More Single View Geometry
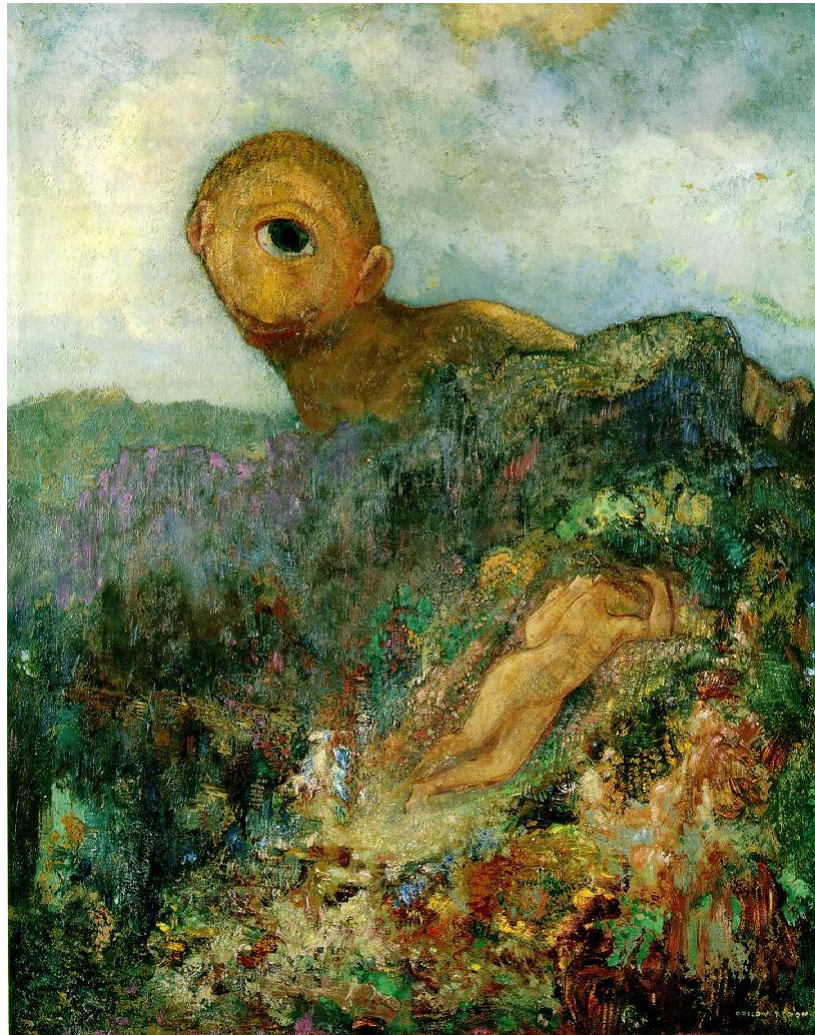
Cyclops **Odilon Redon** 1904

15-463: Computational Photography
Alexei Efros, CMU, Fall 2005

*…with a lot of slides stolen
from Steve Seitz*

Pre-Turkey Issue #1: Midterm

Grades are in!

Undergrads: 63, 54, 56, 61, 66, 53, 54
mean   58.125

Grads: 68, 50, 56
mean   58

Please check my arithmetic!

Projects:
 You are fine, unless marked on the midterm

Pre-Turkey Issue #2: Final Projects

Are coming up fast!

Undergrads can work in pairs, but project must be bigger.

Sample Topics:

- Full 360 panorama construction (spherical or cylindrical)
- Render in synthetic object into real scene
- Automatic Tour into the Picture (can use Pop-up labeling code)
- Implement a paper discussed in class (e.g. Video Textures)
- Come up with art project that uses Comp. Photography
- Etc.

Talk to me before starting!

Pop Quiz: which is 1,2,3-point perspective

Image B

Image A

Image C

Automatic Photo Pop-up
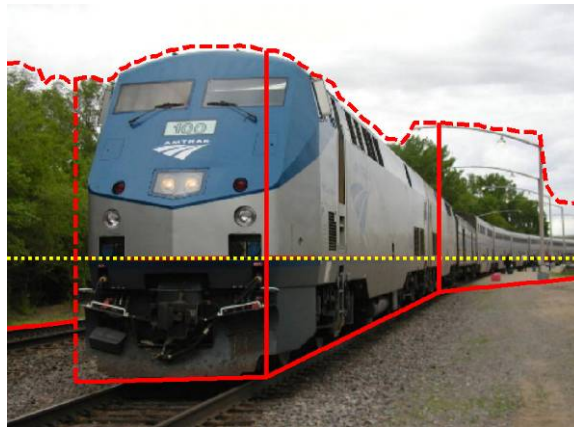

Original Image

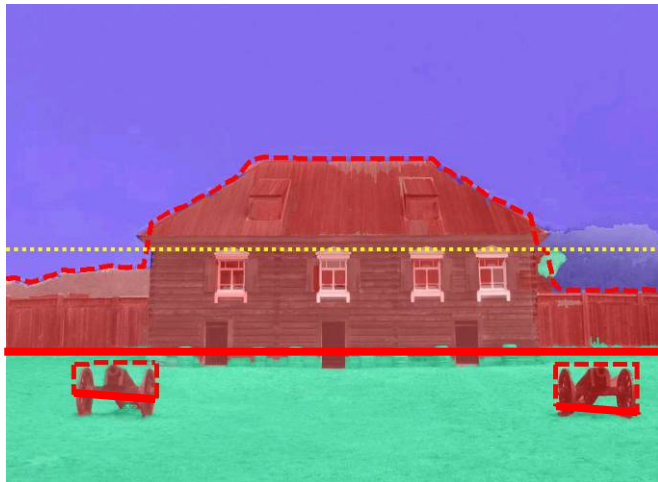Geometric Labels

Fit Segments

Cut and Fold

Novel View

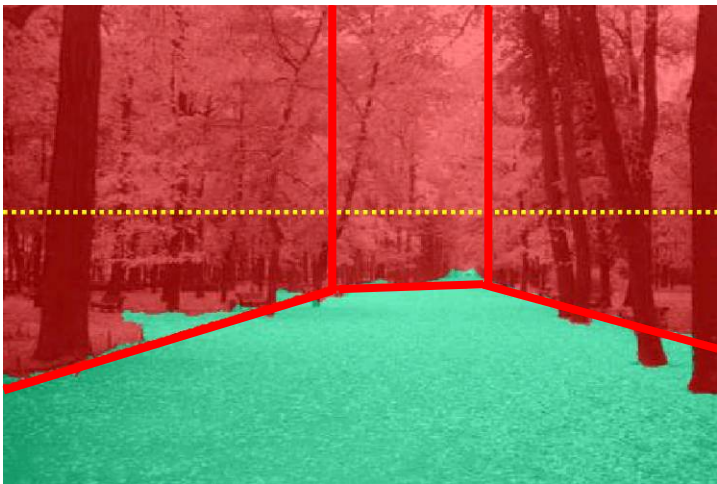# Results

Input Image

Cut and Fold

Automatic Photo Pop-up

Results

Input Image

Cut and Fold

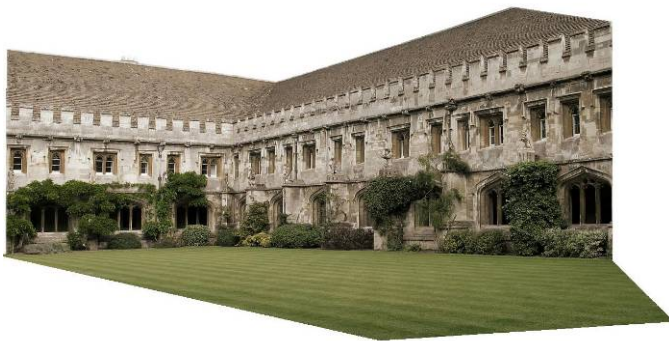Automatic Photo Pop-up
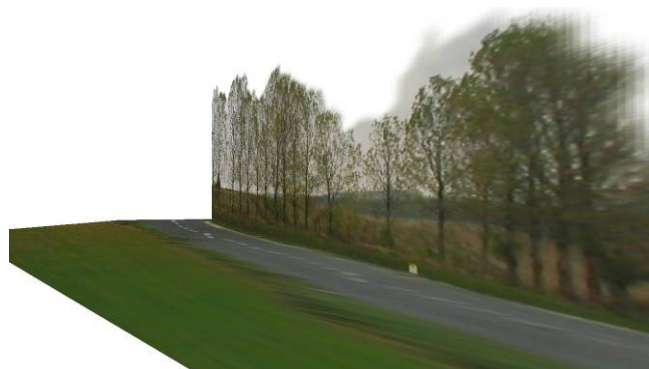
Results

