More Single View Geometry™

15-463: Rendering and Image Processing
Alexei Efros

…with a lot of slides stolen from Steve Seitz and Antonio Criminisi

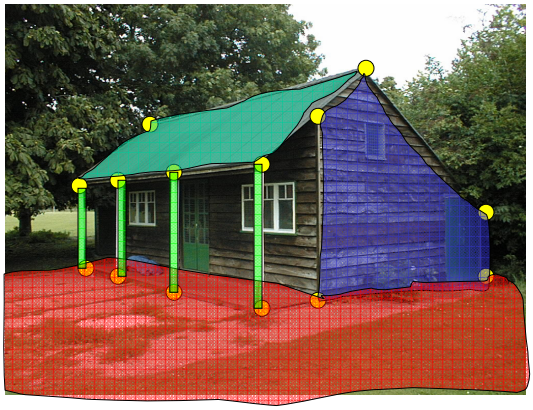
Quiz!

Image B

Image A

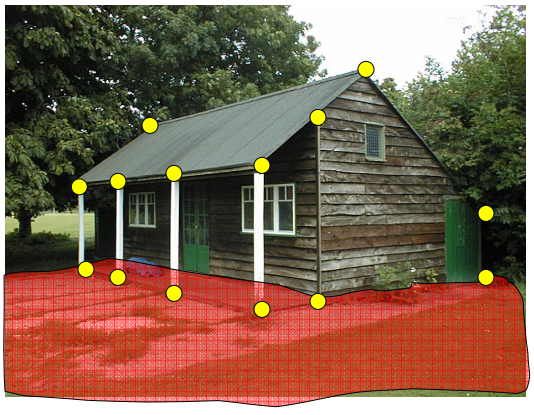
Image C

How can we model this scene?



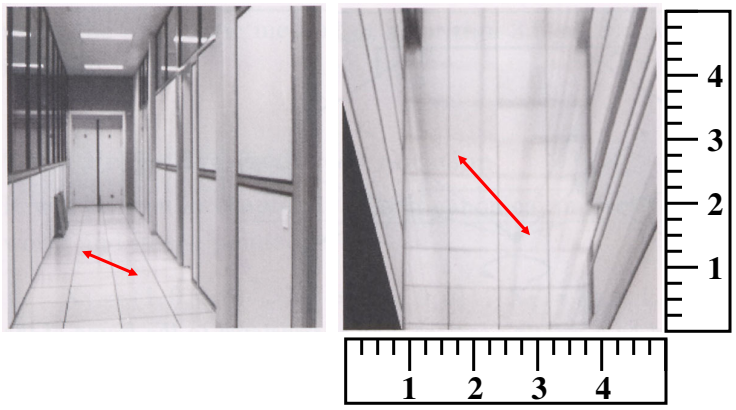
1. Find world coordinates (X,Y,Z) for a few points
2. Connect the points with planes to model geometry
   - Texture map the planes

Finding world coordinates (X,Y,Z)



1. Define the ground plane (Z=0)
2. Compute points (X,Y,0) on that plane
3. Compute the *heights* Z of all other points

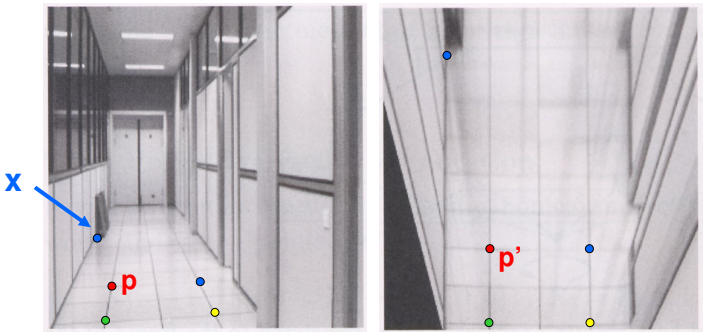
Measurements on planes



Approach: unwarp, then measure

What kind of warp is this?

