## Multiple View Geometry


...with a lot of slides stolen from Steve Seitz and Jianbo Shi

15-463: Computational Photography

## Our Goal



## The Plenoptic Function



$$
P\left(\theta, \phi, \lambda, t, V_{X}, V_{Y}, V_{Z}\right)
$$

How can we compress this into something manageable?

## Stereo Reconstruction

The Stereo Problem

- Shape from two (or more) images
- Biological motivation



## Why do we have two eyes?



Cyclope
vS.
Odysseus

## 1. Two is better than one



## 2. Depth from Convergence



Human performance: up to 6-8 feet

## 3. Depth from binocular disparity


$P$ : converging point

C: object nearer projects to the outside of the $P$, disparity $=+$

F: object farther projects to the inside of the $P$, disparity = -

Sign and magnitude of disparity


## Stereo



## Stereo



## Basic Principle: Triangulation

- Gives reconstruction as intersection of two rays
- Requires
- calibration
- point correspondence


## Stereo correspondence

## Determine Pixel Correspondence

- Pairs of points that correspond to same scene point



## Epipolar Constraint

- Reduces correspondence problem to 1D search along conjugate epipolar lines

Stereo image rectification


## Stereo image rectification

## Image Reprojection

- reproject image planes onto common plane parallel to line between optical centers
- a homography ( $3 \times 3$ transform) applied to both input images
- pixel motion is horizontal after this transformation

- C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision. IEEE Conf. Computer Vision and Pattern Recognition, 1999.


## Stereo Rectification



$$
\sqrt{\square}
$$



## Your basic stereo algorithm



For each epipolar line
For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match windows

- This should look familar...
- Can use Lukas-Kanade or discrete search (latter more common)


## Window size


$\mathrm{W}=3$

$\mathrm{W}=20$
Effect of window size

- Smaller window
$+$
- 
- Larger window


## Stereo results

- Data from University of Tsukuba
- Similar results on other images without ground truth


Scene


Ground truth

## Results with window search



Window-based matching
Ground truth (best window size)

## Better methods exist...



State of the art method
Boykov et al., Fast Approximate Energy Minimization via Graph Cuts, International Conference on Computer Vision, September 1999.

Ground truth

## Depth from disparity


input image (1 of 2)

depth map


3D rendering
[Szeliski \& Kang ‘95]


$$
\text { disparity }=x-x^{\prime}=\frac{\text { baseline } * f}{z}
$$

## Stereo reconstruction pipeline

## Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions


## Stereo matching

## Need texture for matching



Julesz-style Random Dot Stereogram

## Active stereo with structured light



Li Zhang's one-shot stereo


Project "structured" light patterns onto the object

- simplifies the correspondence problem


## Active stereo with structured light



## Laser scanning




Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

Optical triangulation

- Project a single stripe of laser light
- Scan it across the surface of the object
- This is a very precise version of structured light scanning

Portable 3D laser scanner (this one by Minolta)


## Real-time stereo



Nomad robot searches for meteorites in Antartica http://www.frc.ri.cmu.edu/projects/meteorobot/index.html

## Used for robot navigation (and other tasks)

- Several software-based real-time stereo techniques have been developed (most based on simple discrete search)


## Structure from Motion



Reconstruct

- Scene geometry
- Camera motion


## Three approaches

## (b) Hybrid Approach

(a) Geometry-Based



## Outline of a simple algorithm (1)

- Based on constraints
- Input to the algorithm (1): two images



## Outline of a simple algorithm (2)

- Input to the algorithm (2):

User select edges and corners


## Outline of a simple algorithm (3)

- Camera Position and Orientation

Determine the position and orientation of camera


## Outline of a simple algorithm (4)

- Computing projection matrix and Reconstruction



## Outline of a simple algorithm (5)

- Compute 3D textured triangles



## View-Dependant Texture Mapping




Figure 12: The weighting function used in view-dependent texture mapping. The pixel in the virtual view corresponding to the point on the model is assigned a weighted average of the corresponding pixels in actual views 1 and 2 . The weights $w_{1}$ and $w_{2}$ are inversely inversely proportional to the magnitude of angles $a_{1}$ and $a_{2}$. Alternately, more sophisticated weighting functions based on expected foreshortening and image resampling can be used.

## Facade



## Façade (Debevec et al) inputs



## Façade (Debevec et al)