Input Image

Automatic Photo Pop-up

Results

Input Images

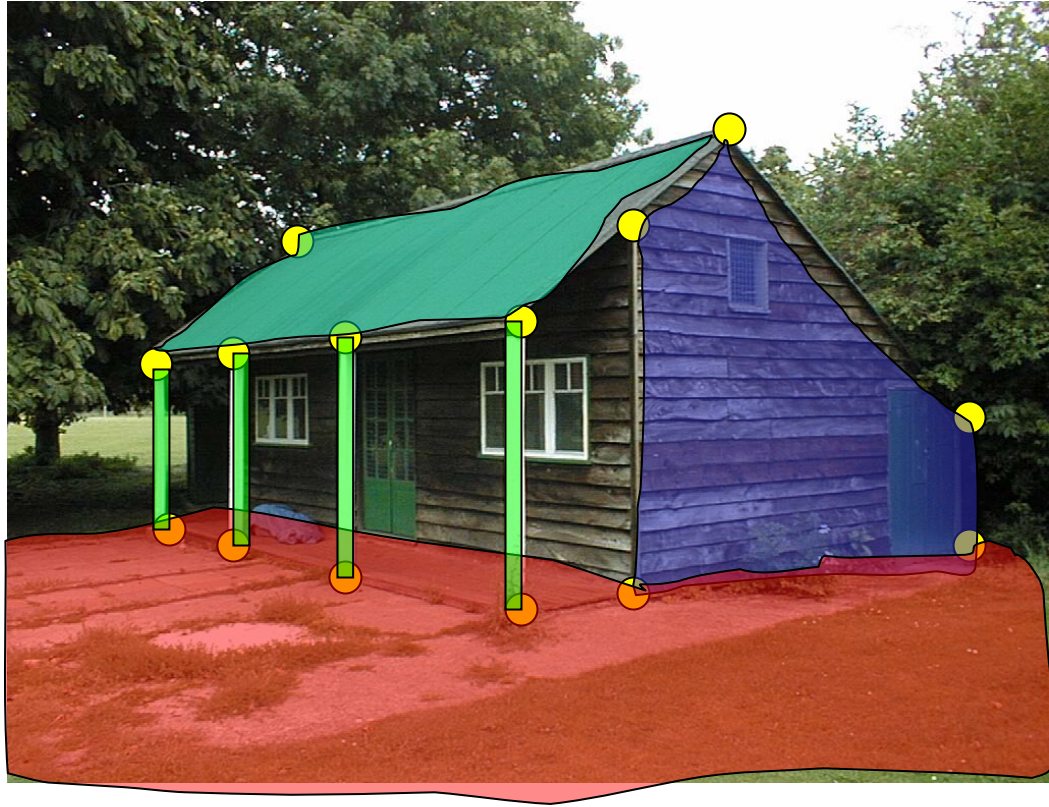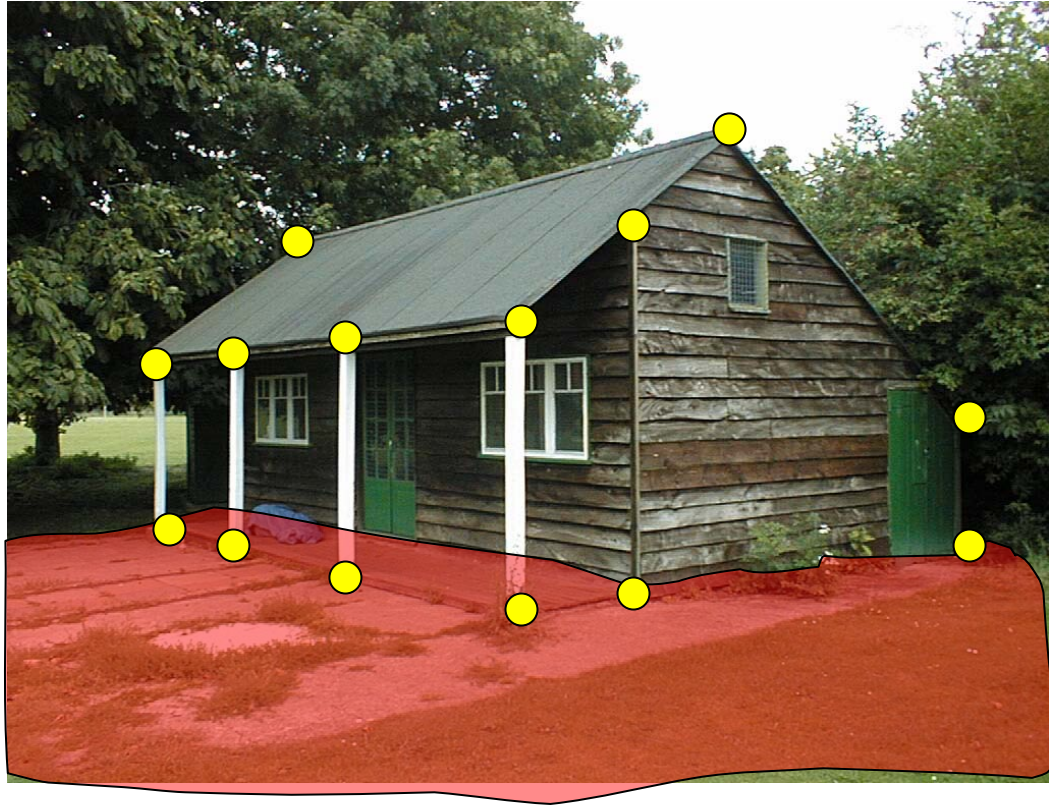Automatic Photo Pop-up

Results

Input Image

Automatic Photo Pop-up

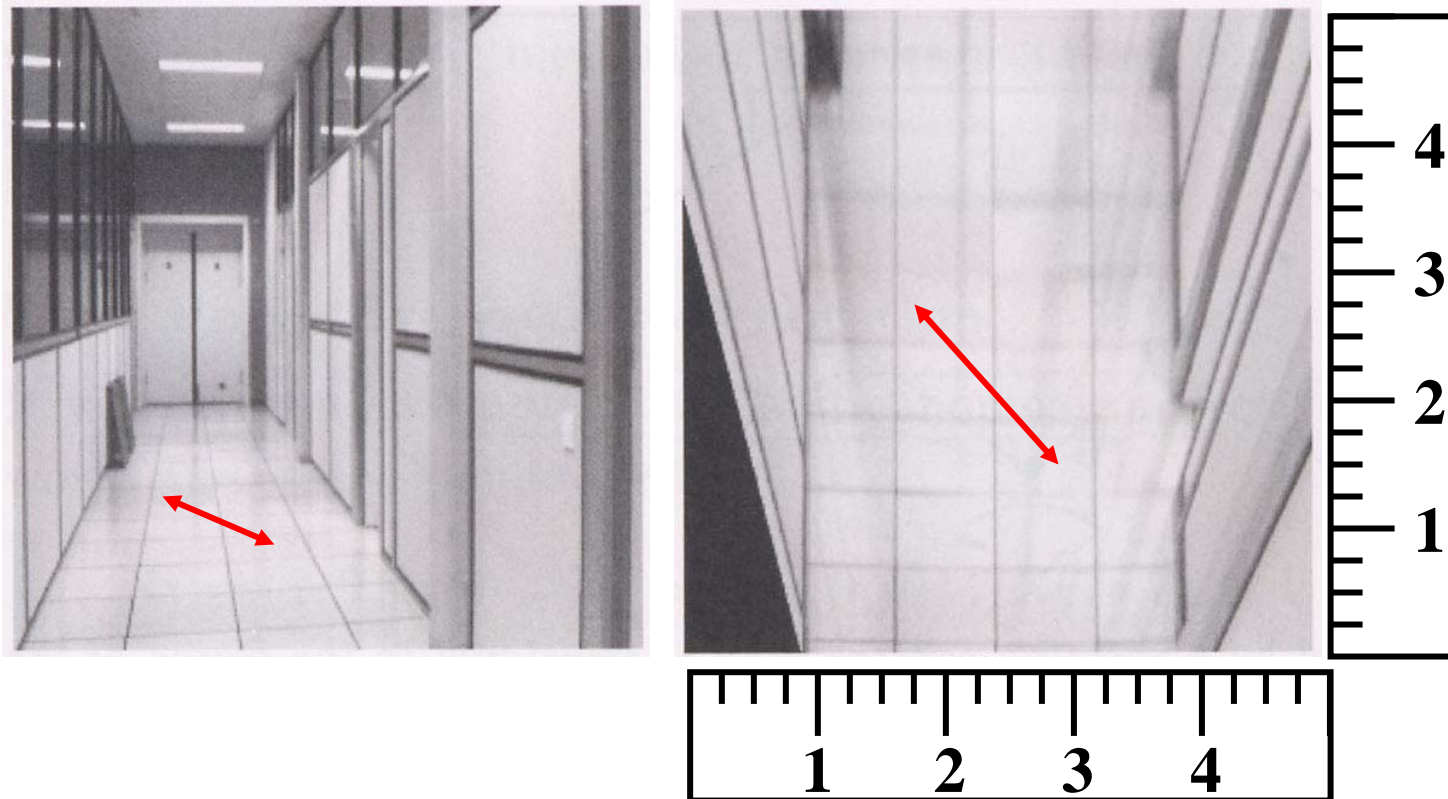# How can we model this scene?

1. Find world coordinates (X,Y,Z) for a few points
2. Connect the points with planes to model geometry
    *   Texture map the planes

Finding world coordinates (X,Y,Z)

1. Define the ground plane (Z=0)
2. Compute points (X,Y,0) on that plane
3. Compute the *heights* Z of all other points
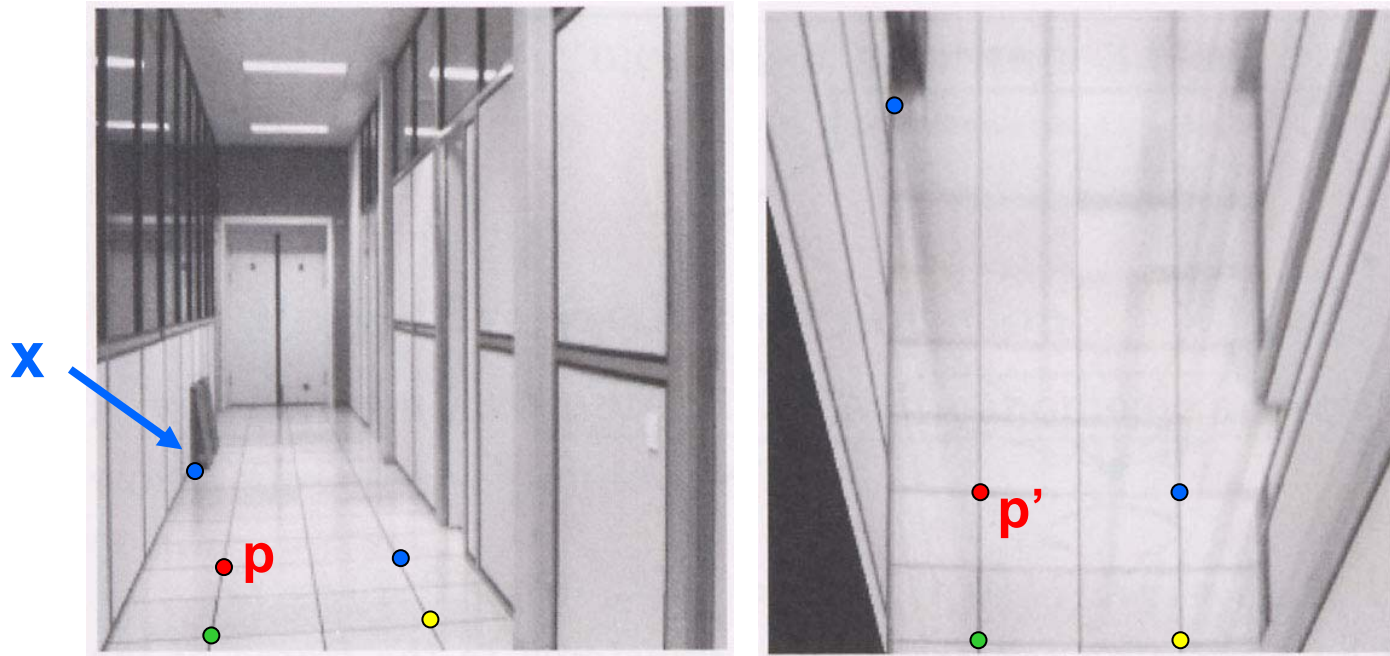
Measurements on planes