Unwarp ground plane



Our old friend – the homography

Need 4 reference points with world coordinates

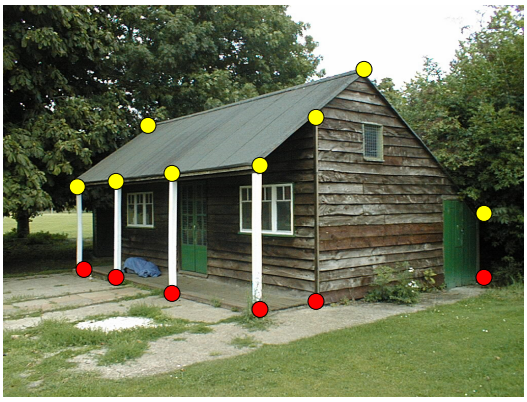
p = (x,y)

p' = (X,Y,0)

Holbein's *Ambassadors*

Can you see something weird?

Finding world coordinates (X,Y,Z)



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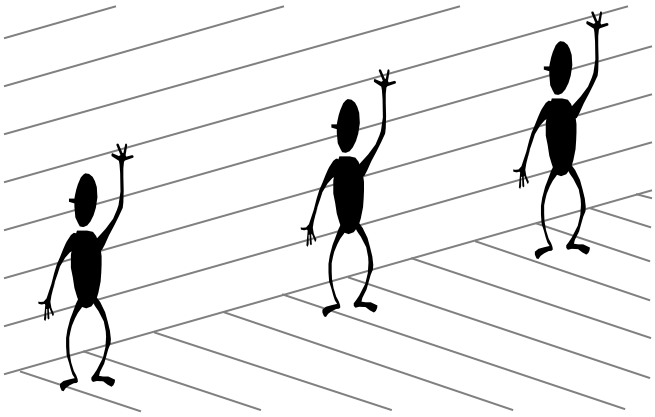
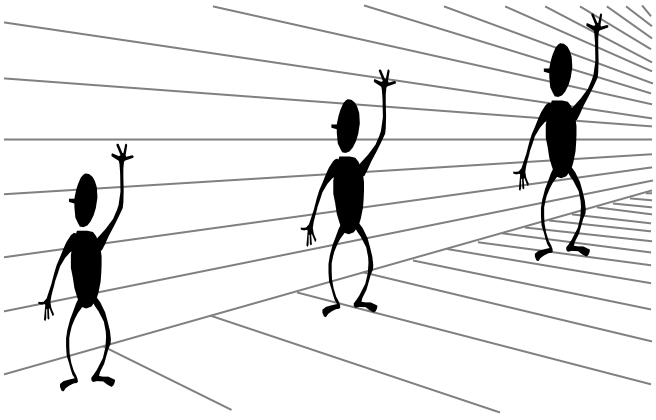
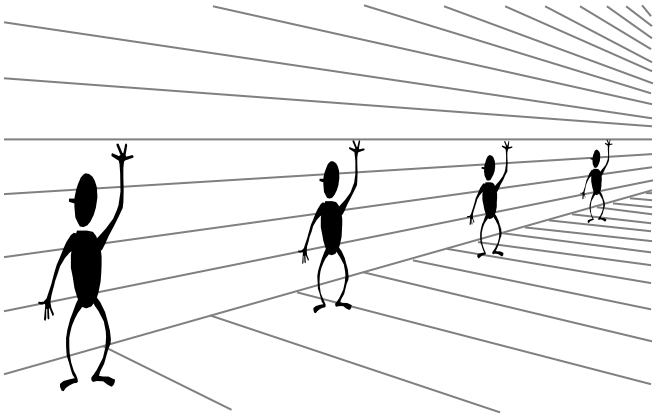
Comparing heights



