Image Filtering
Salvador Dali, “Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976
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Filtering noise

How can we “smooth” away noise in an image?
Mean filtering

\[ F[x, y] \]

\[ G[x, y] \]
Mean filtering

\[ F[x, y] \]

\[ G[x, y] \]
Cross-correlation filtering

Let’s write this down as an equation. Assume the averaging window is $(2k+1)\times(2k+1)$:

$$G[i, j] = \frac{1}{(2k + 1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i + u, j + v]$$

We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

This is called a cross-correlation operation and written:

$$G = H \otimes F$$

$H$ is called the “filter,” “kernel,” or “mask.”

The above allows negative filter indices. When you implement need to use: $H[u+k,v+k]$ instead of $H[u,v]$
Mean kernel

What’s the kernel for a 3x3 mean filter?

\[
\begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

When can taking an un weighted mean be bad idea?

\[H[u, v]\]

\[F'[x, y]\]
Gaussian filtering

A Gaussian kernel gives less weight to pixels further from the center of the window

\[
F[x, y] = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}
\]

This kernel is an approximation of a Gaussian function:

What happens if you increase \( \sigma \)?
Mean vs. Gaussian filtering
Pixelation Fun
Pixelation Fun

http://www.salle.url.edu/~ftorre/
Convolution

A convolution operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

\[ G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i - u, j - v] \]

It is written: \( G = H \ast F \)

Suppose \( H \) is a Gaussian or mean kernel. How does convolution differ from cross-correlation?
Median filters

A **Median Filter** operates over a window by selecting the median intensity in the window.

What advantage does a median filter have over a mean filter?

Is a median filter a kind of convolution?
Comparison: salt and pepper noise

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Gaussian</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x3</td>
<td><img src="3x3_mean.png" alt="Image" /></td>
<td><img src="3x3_gaussian.png" alt="Image" /></td>
<td><img src="3x3_median.png" alt="Image" /></td>
</tr>
<tr>
<td>5x5</td>
<td><img src="5x5_mean.png" alt="Image" /></td>
<td><img src="5x5_gaussian.png" alt="Image" /></td>
<td><img src="5x5_median.png" alt="Image" /></td>
</tr>
<tr>
<td>7x7</td>
<td><img src="7x7_mean.png" alt="Image" /></td>
<td><img src="7x7_gaussian.png" alt="Image" /></td>
<td><img src="7x7_median.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Comparison: Gaussian noise

Mean  Gaussian  Median

3x3

5x5

7x7
Unsharp Masking

So, what does blurring take away?

\[
\begin{align*}
\text{original} & - \quad \text{blurred} & = & \quad \text{low-pass filter} \\
& + \alpha & = & \quad \text{unsharp masking}
\end{align*}
\]
Unsharp Masking (MATLAB)

Imrgb = imread('file.jpg');

im = im2double(rgb2gray(imrgb));

g= fspecial('gaussian', 25,4);

imblur = conv2(im,g,'same');

imagesc([im imblur])

imagesc([im im+.4*(im-imblur)])