Approach: unwarp, then measure
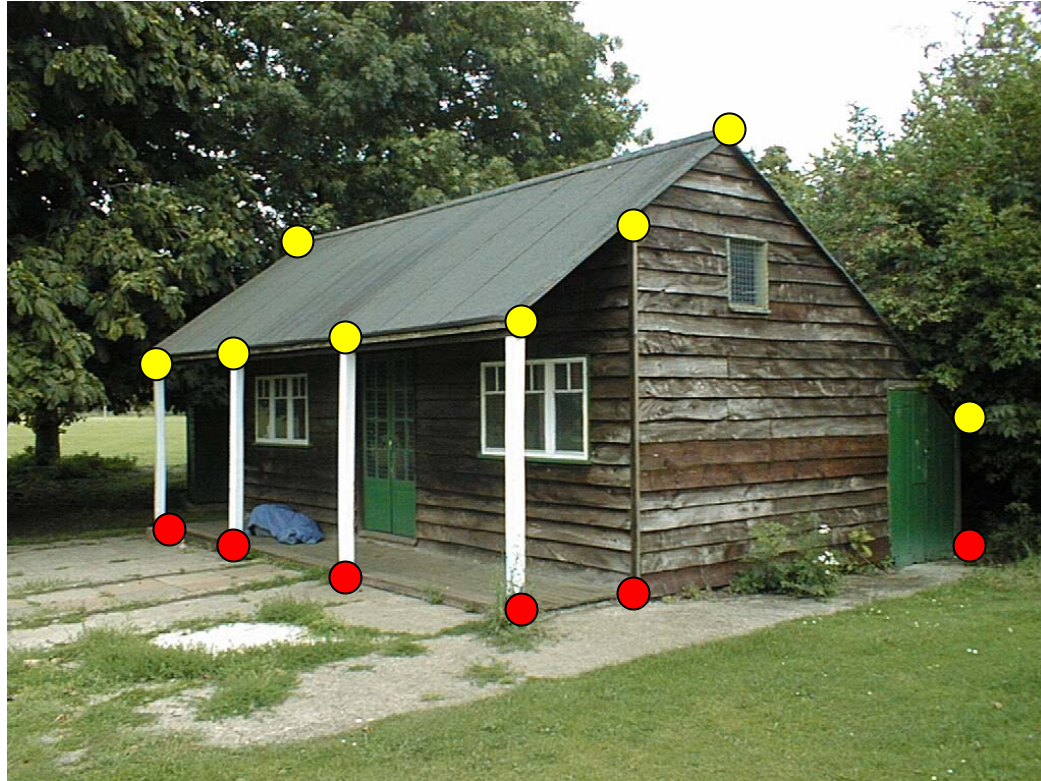
What kind of warp is this?

Unwarp ground plane

Our old friend – the homography

Need 4 reference points with world coordinates

    p = (x,y)
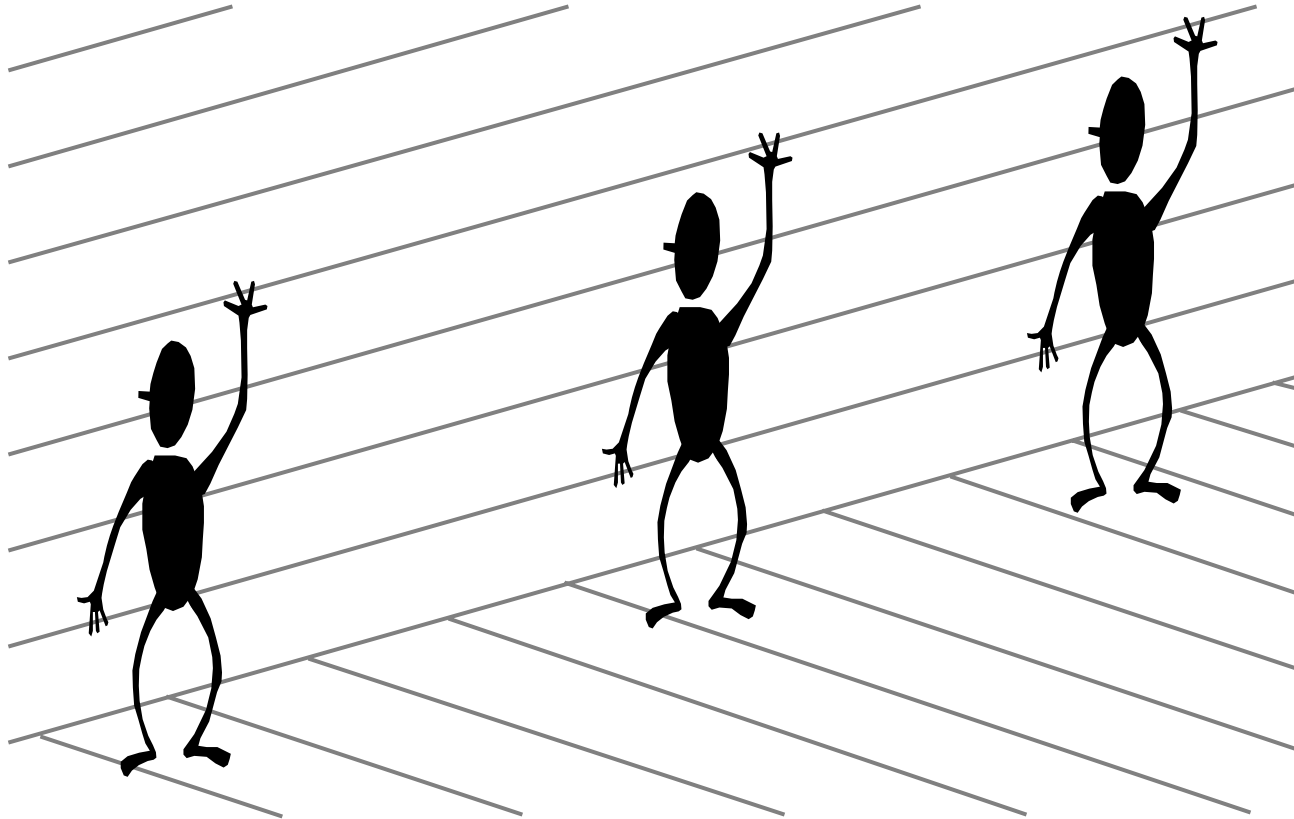
    p' = (X,Y,0)

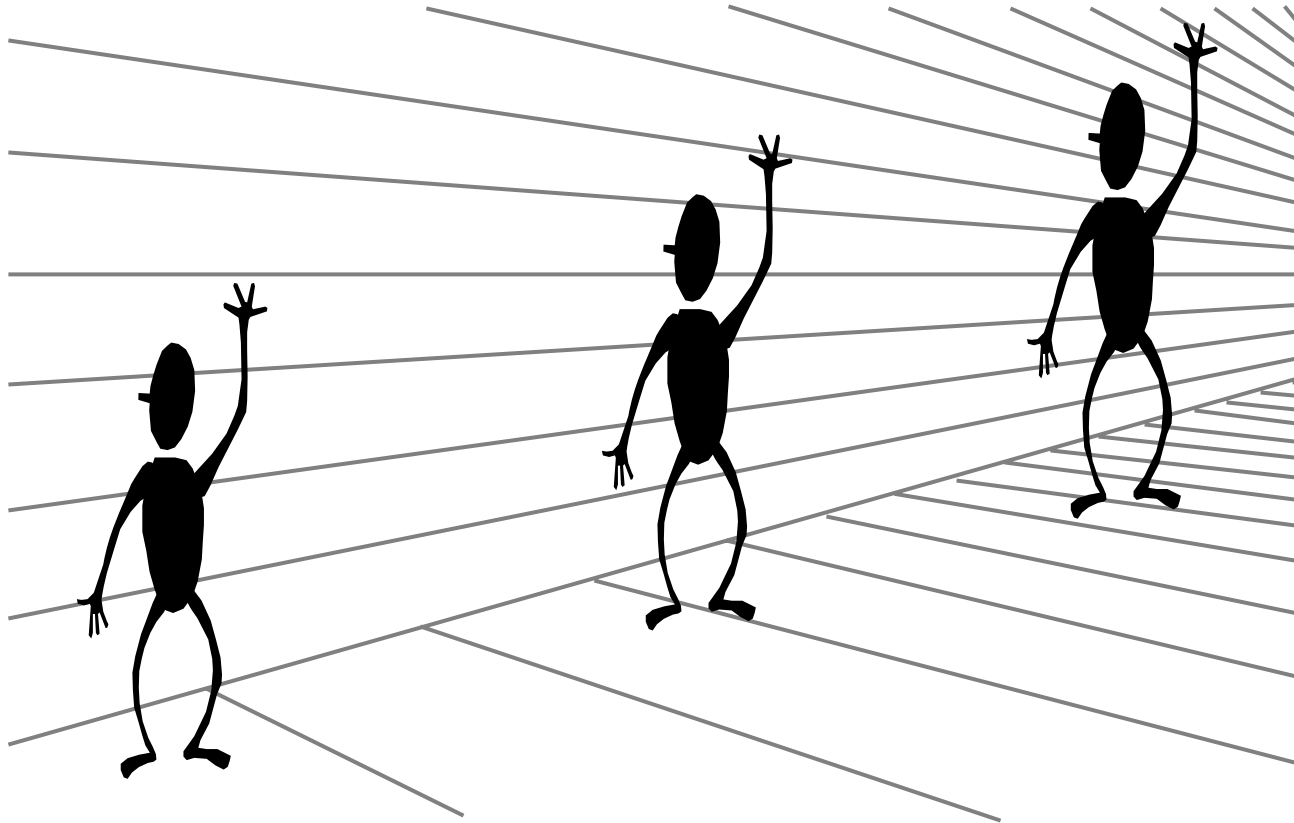# Finding world coordinates (X,Y,Z)