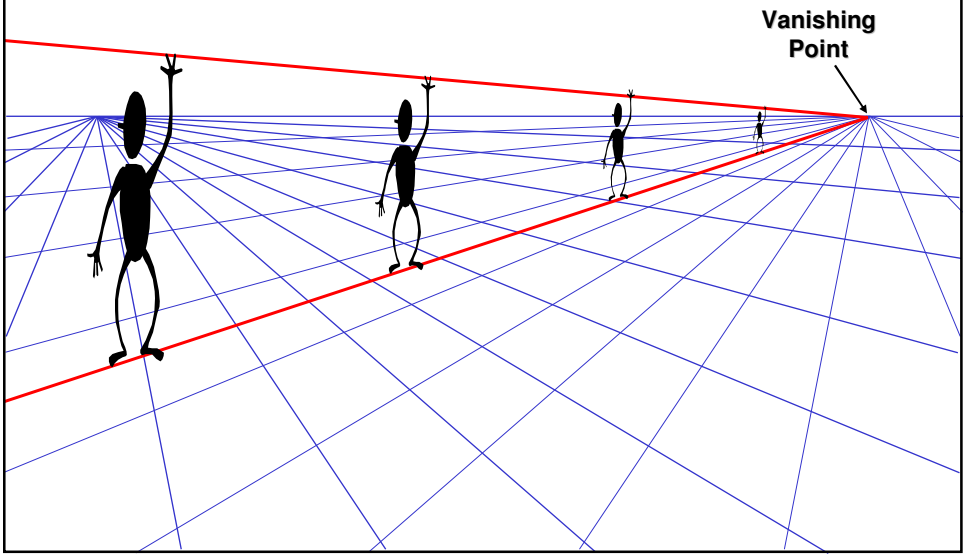
Perspective cues



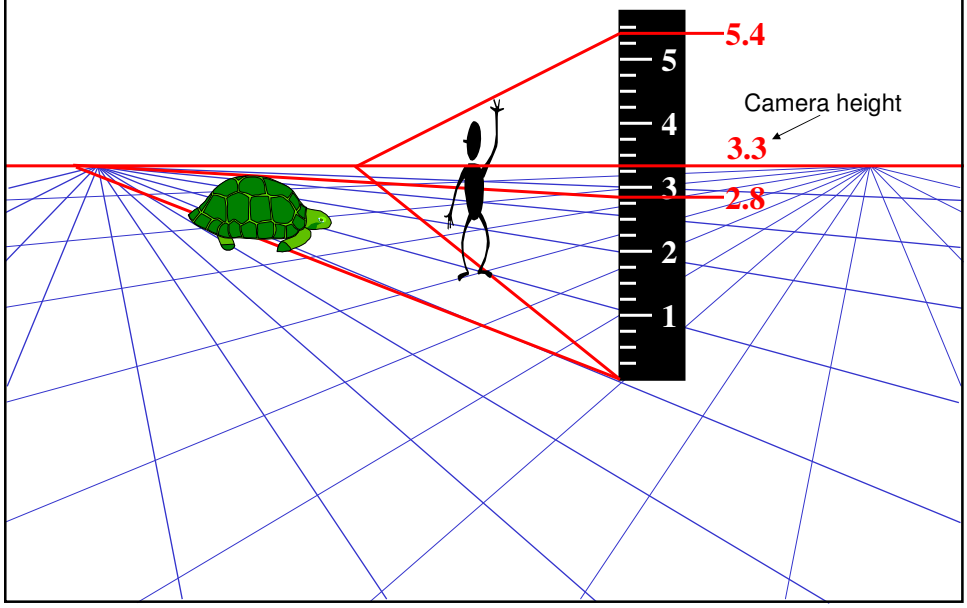
Perspective cues



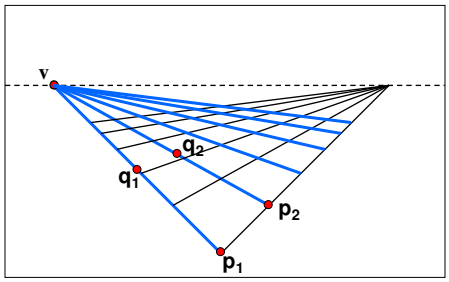
Comparing heights



Measuring height



Computing vanishing points (from lines)



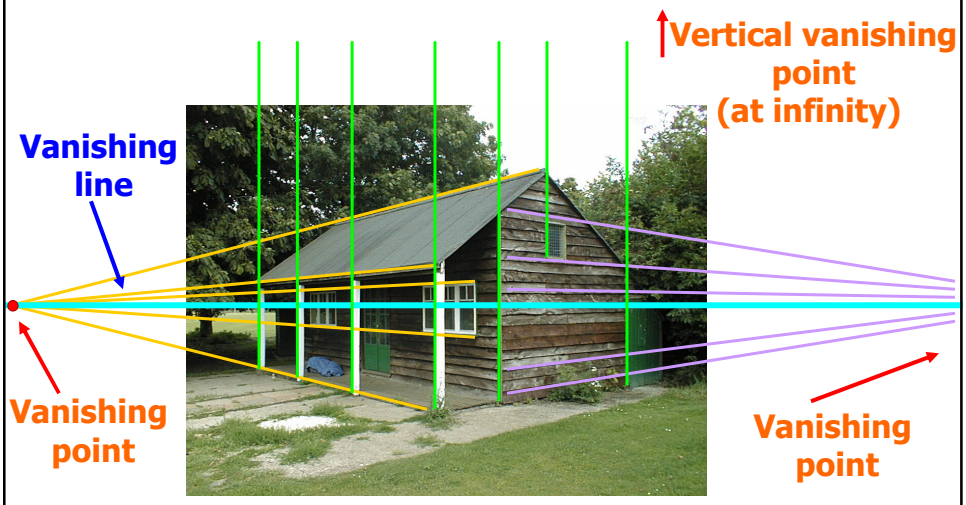
Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

