}{z},$$

$$y' = s\phi\sin\theta = s\frac{y}{r}\tan^{-1}\frac{r}{z},$$

Camera calibration

Determine camera parameters from *known* 3D points or calibration object(s)

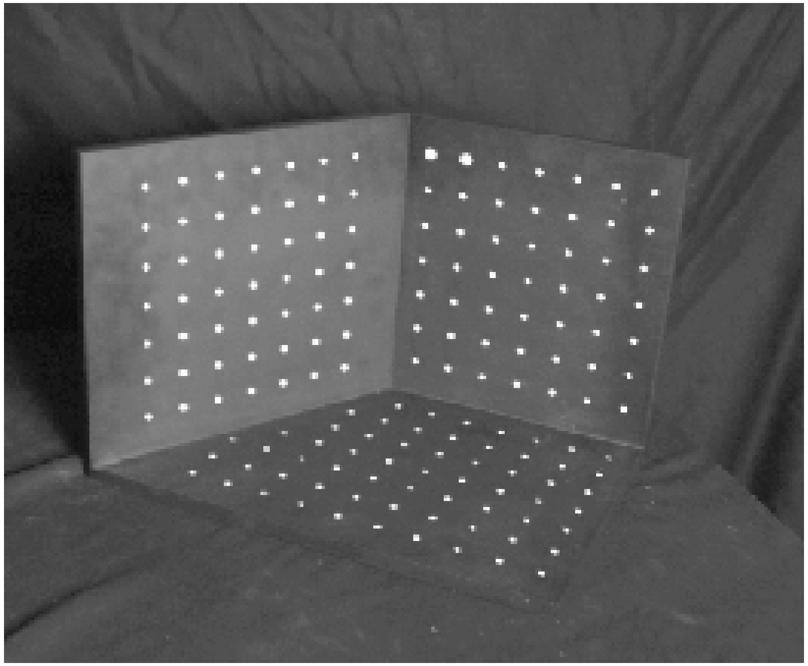
1. *internal* or *intrinsic* parameters such as focal length, optical center, aspect ratio:
   what kind of camera?
2. *external* or *extrinsic* (pose) parameters:
   where is the camera in the world coordinates?
   - World coordinates make sense for multiple cameras / multiple images

How can we do this?

Approach 1: solve for projection matrix

Place a known object in the scene

- identify correspondence between image and scene
- compute mapping from scene to image



$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Solve for Projection Matrix Π using least-squares (just like in homework)

Advantages:

- All specifics of the camera summarized in one matrix
- Can predict where any world point will map to in the image

Disadvantages:

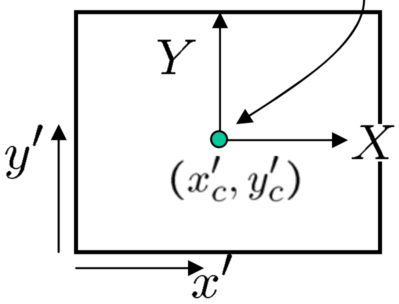
- Doesn't tell us about particular parameters
- Mixes up internal and external parameters
  - pose specific: move the camera and everything breaks

Approach 2: solve for parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation



$$\mathbf{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

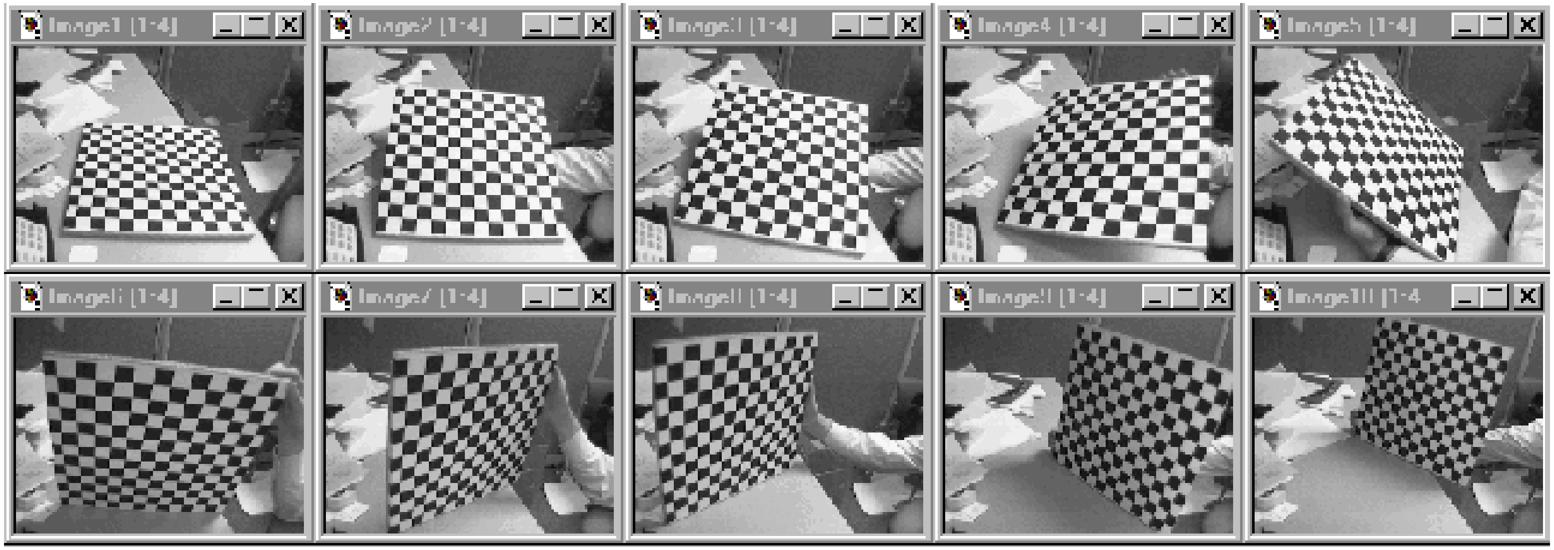
identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

intrinsics projection rotation translation

- Solve using non-linear optimization

Multi-plane calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don’t have to know positions/orientations
- Good code available online!
  - Intel’s OpenCV library: http://www.intel.com/research/mrl/research/opencv/
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Zhengyou Zhang’s web site: http://research.microsoft.com/~zhang/Calib/