1. Define the ground plane (Z=0)
2. Compute points (X,Y,0) on that plane
3. Compute the *heights* Z of all other points

Comparing heights

Perspective cues

Perspective cues
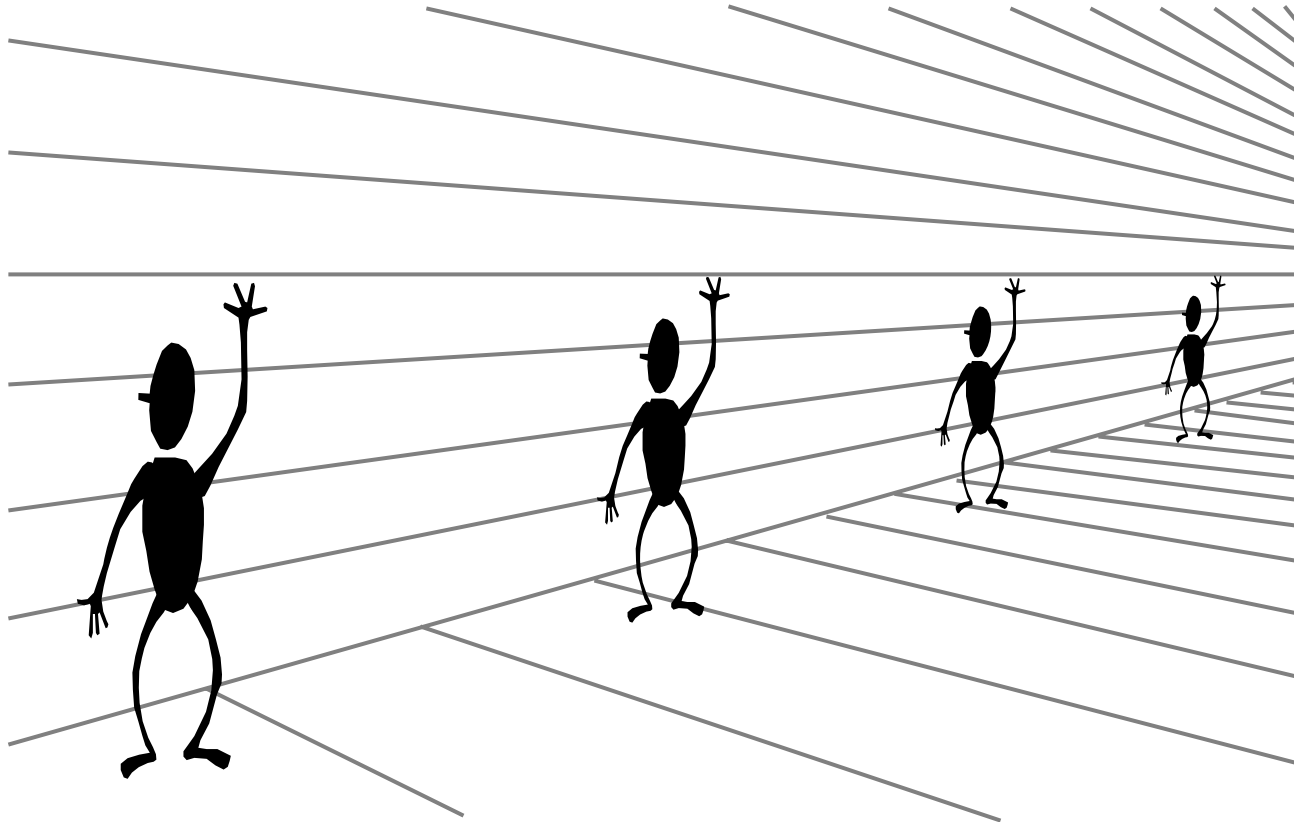
Comparing heights
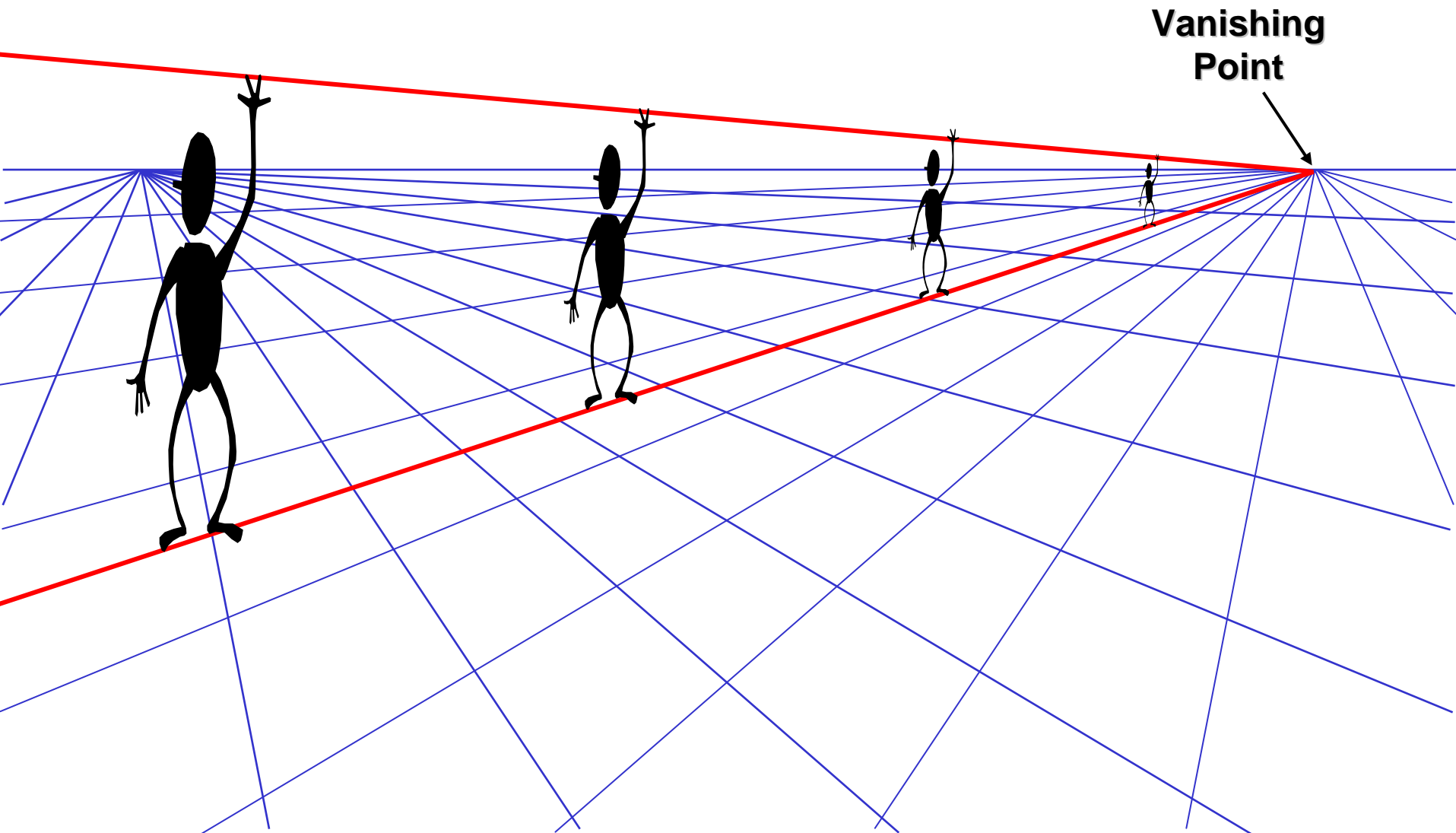
Vanishing Point

Measuring height
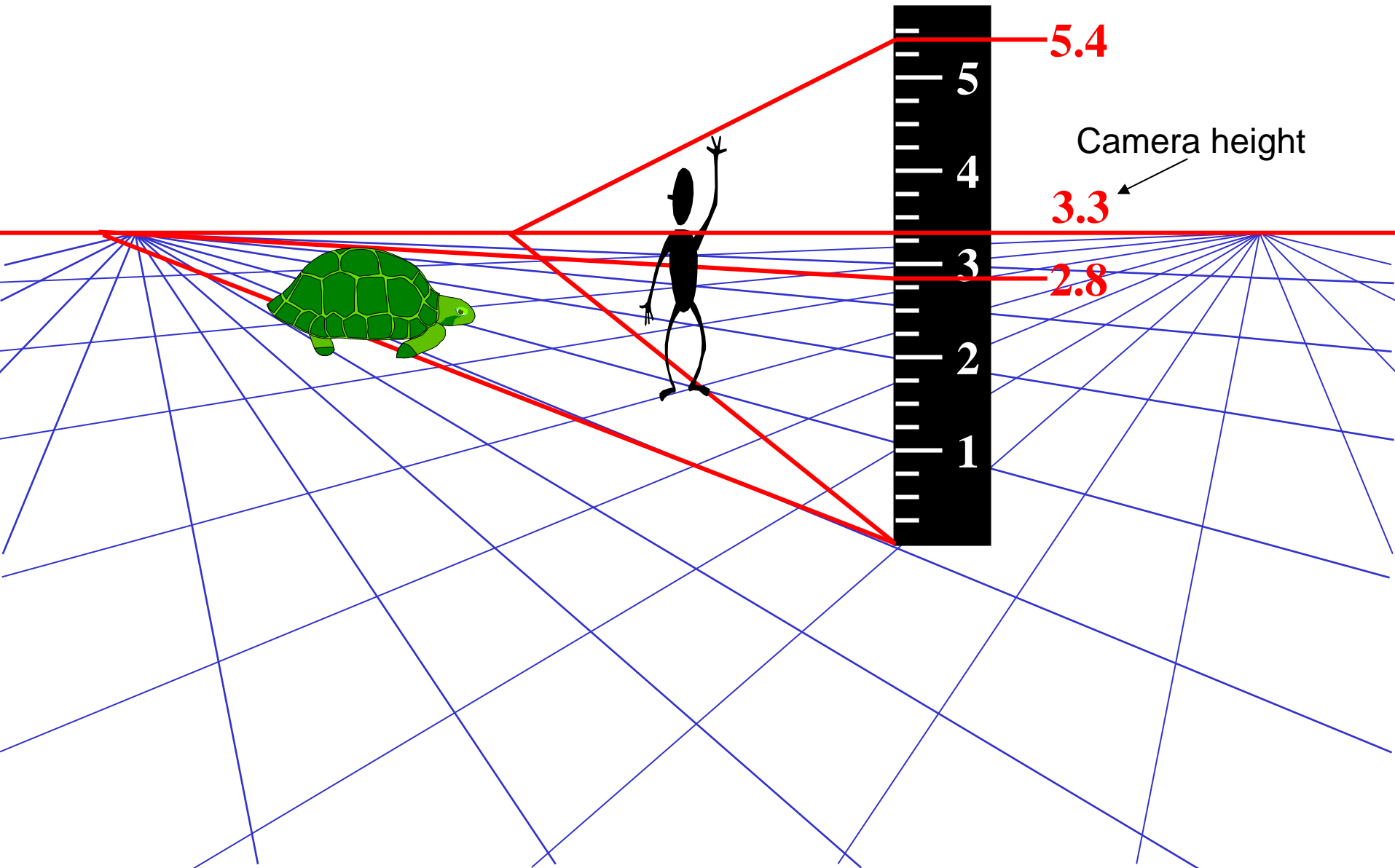
Computing vanishing points (from lines)
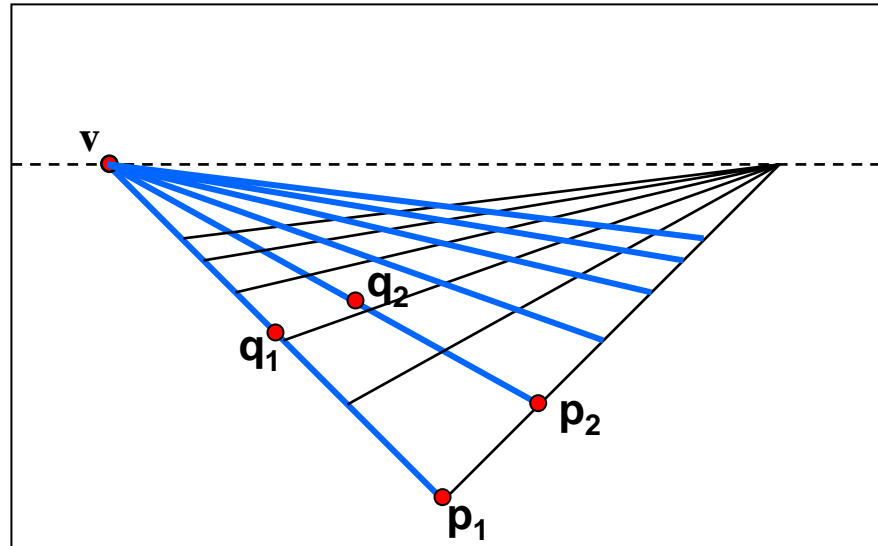
Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the "closest" point of intersection
- See notes by Bob Collins for one good way of doing this:
    - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Criminisi '99