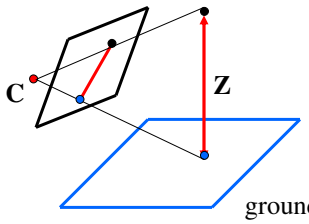
Least squares version

- Better to use more than two lines and compute the “closest” point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Criminisi ’99

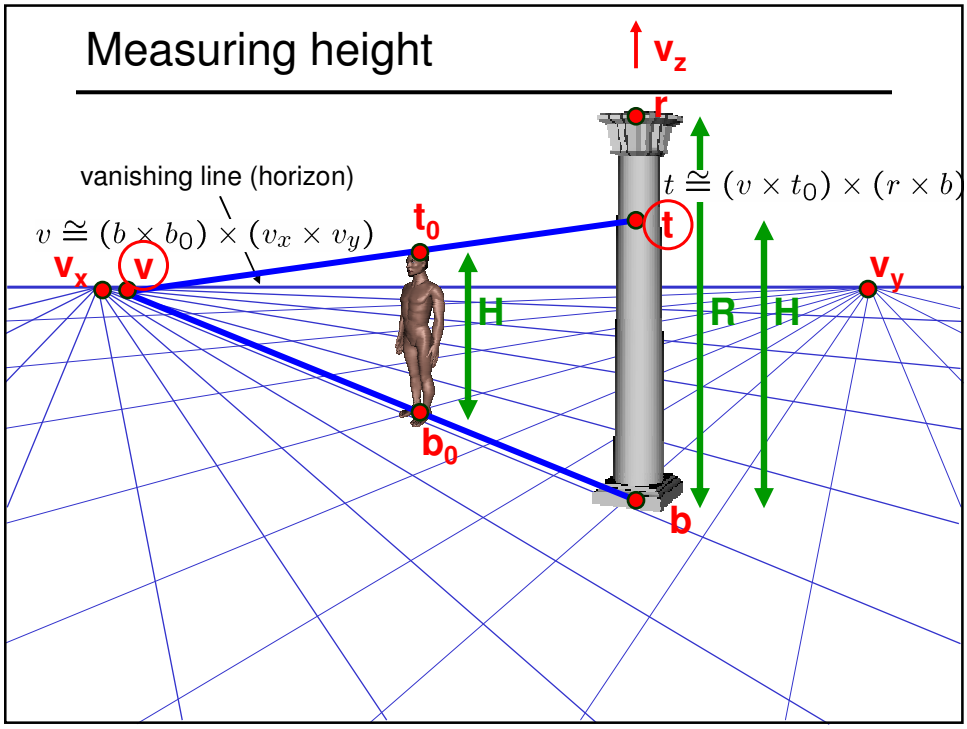


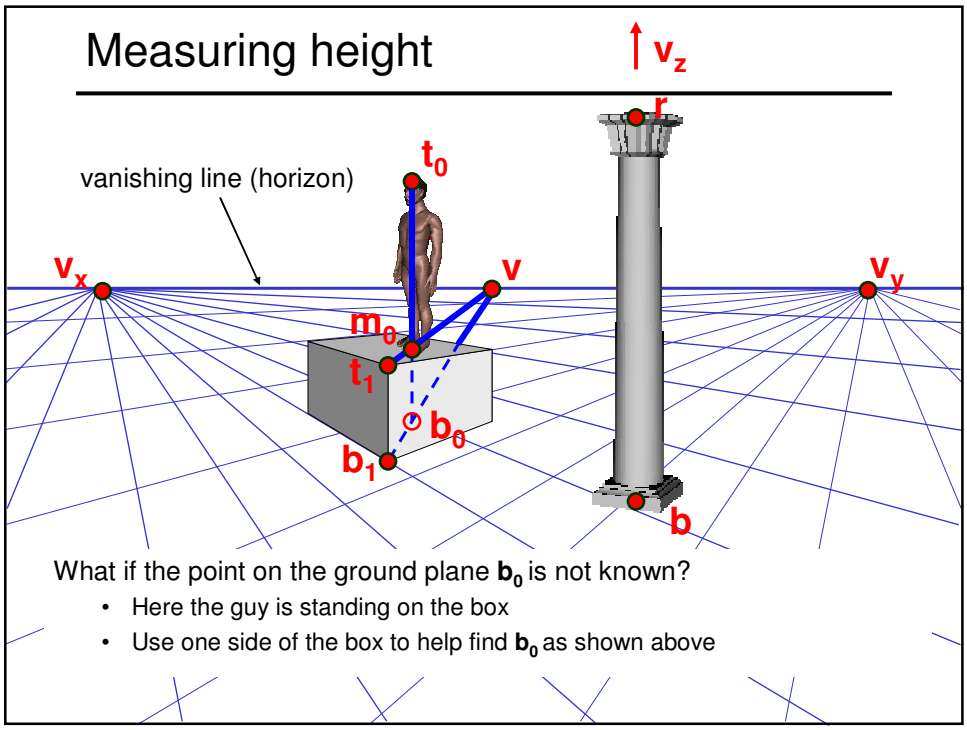
Measuring height without a ruler



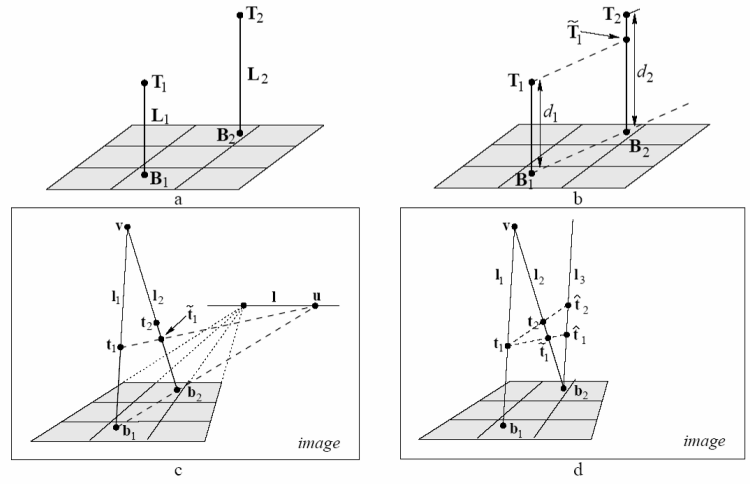
Compute Z from image measurements

- Need more than vanishing points to do this


Measuring height
v_z
r
vanishing line (horizon)
$v \cong (b \times b_0) \times (v_x \times v_y)$
$t \cong (v \times t_0) \times (r \times b)$
t_0
t
v_x
v
v_y
H
R
H
b_0
b


Measuring height
v_z
r
t_0
vanishing line (horizon)
v_x
v
v_y
m_0
t_1
b_0
b_1
b
What if the point on the ground plane $\mathbf{b_0}$ is not known?
• Here the guy is standing on the box
• Use one side of the box to help find $\mathbf{b_0}$ as shown above

What if v_z is not infinity?

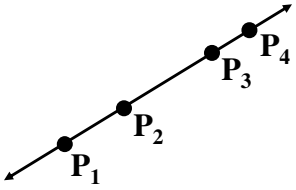


The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



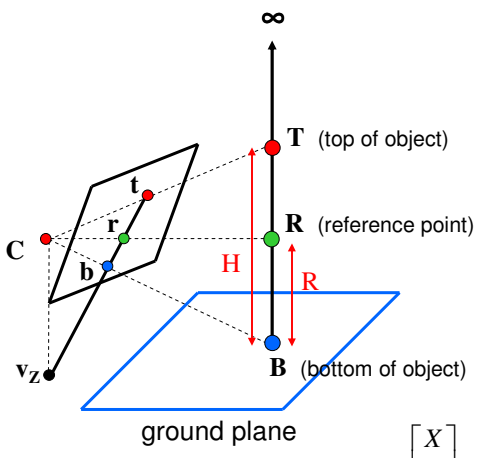
$$\frac{\|\mathbf{P}_3-\mathbf{P}_1\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_3-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_1\|} \qquad \mathbf{P}_i=\begin{bmatrix}X_i\\Y_i\\Z_i\\1\end{bmatrix}$$