Vertical vanishing point (at infinity)

Vanishing line

Vanishing point
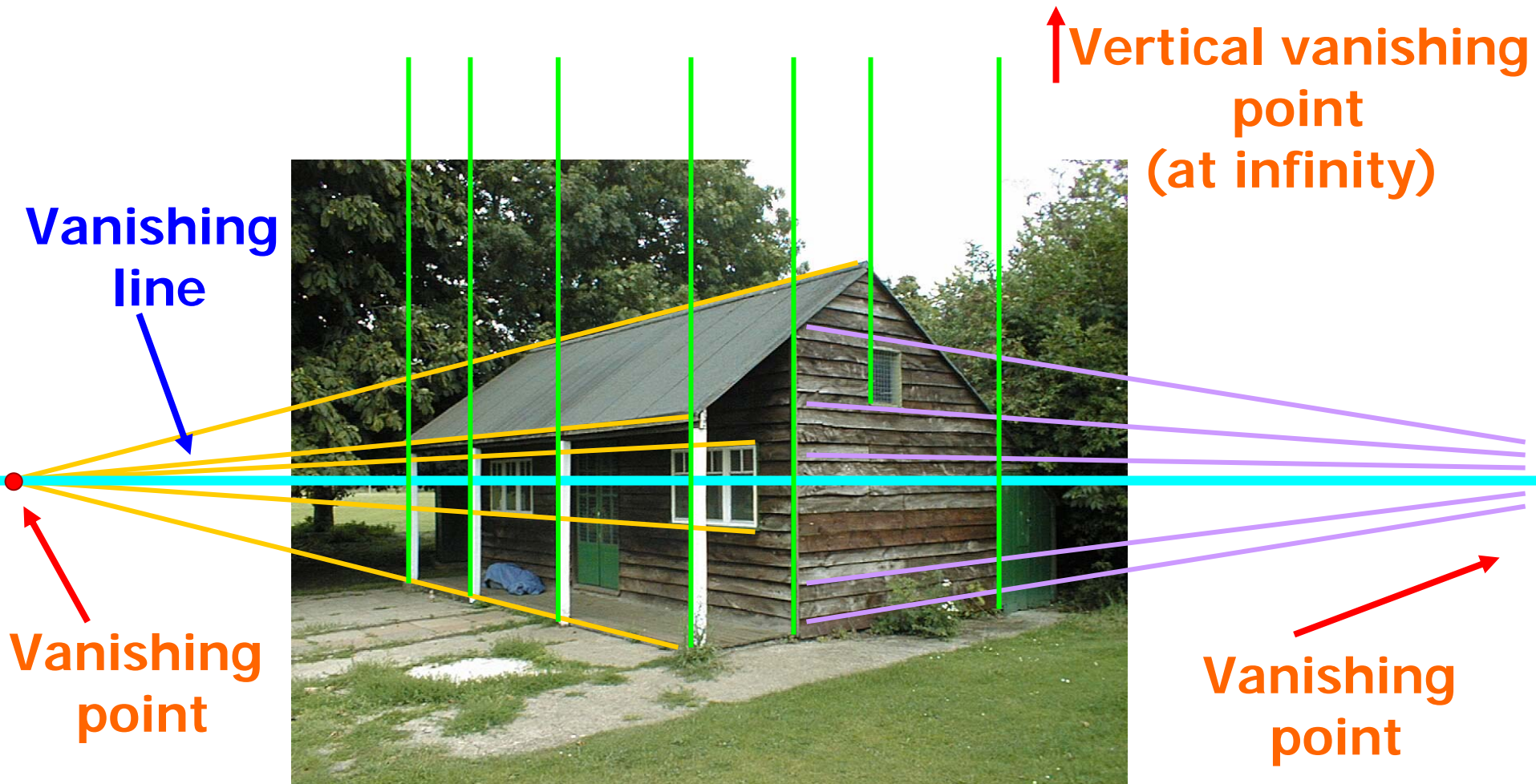
Vanishing point

Measuring height without a ruler
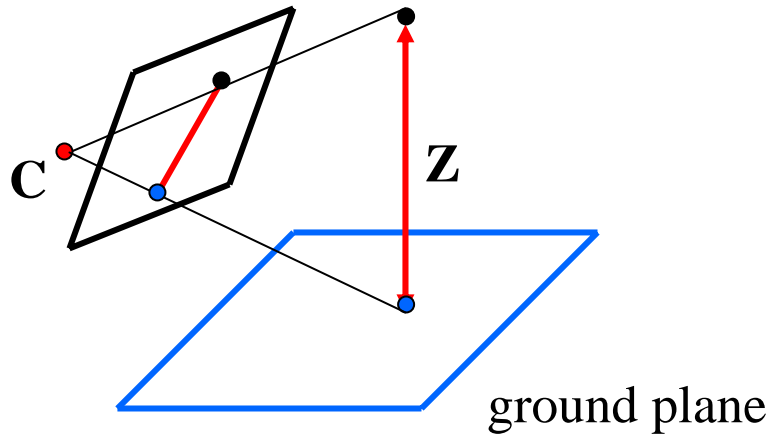
ground plane

Compute Z from image measurements

- Need more than vanishing points to do this

Measuring height

$\uparrow \mathbf{v_z}$

$\mathbf{r}$

vanishing line (horizon)

$t \cong (v \times t_0) \times (r \times b)$

$v \cong (b \times b_0) \times (v_x \times v_y)$

$\mathbf{t_0}$

$\mathbf{t}$

$\mathbf{v_x}$ $\mathbf{v}$

$\mathbf{v_y}$

$\mathbf{H}$

$\mathbf{R}$ $\mathbf{H}$

$\mathbf{b_0}$

$\mathbf{b}$

Measuring height

$\uparrow v_z$

vanishing line (horizon)