Can permute the point ordering $\dfrac{\|\mathbf{P}_1-\mathbf{P}_3\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_1-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_3\|}$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

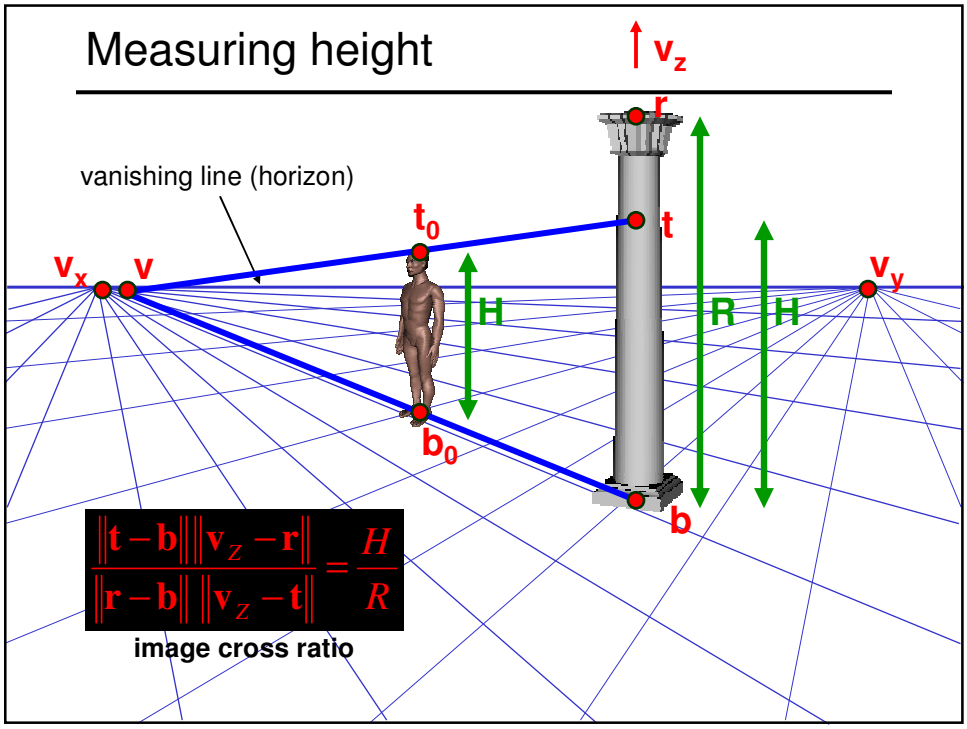
scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P}=\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix}x\\y\\1\end{bmatrix}$

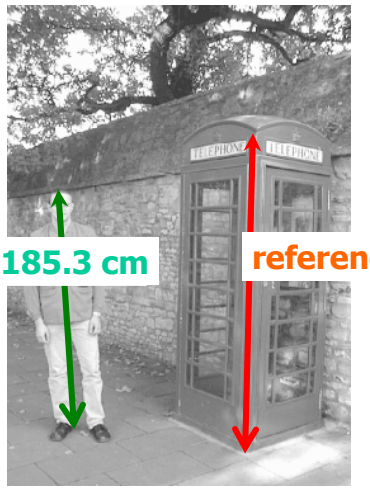
Measuring height



$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

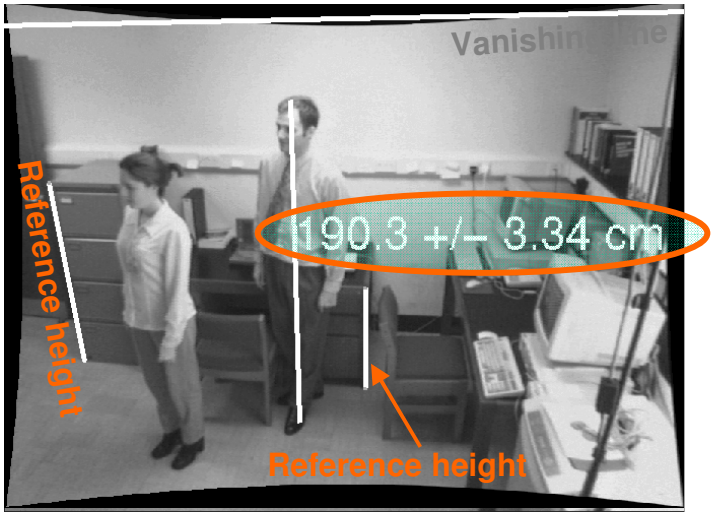
image cross ratio

Measuring heights of people



Here we go !

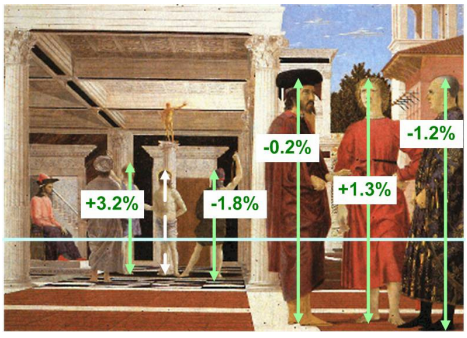
Forensic Science: measuring heights of suspects



Assessing geometric accuracy

Are the heights of the 2 groups of people consistent with each other?

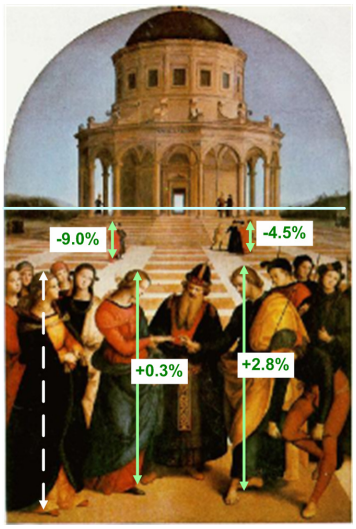
***Flagellation*,**
Piero della Francesca



Estimated relative heights

Assessing geometric accuracy

***The Marriage of the Virgin*, Raphael**



Estimated relative heights

Criminisi et al., ICCV 99

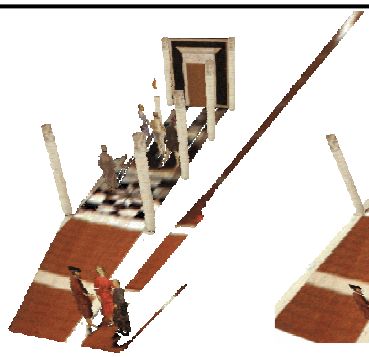
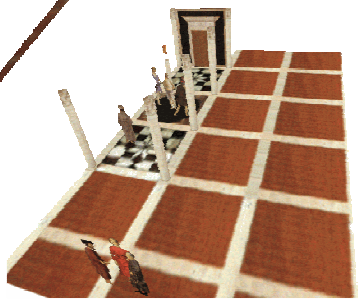
Complete approach

- Load in an image
- Click on lines parallel to X axis
  - repeat for Y, Z axes
- Compute vanishing points
- Specify 3D and 2D positions of 4 points on reference plane
- Compute homography H
- Specify a reference height
- Compute 3D positions of several points
- Create a 3D model from these points
- Extract texture maps
  - Cut out objects
  - Fill in holes
- Output a VRML model

Interactive silhouette cut-out



Occlusion filling

Geometric filling by exploiting:

- symmetries
- repeated regular patterns

Texture synthesis

- repeated stochastic patterns

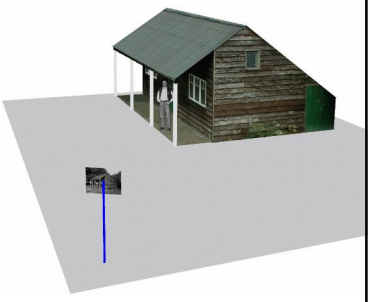
Occlusion Filling: texture synthesis.

Non parametric texture synthesis to fill in removed areas.

My son cannot walk but he can fly☺

Complete 3D reconstruction

Single View algorithms



Single image

- **Planar measurements**
- **Height measurements**
- **Automatic vanishing point/line computation**
- **Interactive segmentation**
- **Occlusion filling**
- **Object placement in 3D model**

3D model

Reconstruction from single photographs

hut

Reconstruction of the garden Hut from a single image

Virtual Museum DEMO