t_0

v_x

v

v_y

m_0

t_1

b_0

b_1

r

b
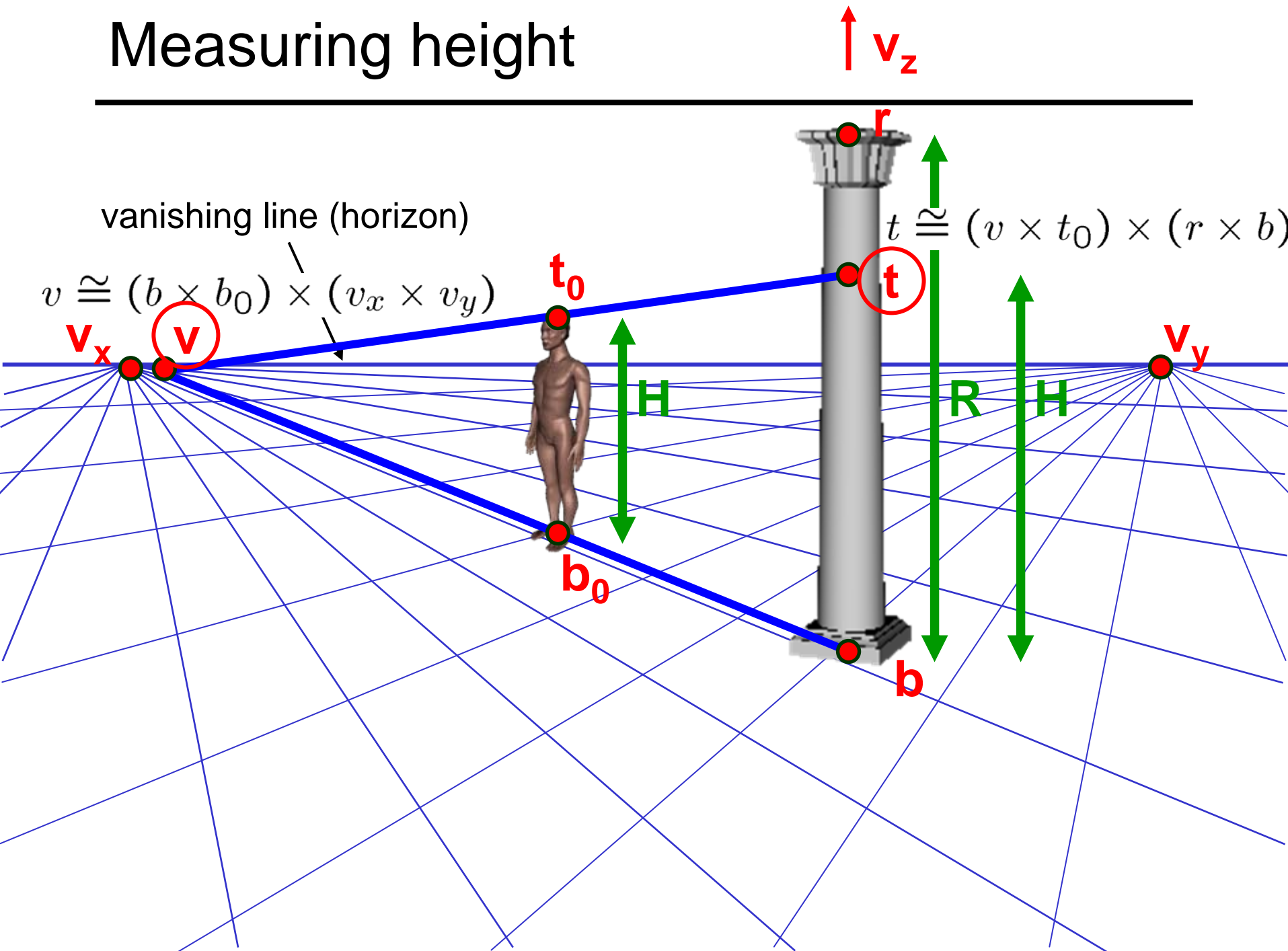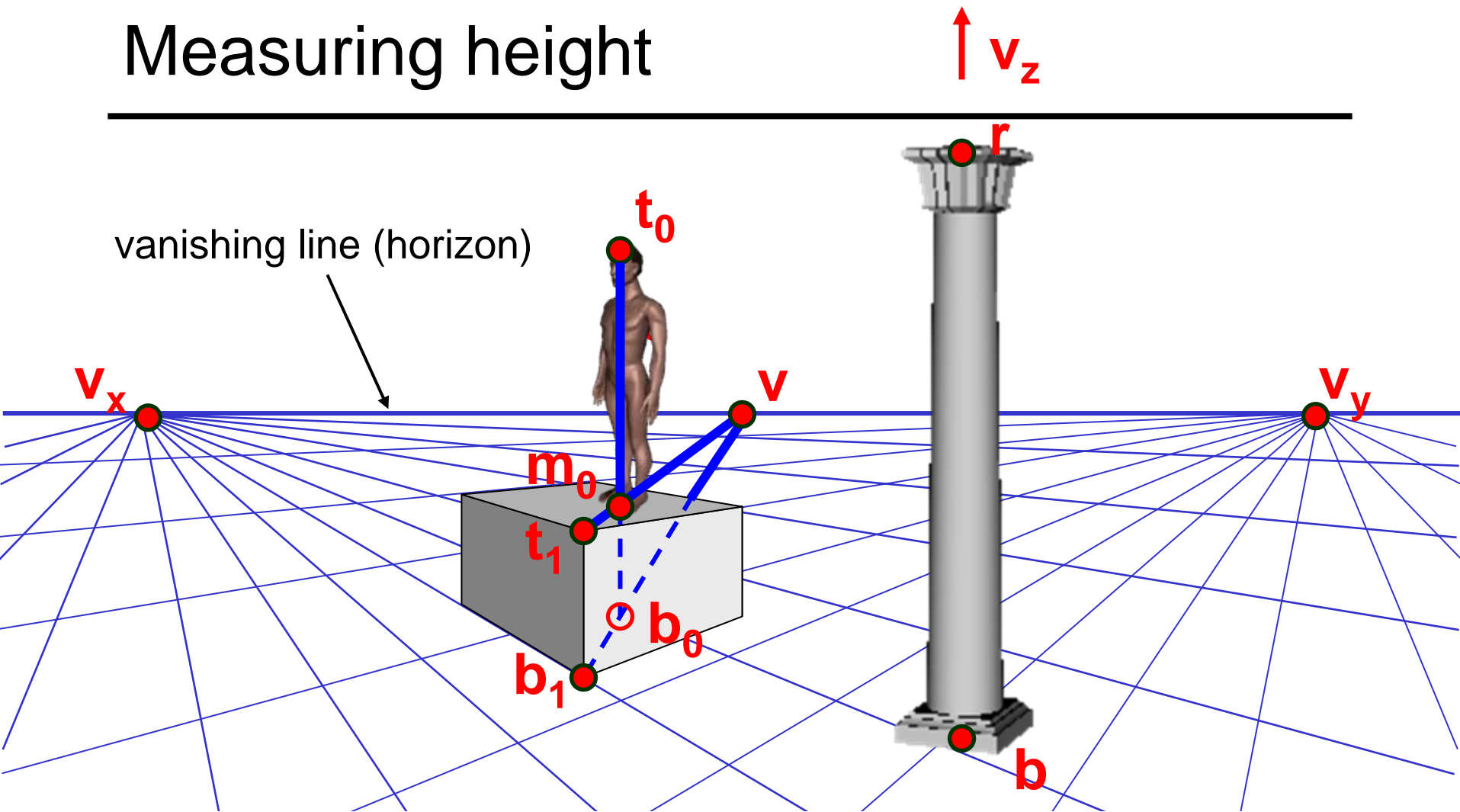
What if the point on the ground plane b_0 is not known?

- Here the guy is standing on the box
- Use one side of the box to help find b_0 as shown above

What if v_z is not infinity?

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

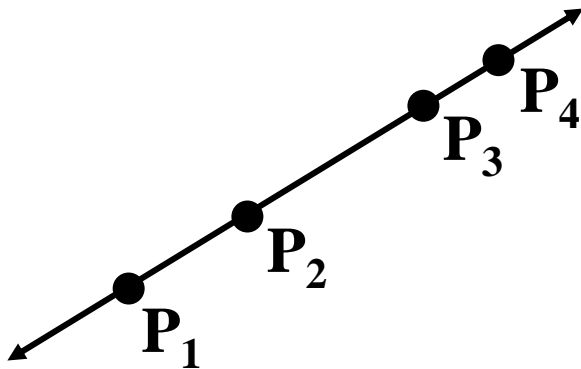$$\frac{\left\|\mathbf{P}_3 - \mathbf{P}_1\right\| \left\|\mathbf{P}_4 - \mathbf{P}_2\right\|}{\left\|\mathbf{P}_3 - \mathbf{P}_2\right\| \left\|\mathbf{P}_4 - \mathbf{P}_1\right\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Can permute the point ordering

$$\frac{\left\|\mathbf{P}_1 - \mathbf{P}_3\right\| \left\|\mathbf{P}_4 - \mathbf{P}_2\right\|}{\left\|\mathbf{P}_1 - \mathbf{P}_2\right\| \left\|\mathbf{P}_4 - \mathbf{P}_3\right\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height

∞

T (top of object)

t

r

C

b

H

R

R (reference point)

B (bottom of object)

$\mathbf{v_z}$

ground plane

$$\frac{\|\mathbf{T}-\mathbf{B}\|}{\|\mathbf{R}-\mathbf{B}\|}\frac{\|\infty-\mathbf{R}\|}{\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|}{\|\mathbf{r}-\mathbf{b}\|}\frac{\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$
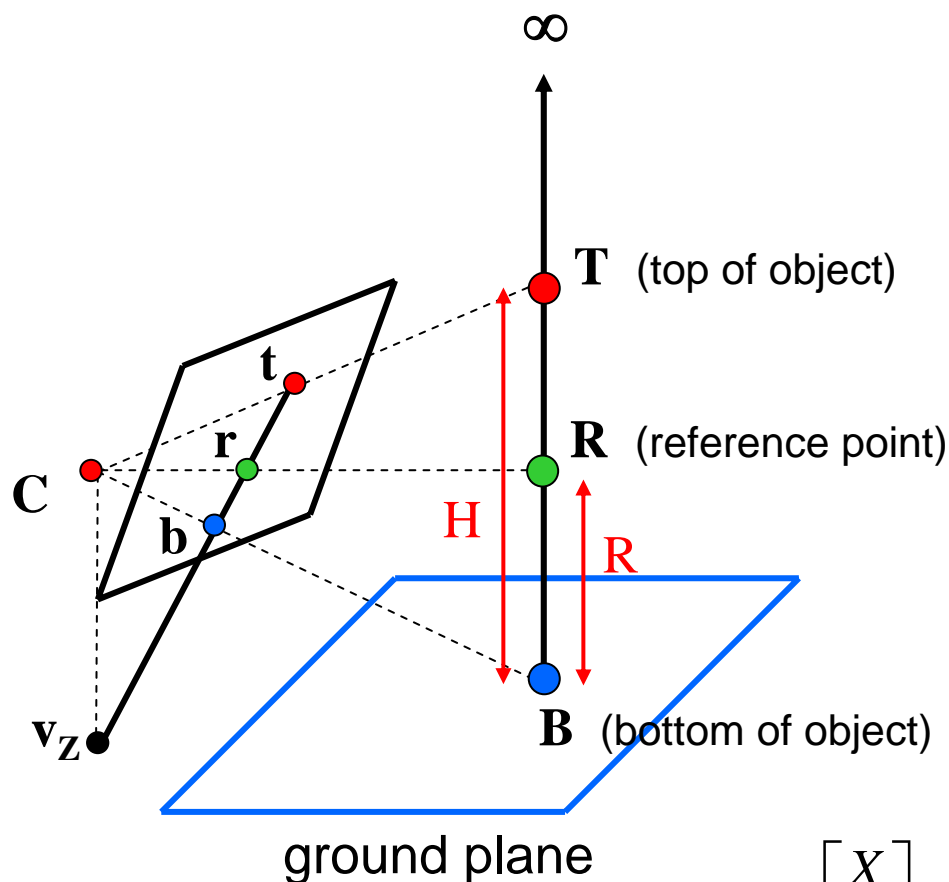
image cross ratio

scene points represented as $\mathbf{P}=\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Measuring height
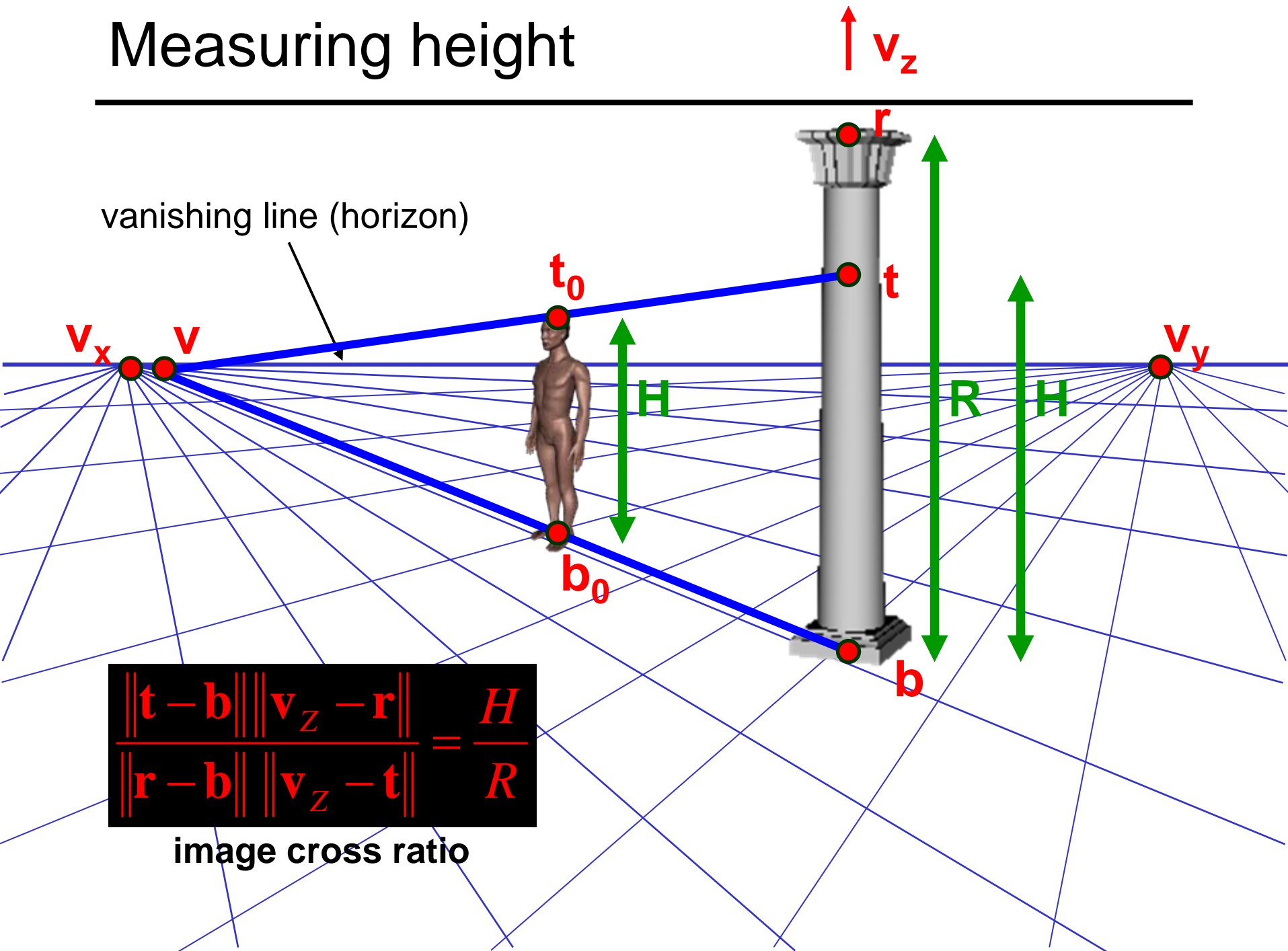

$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

Measuring heights of people

Here we go !

185.3 cm

reference
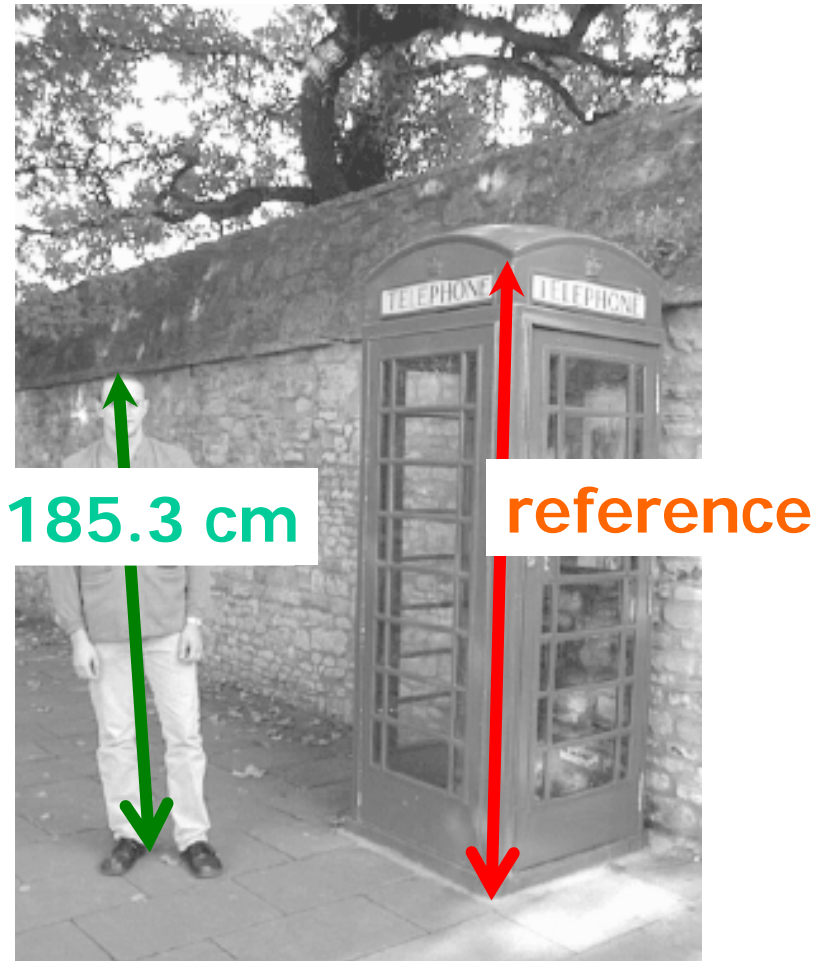
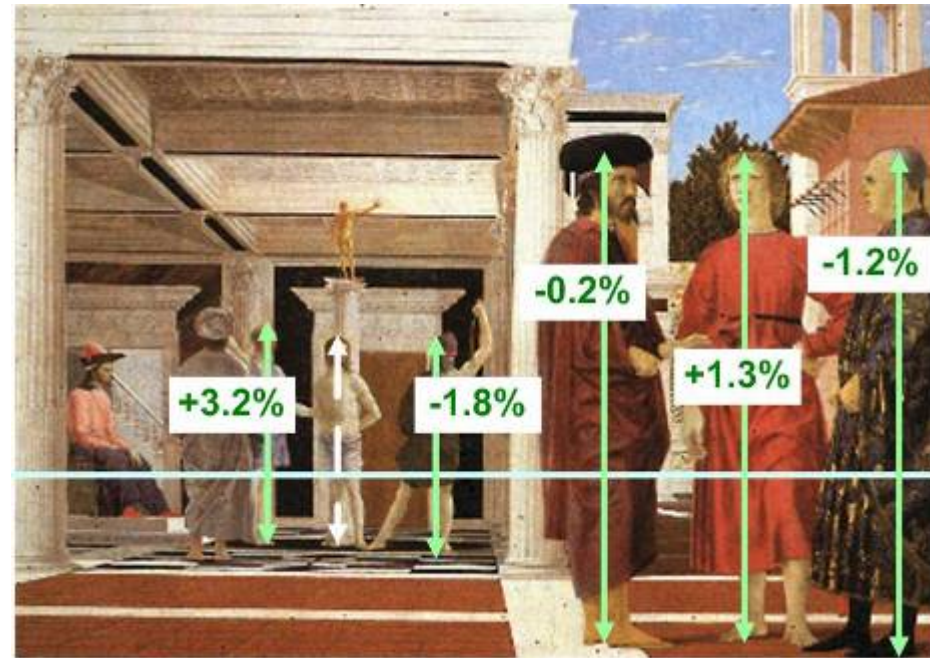# Forensic Science: measuring heights of suspects
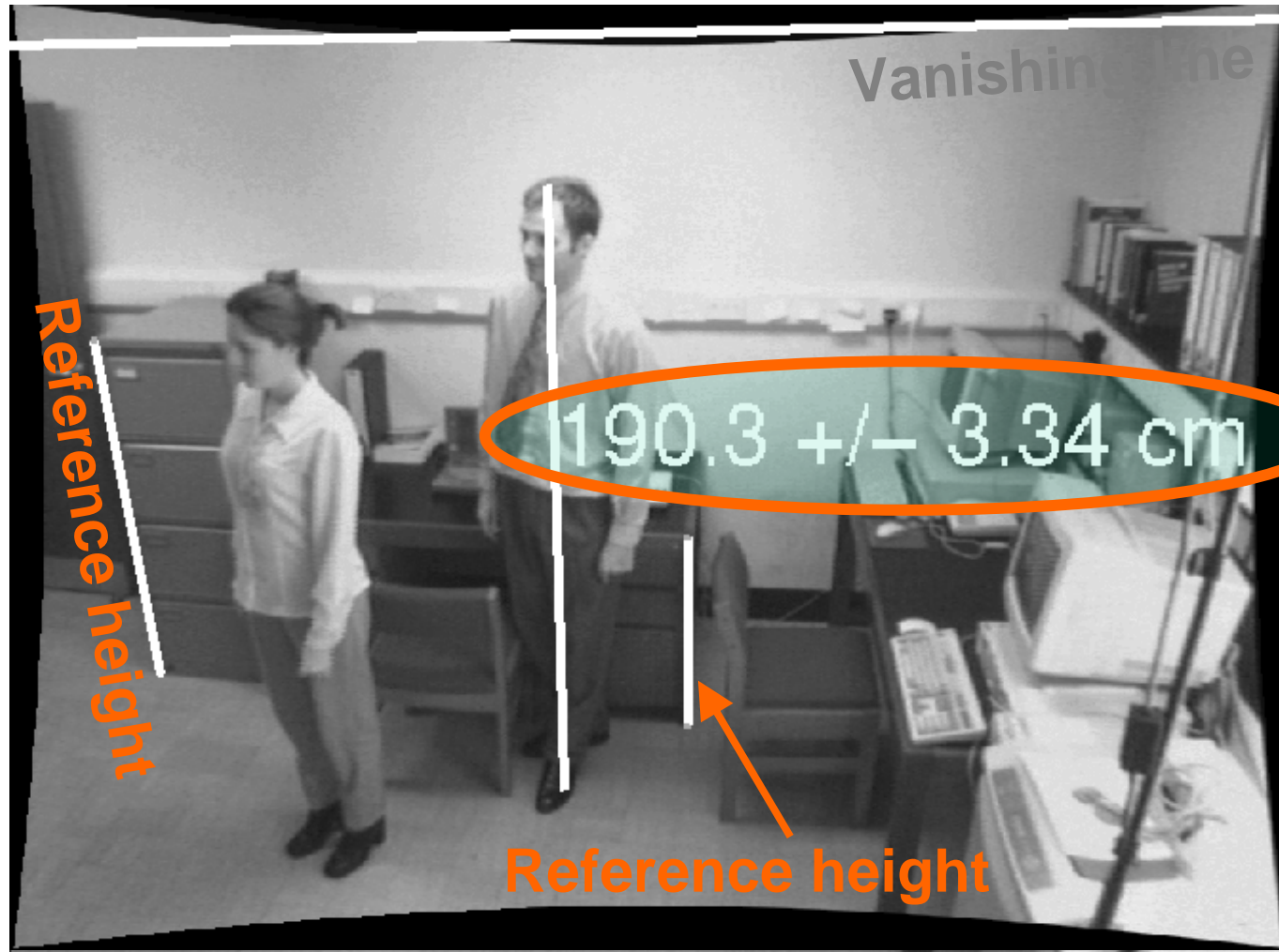
Assessing geometric accuracy

Are the heights of the 2 groups of people consistent with each other?
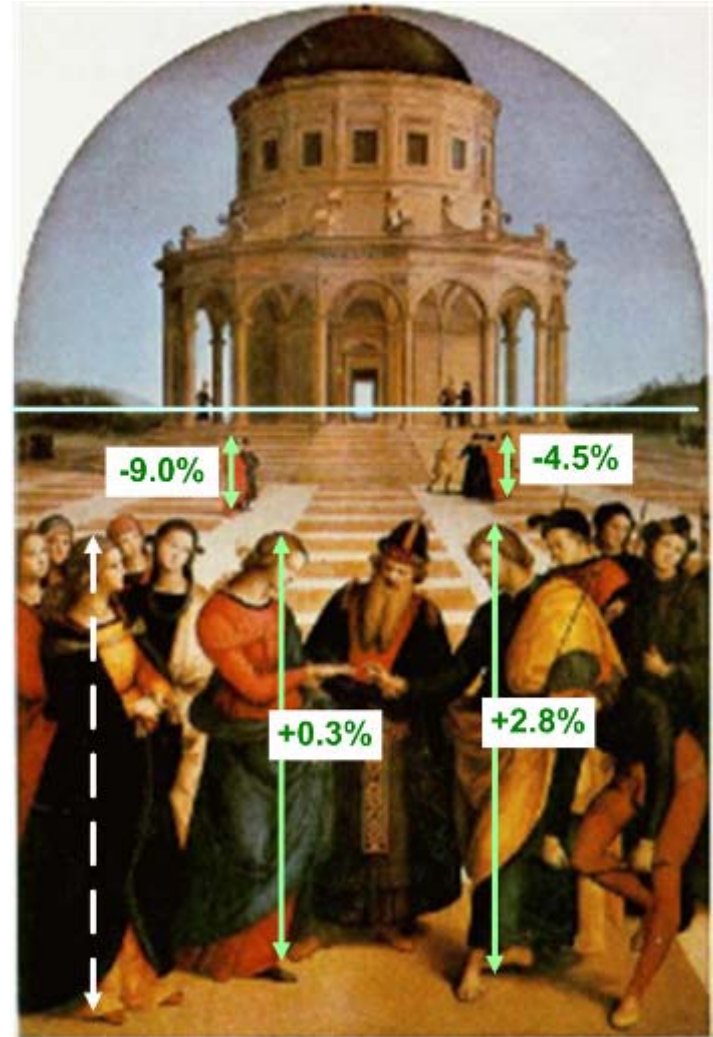

Flagellation,
Piero della Francesca

Estimated relative heights

Assessing geometric accuracy

The Marriage of the Virgin, Raphael

Estimated relative heights

Criminisi et al., ICCV 99

Complete approach

- Load in an image
- Click on lines parallel to X axis
  - repeat for Y, Z axes
- Compute vanishing points
- Specify 3D and 2D positions of 4 points on reference plane
- Compute homography H
- Specify a reference height
- Compute 3D positions of several points
- Create a 3D model from these points
- Extract texture maps
  - Cut out objects
  - Fill in holes
- Output a VRML model

Interactive silhouette cut-out
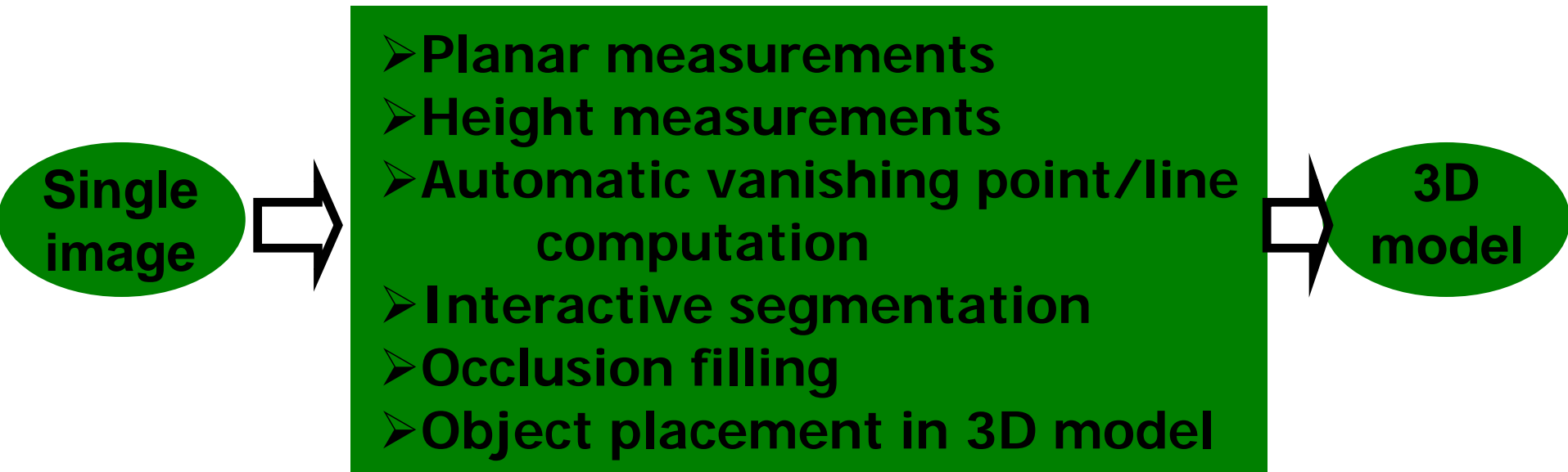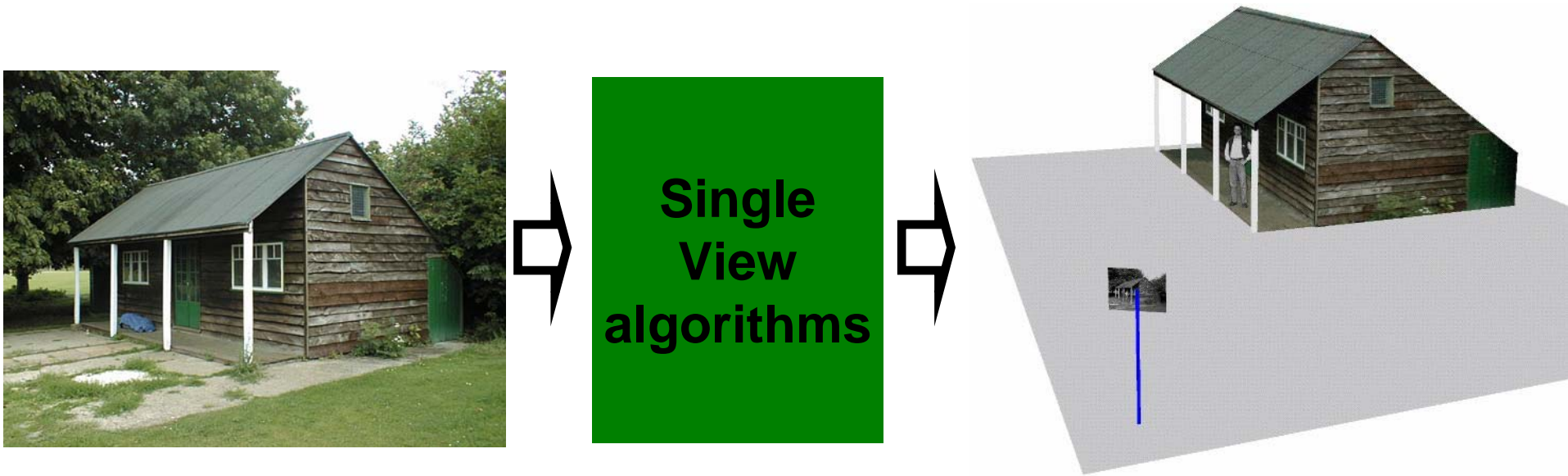
Occlusion filling
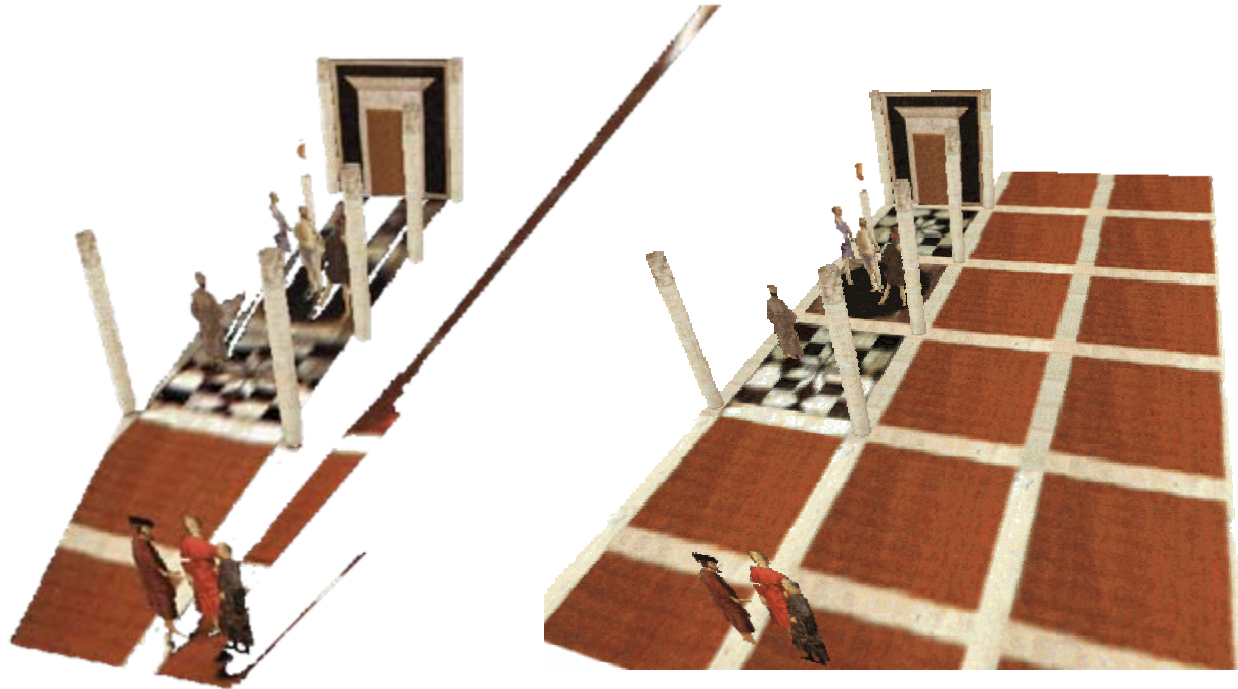
Geometric filling by exploiting:

- symmetries

- repeated regular patterns

Texture synthesis

- repeated stochastic patterns

Complete 3D reconstruction

Single View algorithms

Single image ➡ **Planar measurements**
Height measurements
Automatic vanishing point/line computation
Interactive segmentation
Occlusion filling
Object placement in 3D model ➡ **3D model**

Reconstruction from single photographs

hut

**Reconstruction of the garden
Hut from a single image**

A virtual museum @ Microsoft